IMPROVEMENT IN HSPF’S LOW FLOW PREDICTIONS BY IMPLEMENTATION OF A POWER LAW GROUNDWATER STORAGE-DISCHARGE RELATIONSHIP

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ABSTRACT: We have enhanced the ability of a widely-used watershed model, Hydrologic Simulation Program – FORTRAN (HSPF), to predict low flows by reconfiguring the algorithm that simulates groundwater discharge. During dry weather periods, flow in most streams consists primarily of base flow, that is, groundwater discharged from underlying aquifers. In this study, HSPF’s groundwater storage-discharge relationship is changed from a linear to a more general nonlinear relationship which takes the form of a power law. The nonlinear algorithm is capable of simulating streamflow recession curves that have been found in some studies to better match observed dry weather hydrographs. The altered version of HSPF is implemented in the Chesapeake Bay Program’s Phase 5 Model, an HSPF-based model that simulates nutrient and sediment loads to the Chesapeake Bay, and is tested in the upper Potomac River basin, a 29,950 square kilometer drainage area that is part of the Bay watershed. The nonlinear relationship improved median Nash-Sutcliffe efficiencies for log daily flows at the model’s 45 calibration points. Mean absolute percent error on low flow days dropped in five major Potomac River tributaries by up to 12 percentage points, and in the Potomac River itself by 4 percentage points, where low flow days were defined as days when observed flows were in the lowest 5th percentile

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range. Percent bias on low flow days improved by 8 percentage points in the Potomac River, from -11 to -3 percent.

(KEY TERMS: drought, rivers/streams, watersheds, simulation, HSPF, base flow, nonlinear recession, Potomac)

INTRODUCTION

Water supply managers are in need of quantitative tools for forecasting the low streamflows that occur during prolonged periods of dry weather. Low flow forecasting models can assist both in long-term water supply planning in the face of rising demands and global climate change and also in real-time drought operations. Low flow forecasting is particularly relevant in the upper Potomac basin, the portion of the Potomac River drainage area upstream of Washington, DC, where the majority of the population relies on water supplies from surface water sources (Steiner et al., 2000). Both streamflows and water levels in the basin’s fractured rock aquifers tend to be lowest in late summer and early fall. This seasonal pattern is the result of the higher summertime evapotranspiration rates in the Mid-Atlantic region of the United States (Thornthwaite and Mather, 1955; Neff et al., 2000; Maxwell et al., 2011). An extended dry period during these months may result in a falloff of streamflows to levels that strain water supplies of some upland communities and require releases of water from storage impoundments to meet demand in the Washington, DC, metropolitan area (Hagen et al., 2005).

The study of dry weather flows has a long rich history, focused largely on analyses at the catchment scale of base flow recessions (see reviews by Hall, 1968; Tallaksen, 1995). Stream
base flow is defined as the portion of streamflow that results from the discharge of groundwater by surrounding aquifers. During extended periods of dry weather, when flow in most streams consists primarily of base flow, the gradual recession, that is, falloff, of flow over time can often be approximated by a simple characteristic curve, called the recession curve. The most widely used model of flow recession is the exponential decay function, which implies that during periods of no recharge discharge of groundwater to a stream is governed by a single constant parameter, the decay rate. As modeled by the exponential decay function, the recession rate, defined in this study as the rate of change with respect to time of the logarithm of flow, is constant and a plot of the logarithm of flow with respect to time is a straight line with negative slope.

For many applications, the exponential decay function has been found to provide a good approximation to observed recessions. But in some cases this simple function is unable to capture recession characteristics that may be important for water supply and other low flow applications, such as environmental flows studies (Buchanan et al., forthcoming). It has been observed that some streams in the upper Potomac River basin have recession curves whose recession rates are not constant, but rather vary with flow magnitude (Trainer and Watkins, 1974; 1975; Rutledge and Mesko, 1996). This has implications for Washington, DC metropolitan area water supply, for which the upper Potomac River is the primary source (Ahmed et al., 2010). During periods of drought, flows drop relatively rapidly in many basin tributaries, but are better sustained in others, in particular, in streams located in the karst-dominated Great Valley region. An alternative model for recession is a two-parameter family of curves which has a flow-dependent recession rate. This recession curve, based on a nonlinear storage-discharge relationship which is in the form of a power law, has been shown in some studies to provide a
good match to the dry weather portion of stream hydrographs (Brutsaert and Nieber, 1977; Chapman, 1999; Wittenburg, 1999; Tague and Grant, 2004). Both the exponential decay function and the two parameter recession curve have been used by hydrologists to compute estimates of catchment scale geomorphic characteristics from streamflow recession data (Rorabaugh, 1960; Bevans, 1986; Vogel and Kroll, 1992; Brutsaert and Nieber, 1977; Brutsaert and Lopez, 1998; Szilagyi et al., 1998).

Watershed models aggregate flows at the catchment scale to predict flows from larger drainage areas, and are obvious candidates for low flow forecasting tools. In the case of the Potomac River basin, a watershed model based on the Hydrologic Simulation Program – FORTRAN (HSPF) (Bicknell et al., 2001) is available as part of an open-source community model constructed to simulate nutrient and sediment loading to the Chesapeake Bay, the Chesapeake Bay Program’s Phase 5 Watershed Model (Linker et al., 2002; USEPA, 2010; Shenk et al., 2012). HSPF is a conceptual, physically-based model which can simulate at an intra-daily time step the hydrologic and water quality processes that occur in a watershed. It is a “semi-distributed” model, in that the spatial domain is divided into user-defined sub-areas in which all model inputs, processes and outputs are assumed to be uniform. The accurate simulation of high flow events is of primary importance in many applications of HSPF, including studies on the impact of urbanization on flood events (Lohani et al., 2002; Ng and Marsalek, 1989) and on channel erosion (Im et al., 2003), and on the estimation of pollutant loads from non-point sources (Bergman et al., 2002; Lin, 2011). There have been fewer applications of HSPF in cases where dry weather periods and low flow conditions are of concern (Ackerman et al., 2005; He and Hogue, 2011), and few uses to date of HSPF to assist in water supply management decisions (Berris et al., 1998; Xu et al., 2007).
We report on an effort to enhance the ability of HSPF to simulate low flows by reconfiguring the portion of the model that governs base flow recessions. We replace the groundwater discharge algorithm in HSPF, based on the exponential decay function, with a more general algorithm, based on a power law storage-discharge relationship, which includes the linear recession model as a special case. This change is implemented in the Chesapeake Bay Program’s Phase 5 model, and Phase 5 is used to compare the ability of the original and the altered versions of HSPF to simulate flows in upper Potomac River basin streams. We evaluate the performance of the altered model using a suite of metrics, including Nash-Sutcliffe efficiencies for daily, log daily, and monthly mean flow and base flow, and two metrics devised for this study to assess model performance during low flow periods: low flow mean absolute percent error and low flow percent bias. We find that the power law storage-discharge relationship improves low flow predictions with no degradation in high flow predictions.

BACKGROUND

Study Area

The upper Potomac River basin is defined in this study as the 29,950 square kilometer (km²) drainage area of the fresh water portion of the Potomac River. Discharge from the upper basin is measured at the US Geological Survey’s stream gage on the Potomac River at Little Falls near Washington, DC. Several kilometers downstream of that location, the river discharges into the tidally influenced waters of the Potomac Estuary. The upper Potomac River basin is located within the larger Chesapeake Bay watershed, and includes portions of four states: Maryland, Pennsylvania, Virginia and West Virginia (Figure 1). Major tributaries in the upper basin are the
North Branch and the South Branch of the Potomac River, the Shenandoah River, the Cacapon River, the Monocacy River, and Conococheague Creek (Figure 2). Land use in the upper basin includes approximately 59 percent forest, 32 percent agriculture, and 3 percent developed based on the 2001 U.S. National Land Cover Database (Homer et al., 2007).

The Potomac River supplies water to the over 4.3 million people in the Washington, DC metropolitan area, which has an annual average water demand of 21 cubic meters per second ($m^3/s$) (based on 2005 through 2008 data), approximately three quarters of which is withdrawn from the Potomac (Ahmed et al., 2010). The river and its tributaries are also the primary source of water supply for the upper basin (Steiner et al., 2000), which has a population of 1.3 million (based on 2010 US Census data), excluding the Washington, DC metropolitan area.

Flow in the river is highly variable and exhibits strong seasonality. With few large impoundments in the basin, the Potomac is one of the most unregulated rivers in the Eastern United States (Cummins et al, 2011). Precipitation in the upper basin is distributed fairly evenly throughout the year, averaging 990 millimeters (mm) per year (Middle Atlantic River Forecast Center precipitation departure maps at [http://www.erh.noaa.gov/marfc/Precipitation/Departures/](http://www.erh.noaa.gov/marfc/Precipitation/Departures/)), and ranging from less than 890 mm per year in parts of the South Branch sub-basin to over 1270 mm per year in the headwaters of the North Branch sub-basin (National Weather Service’s Advanced Hydrologic Prediction Service, map of normal precipitation, at [http://water.weather.gov/precip/](http://water.weather.gov/precip/)). Higher summertime evapotranspiration rates reduce both groundwater recharge and surface runoff rates, resulting in a pronounced seasonal fluctuation in flows (Trainer and Watkins, 1975; Maxwell et al., 2011). Accumulations of snow can be significant in the mountainous area of the basin and contribute to
slightly higher median flows in March and April, but snowmelt does not have a major impact on summertime flows (Cummins et al., 2011).

In the upper Potomac River basin, stream base flow is provided primarily by the discharge of the groundwater from the upper layers of the fractured bedrock and the overlying layer of unconsolidated material, which consists of soil, weathered rock, and alluvium deposited locally along stream channels. Base flow characteristics have been related to a watershed’s predominant rock type and to the thickness of the unconsolidated layer, which ranges from approximately 1 to 10 or more meters (Trainer and Watkins, 1975). The upper basin lies in four physiographical provinces: the Appalachian Plateau, the Ridge and Valley, the Blue Ridge, and the Piedmont (Fenneman, 1938), as shown in Figure 2. The mountainous Appalachian Plateau, to the northwest, is underlain by deeply dissected horizontally bedded sedimentary rock, including shale, sandstone, limestone and coal. Aquifer permeability, which is determined by the rock fractures and the thickness of the overburden, is generally low, and surface runoff is high, except in some areas with significant carbonate rock. The Valley and Ridge, which constitutes more than half of the upper Potomac basin, is underlain by folded layers of sedimentary rock which have been eroded to form northeast – southwest trending valleys and ridges (Swain et al., 2004). The hydrology of the western part of this province, which is characterized by narrow valleys and ridges underlain by fairly impermeable shales and sandstones, has generally low aquifer permeability and high runoff during storm events. The eastern part of the Valley and Ridge, or Great Valley, is a wide valley drained by the Shenandoah River to the south and Conococheague Creek to the north. Low-lying areas in Great Valley have significant amounts of carbonate rock, with large solution fractures and cavities and a relatively thick unconsolidated layer. In the Great Valley, a relatively high portion of precipitation infiltrates into the subsurface as groundwater
recharge, and stream base flow is better sustained during dry weather periods. The eastern portion of the upper Potomac basin lies in the Piedmont, an area of low relief underlain to the west by crystalline rock and to the east by a sedimentary basin of sandstone, shale, and carbonates. The mountains of the Blue Ridge, composed predominantly of crystalline rock, form the topographic divide between the Great Valley and the Piedmont.

**Base flow Recession Curves**

The most widely used model of base flow recession, and the model implemented in the standard version of HSPF, is the exponential decay function,

\[
Q = Q_0 e^{-k(t-t_0)} \tag{1a}
\]

sometimes written in the equivalent form (Barnes, 1939),

\[
Q = Q_0 \kappa^{(t-t_0)} \tag{1b}
\]

where flow, \(Q = Q(t)\), is a function of time, \(Q_0\) is flow at time, \(t = t_0\), \(k\) is the decay constant and \(\kappa\) is the “recession constant”, with the relationship between \(k\) and \(\kappa\) given by

\[
\kappa = e^{-k} \tag{2}
\]

A semilogarithmic plot of equation (1a) is a straight line with slope, \(-k\). Numerous methods have been developed to estimate the constant, \(k\), or alternatively, \(\kappa\), for a given stream from dry conditions.

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weather flow data. The inverse of $k$, which has units of time, provides a measure of a stream’s ability to sustain flows during dry weather. For example, the quantity, $K = \ln 10/k$, sometimes referred to as the recession index, is the length of time it takes, according to equation (1a), for flow to decrease by a factor of 10. Estimates of $K$ for streams in the upper Potomac River basin have ranged from 35 to 200 days (Trainer and Watkins, 1975).

An alternative model of base flow recession is the two-parameter family of curves,

$$Q = \left[Q_0 (1 - \beta)^{1 - \beta} - (1 - \beta)k^{1/\beta}(t - t_0)^{\beta/(1 - \beta)} \right]$$  \hspace{1cm} (3)

Both functional forms, equations (1a), and (3) in the special case $\beta = 2$, were obtained by Boussinesq (1877; 1904) as solutions of a nonlinear equation formulated to describe groundwater discharge to a stream channel from an unconfined aquifer bounded below by a horizontal impermeable layer, with the Dupuit assumption. Brutsaert and Nieber (1977) introduced a graphical method to estimate the parameters in (3), and found that $\beta = 2$ provided a good description of the low flow portion of hydrographs for six streams underlain by shale and limestone formations in the Finger Lakes region of New York. Other investigators have used equation (3) to analyze low flow data and have estimated a wide range of values for $\beta$ (Chapman, 1999; Wittenburg, 1999; Tague and Grant, 2004). Numerical simulations of the nonlinear 2-dimensional Boussinesq equation have demonstrated that a relaxation of Boussinesq’s original assumptions, to slightly irregularly shaped aquifers, sloping aquifers, or inhomogeneous aquifers with varying hydraulic conductivities, results in solutions described by the recession curve given by equation (3), with values of $\beta$ deviating from $\beta = 2$ (Rupp and Selker, 2006; Harman and Sivapalan, 2009; Harman et al., 2009).
The recession curve given by equation (3) provides a model of streamflow recessions with flow-dependent recession rates, as discussed in more detail in the next section. Storms are frequent in the Potomac basin, so recession rates for a given stream are estimated from empirical recession curves, commonly constructed by examining a large number of relatively short recession events that occur over a multi-year time period. Hydrogeologic studies of the upper Potomac basin (Trainer and Watkins, 1975) and of the Valley Ridge, Blue Ridge, and Piedmont provinces (Rutledge and Mesko, 1996) have found that flow-dependent recession rates are relatively common. In these studies, recession rates that decrease with decreasing flow were found to be associated with areal geologic heterogeneities, and in particular, with the presence of carbonate rock beneath a portion of a stream’s catchment, and were attributed to the discharge to a stream of two or more aquifers with very different hydrogeologic characteristics. Recession rates which increase with decreasing flow have been observed in more than half of streams in the Piedmont province, and attributed at least partially to decreases in specific yield with depth in fractured crystalline rock aquifers (Rutledge and Mesko, 1996).

Lumped storage-discharge model

An alternative and equivalent formulation of the base flow recession models discussed above is the simple lumped storage model for groundwater discharge from an aquifer into a stream (Coutagne, 1948; Brutsaert and Nieber, 1977). The storage model formulation explicitly uses the water balance equation, so it can be implemented in a straightforward fashion into semi-distributed water balance models such as HSPF.

Both recession curves, equations (1) and (3), can be derived from a pair of equations for a storage model which describes the discharge of groundwater from a simple storage reservoir.
Let the relationship between active groundwater storage, S, and discharge, Q, take the form of a power law, that is,

\[ Q = k S^\beta \]  \hspace{1cm} (4)

where k and \( \beta \) are constants. Storage and discharge also satisfy a water balance equation,

\[ \frac{dS}{dt} = -Q - E + R \]  \hspace{1cm} (5)

with R representing groundwater recharge and E representing extractions from storage, such as well withdrawals and riparian evapotranspiration. Making the assumption that recharge is absent in a period of recession and that extractions are negligible, that is, \( R = E = 0 \), the solution of the two equations describing the lumped storage model, (4) and (5), is, in the linear case, \( \beta = 1 \), the exponential decay model given by equation (1a). In the nonlinear case, \( \beta \neq 1 \), the solution is the two parameter recession curve given by equation (3). This can be seen by solving equation (4) for S and using the result to eliminate S from equation (5), which then becomes an ordinary differential equation for Q, that is,

\[ \frac{dQ}{dt} = -k^{1/\beta} \beta Q^{2-1/\beta} \]  \hspace{1cm} (6)

In the simple case, \( \beta = 1 \), equation (6) is linear, and its solution is the exponential decay function, equation (1a). In the case, \( \beta \neq 1 \), equation (6) is a nonlinear differential equation, but it can be

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linearized with the transformation, \( u = Q^{(1-\beta)/\beta} \), and solved, resulting in the nonlinear recession curve given by equation (3).

A slight rearrangement of equation (6) shows the explicit flow-dependence of the recession rate in this model, where in this study the recession rate is defined as the magnitude of the slope of a semilogarithmic plot of the recession curve, that is, the absolute value of the derivative with respect to time of the natural logarithm of flow. Dividing both sides of equation (6) by \( Q \) and taking absolute values, the following expression is obtained for the recession rate,

\[
|d(\ln Q)/dt| = |Q^1 dQ/dt|,
\]

which has units of inverse time,

\[
\left| \frac{d(\ln Q)}{dt} \right| = k^{1/\beta} \beta Q^{1-1/\beta} \tag{7}
\]

In the linear case, \( \beta = 1 \), the right-hand-side of this equation reduces to \( k \), that is, the recession rate is constant with respect to flow, \( Q \). In the nonlinear case, \( \beta \neq 1 \), equation (7) predicts that the recession rate depends on flow, increasing as flow increases (or equivalently, decreasing as flow decreases) if \( \beta > 1 \), and decreasing as flow increases if \( \beta < 1 \).

It’s long been recognized that groundwater extractions change recession curves, causing recession rates to increase (Riggs, 1963), and conversely, that minor amounts of recharge occur during a recession period will decrease recession rates. For this reason, summertime data is often excluded from recession analyses to limit the impact of riparian evapotranspiration on results. The impact of non-zero recharge and extractions on recession rates is predicted by the lumped storage model, equations (4) and (5), and has been investigated in the nonlinear case by Wittenburg and Sivapalan (1999) and by Wang and Cai (2009).
HSPF

The Chesapeake Bay Program’s Phase 5 Watershed Model, an HSPF-based modeling tool, was used in this study to test the ability of the nonlinear storage-discharge model to improve HSPF streamflow simulations in the upper Potomac River basin. The Potomac is the second largest river in the Chesapeake Bay watershed. HSPF is a computer model that simulates flows and pollutant concentrations in watershed streams and water bodies. The primary inputs of HSPF’s hydrologic component, which is the focus of this study, are meteorologic time series. These are typically at an hourly or finer time step, and include precipitation, air temperature, wind speed, solar radiation, and potential evapotranspiration. HSPF simulates the hydrologic processes that occur in a watershed, including surface runoff, evapotranspiration, infiltration, soil and aquifer zone storage, and flow routing in stream channels.

HSPF has a modular structure that allows the user to select modules for a given run based on the processes to be simulated. The modules PERLND and IMPLND simulate the water budget and water quality-related processes that occur on pervious land areas and impervious land areas, respectively. The module RCHRES simulates processes that occur in stream reaches or lakes.

The discharge of groundwater from aquifer storage to stream reaches is simulated in the PERLND module of HSPF. In the standard version of HSPF, discharge is based on the base flow recession curve given by equation (1), with an option to model variable recession rates resulting from recent groundwater recharge. HSPF associates with each land segment a single state variable representing groundwater storage and two input parameters that govern the rate of discharge from that storage. In the standard version of HSPF, the first of these parameters, AGWRC, represents the recession constant of the linear recession equation, $\kappa = e^{\kappa t}$, in equation (1), and a second parameter, KVARY, is provided to model variable recession rates resulting
from recent groundwater recharge. As discussed above, equation (1) can be derived from the linear storage model, equations (4) and (5) in the special case, $\beta = 1$, under the assumption that no aquifer recharge and no extractions from groundwater are occurring. Estimates of recharge and evapotranspiration from groundwater are also included in HSPF’s water balance computations.

**Chesapeake Bay Program Phase 5 Watershed Model**

The first version of the Chesapeake Bay Program’s Chesapeake Bay Watershed Model was developed in 1982 to estimate point and nonpoint source loads to the Chesapeake Bay from the Bay watershed (Linker et al, 2002). Numerous upgrades have been made over the past decades (Linker et al, 2002; USEPA, 2010; Shenk et al., 2012). Since 1985, the model has been based on HSPF. Phase 5, the latest generation of the watershed model, includes a powerful suite of data management, automated calibration, and output analysis tools, implemented via a set of UNIX scripts which run HSPF modules and other model components. Phase 5 uses over 20 land use categories and a model simulation period extending from 1984 to 2005. Unlike stand-alone versions of HSPF, Phase 5’s UNIX-based framework has been designed to simulate time-varying rather than static land-use inputs. This model is used, in conjunction with an airshed model and an estuarine water quality model, to assist in management decisions aimed at improving water quality and ecosystem health in the Bay. The model is currently being used to develop total maximum daily load allocations of nutrients and sediments to the Bay, and to evaluate best management practices implementation plans developed by Bay watershed jurisdictions (Shenk et al., 2012).
Phase 5 allows model calibration and scenario runs to be made on user-selected sub-watersheds. For this study, version 5.2 of the model was used to simulate streamflows in the upper Potomac River basin. Streams in the upper Potomac River basin are represented by 123 distinct river reaches and associated drainage areas, or “river segments”. In Phase 5, calibration parameters are assigned by model "land segment", delineated primarily based on county boundaries in order to make use of county-based data sets, though in some cases counties have been sub-divided based on topographic and physiographic characteristics. The Potomac River basin is represented by 53 land segments, shown in Figure 3. Hydrologic characteristics are assumed to be homogenous within each of the land segments. Areas formed by the intersection of the river segments and the land segments, referred to as “land-river segments,” are used in Phase 5 to appropriately route flows and loads from HSPF’s land process simulation to river reaches.

Phase 5, because of the size and complexity of the modeled domain, uses a multi-objective function automated calibration routine to adjust key parameters that govern the hydrologic response of pervious land surfaces. This calibration strategy is based on studies which have investigated the sensitivity of HSPF's simulated hydrology to model parameters, and found that calibration statistics can be identified which are primarily sensitive to a single calibration parameter type (Lumb et al., 1994; Doherty and Johnston, 2003). Further experimentation identified a set of statistics, each of which was primarily sensitive to changes in one of the seven types of parameters that are varied in Phase 5’s hydrology model calibration. The statistics used in the hydrology model calibration, listed in Table 1, are measures of the bias between paired simulated and observed flows and flow-related quantities, such as total flow, base flow and peak flow. The bias of a flow-related quantity is defined as the average difference between the
simulated and observed values during of the calibration period, normalized by the average observed value, (USEPA, 2010), that is,

\[
bias = \frac{\sum_{i=1}^{N} (sim_i - obs_i)}{\sum_{i=1}^{N} obs_i} \tag{8}
\]

where \(N\) is the total number of days in the calibration period and \(sim_i\) and \(obs_i\) represent the simulated and observed flow-related quantity on day \(i\).

In Phase 5's automated calibration routine, separate objective functions have been constructed from the identified statistics, and these objective functions are individually optimized during the iterative calibration procedure using corresponding parameter types. A land segment typically drains to more than one gaged stream location, and the objective functions for each land segment’s set of calibration parameters have been designed to include simulation results from all of these downstream gages, with calibration results for a given gage weighted based on the relative area of the land segment in the gaged drainage area.

Phase 5 output includes additional statistics which provide information on the model’s ability to simulate observed conditions. Output statistics which are not used in model calibration and which provide an independent verification of model performance include Nash-Sutcliffe efficiency coefficients (Nash and Sutcliffe, 1970). Phase 5 computes separate Nash-Sutcliffe efficiencies for daily flows, log of daily flows, and monthly mean flows.

The option to model variable recession rates was not used in original version of Phase 5, that is, \(KVARY\) was set equal to zero for all model land segments, because past efforts to improve
model performance by considering non-zero values of KVARY had not proved successful (personal communication, Gary Shenk, Chesapeake Bay Program Office).

METHODS

The flow-dependent recession model based on the power law storage-discharge relationship, equation (4), was implemented in the HSPF modules used by the Chesapeake Bay Program’s Phase 5 Watershed Model (version 5.2), as described below. The altered version of HSPF is capable of simulating recessions described by equation (1), which are governed by the linear storage-discharge relationship, i.e., the special case $\beta = 1$, and also recessions described by equation (3), with the nonlinear storage-discharge relationship, $\beta \neq 1$.

Implementation of the power law storage-discharge relationship

To allow simulation of flow-dependent recessions, the lumped storage model with a power law storage-discharge relationship, equations (4) and (5), was implemented in HSPF by altering the set of subroutines that simulate groundwater discharge to streams, located in the PWATER section of HSPF’s PERLND module, and by redefining the two sets of HSPF parameters which govern the rate of groundwater discharge, AGWRC and KVARY. More details on corresponding changes made to the HSPF FORTRAN code are given in the Appendix.

Because the power law storage-discharge model requires two parameters, k, and $\beta$, the original approach to model variable recession rates was removed from the code, and HSPF’s two groundwater discharge parameters were redefined as
AGWRC = k \quad (9)

and

KVARY = \beta \quad (10)

The standard version of HSPF was altered to incorporate the power law relationship between groundwater discharge and storage, equation (4), by changing code in HSPF’s PWATER section. HSPF uses a discretized version of the water balance equation, (5), to keep track of changes in groundwater storage associated with each model land segment due to inflows and outflows to and from the model’s groundwater storage reservoirs. In the standard version of PWATER, the expression for outflow from groundwater storage is given by the change in storage, \( S_i - S_{i+1} \) over the time step, \( \Delta t = t_{i+1} - t_i \), where the time step used by Phase 5 is hourly (1/24 day), and where both storage and groundwater outflow over the time step are in units of length. Since in the standard exponential decay model, \( S_{i+1} = S_i e^{-k \Delta t} \), the original PWATER code computes groundwater outflow over the time period, \( t_i \) to \( t_{i+1} \), as

\[
groundwater\ outflow_i = (1 - e^{-k \Delta t}) S_i \quad (11)
\]

where, as described above, it’s been assumed that HSPF’s native variable groundwater recession rate parameter, originally represented by KVARY, is zero. For \( k \) and \( \Delta t \) both small, as is the case in this study, \( k \Delta t << 1 \), and \( e^{-k \Delta t} \) can be approximated by its Taylor series expansion, that is, \( e^{-k \Delta t} \sim (1 - k \Delta t) \). In this approximation, the expression for groundwater outflow in the
standard version of PWATER is equivalent to the expression for outflow given by the storage-discharge relationship, equation (4), in the linear case $\beta = 1$, that is,

$$\left(1 - e^{-k \Delta t}\right) S_i \approx k S_i \Delta t$$  \hspace{1cm} (12)$$

To implement the nonlinear groundwater storage-discharge model in HSPF, the expression for groundwater outflow in the PWATER code was replaced with a new expression based directly on the power law storage-discharge relationship, equation (4), that is,

$$\text{groundwater outflow}_i = k S_i^\beta \Delta t$$  \hspace{1cm} (13)$$

The altered algorithm, (13), is capable of representing groundwater discharge in both the linear case, $\beta = 1$, and the nonlinear case, $\beta \neq 1$. Since both groundwater outflow and storage have units of length, L, in HSPF, then equation (13) implies that $k$ has units of $L^{1-\beta}$ per unit time. The parameter $\beta$ is unitless.

As discussed above, the original PWATER code involving KVARY, as used in by the standard HSPF model to simulate changes in the recession rate due to recent groundwater recharge, was not used. However, the presence of the recharge term, $R$, in the water balance equation, (5), that is implemented in HSPF should automatically account for much of the impact of recharge on recession curves, as discussed above.

Preliminary testing of Phase 5 verified that simulated streamflows from the enhanced version of the HSPF model, which uses equation (13) to compute outflow from groundwater

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storage, were, in the linear case, \( \beta = 1 \), essentially identical to results from the standard model, which is based on equation (11).

Changes to Phase 5’s automated calibration routine

The alterations made to Phase 5 in this study included creation of several new flow-related quantities and corresponding statistics to assist in the calibration of the model parameters, \( k \) and \( \beta \). The first of these is a measure of the base flow recession rate and is used to calibrate \( k \). The parameter, \( k \), by the relationship given by equation (4), determines the magnitude of groundwater discharge for a given value of storage, \( S \). In the standard HSPF model, \( k \) is stored in the input parameter, AGWRC, where AGWRC = \( e^{-k} \). The statistic originally used by Phase 5 to calibrate AGWRC was based on the ratio of stream base flow, \( Q_{BF} \), computed on two successive days, day \( i \) and day \( i+1 \), that is, \( Q_{BF_{i+1}}/Q_{BF_{i}} \), for all pairs of successive days identified by the automated calibration routine to be in base flow recession periods. Assuming that the linear recession model, equation (1), is valid for observed and simulated base flows during recession periods, each of these ratios provides an estimate of \( e^{-k} \). Stream base flow is estimated in Phase 5 using code from the USGS’s automated base flow separation software, PART (Rutledge, 1998), and base flow recession periods are defined as periods in which base flow decreases every day for three or more successive days. In the altered version of HSPF used in this study, the definition of AGWRC has been changed to AGWRC = \( k \). The quantity used to compute Phase 5’s original calibration statistic for AGWRC, \( Q_{BF_{i+1}}/Q_{BF_{i}} \), becomes a highly nonlinear function of \( k \) in this case, since base flow during a period of recession is now given by (3). Therefore, a new statistic was devised from an alternative quantity that has a simpler relationship to \( k \), the average of the recession rate, \( |d(\ln Q_{BF})/dt| \), given by equation (7), and estimated using a difference
approximation to the derivative as \( \frac{Q_{BF,i+1} - Q_{BF,i}}{Q_{BF,i}} \). As defined here, the recession rate has units of \( \text{days}^{-1} \). In the linear case, \( \beta = 1 \), the recession rate is simply \( k \). As indicated in Table 1, the redefined AGWRC is calibrated using the ratio of simulated to observed average base flow recession rates.

A second new statistic was added to Phase 5’s automated calibration routine for calibration of the parameter, \( \beta \). Preliminary model runs were conducted to test the sensitivity of simulated flows to the new parameter, \( \beta \), and to investigate the dependence of recession rates, as defined by equation (7), on flow. It was verified that recession rates were flow dependent in Potomac basin streams, increasing in most cases with increasing flow, and that the altered version of HSPF with \( \beta \neq 1 \), could capture this behavior but that the linear case, \( \beta = 1 \), could not. Thus, a new calibration statistic was constructed based on a flow-related quantity which measures the difference between base flow recession rates during average flow periods and during low flow periods. The average flow base flow recession rate was defined as in the preceding paragraph. The low flow base flow recession rate was defined similarly, but with recession rates contributing to the average only in cases where the base flow, \( Q_{BF,i} \), is less than the average base flow, \( \bar{Q}_{BF} \). New code was written into Phase 5’s automated calibration procedure to iteratively adjust the parameter, \( \beta \), based on the magnitude of the bias between simulated and observed values of this quantity.

Additional statistics to evaluate model performance

Additional statistics were created to help evaluate model performance during low flow periods, since the values of many standard statistics are largely determined by high flow simulation errors. Statistics were defined based on two commonly used statistics, mean absolute
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percent error (MAPE) and percent bias, but computed for low flow periods only. Low flow
MAPE and low flow percent bias are defined as follows, with summations restricted to the
lowest 5\textsuperscript{th} percentile of observed flows, that is,

\[
\text{low flow MAPE} = \frac{100}{N_{95}} \sum_{Q_{obs,i} < Q_{95obs}} \frac{\text{abs}(Q_{sim,i} - Q_{obs,i})}{Q_{obs,i}}
\]

\[
\text{low flow percent bias} = 100 \times \frac{\sum_{Q_{obs,i} < Q_{95obs}} (Q_{sim,i} - Q_{obs,i})}{\sum_{Q_{obs,i} < Q_{95obs}} Q_{obs,i}}
\]

where \(Q_{sim,i}\) and \(Q_{obs,i}\) are simulated and observed daily flows on day \(i\), \(Q_{95obs}\) is the value of
observed daily flow which is exceeded 95 percent of the time over the model calibration period,
and \(N_{95}\) is the number of days in which \(Q_{obs,i} < Q_{95obs}\).

Because the aim of this study was to improve low flow simulations by enhancing HSPF’s
ability to simulate stream base flow, Nash-Sutcliffe efficiencies were also calculated for daily
base flow, log of daily base flow, and monthly mean base flow. Base flows were computed
using the USGS base flow separation program, PART, for consistency with other computations
done by Phase 5’s automated calibration routine.

RESULTS

Past studies have evaluated the ability of the power law groundwater storage-discharge
relationship to simulate recession flows in individual streams draining single catchment areas,
with determination of the values of the parameters k and beta accomplished using simple 
optimization approaches. Calibration of a model simulating flows in multiple streams in an area 
the size of the upper Potomac River basin is more challenging, and results are bound to depend, 
in part, on the calibration method selected. This study has relied on an altered version of the 
Phase 5 model’s multi-objective function automated calibration routine, described above, to 
estimate appropriate values of k and $\beta$ for each of the 53 Phase 5 land segments of the upper 
Potomac basin.

The altered version of HSPF described above was calibrated using Phase 5’s automated 
routine in both the linear case, $\beta = 1$, which leads to the standard exponential decay model of 
recessions, and in the more general case which allows for recessions governed by the nonlinear 
storage-discharge relationship, $\beta \neq 1$. To evaluate the ability of the enhanced version of HSPF to 
simulate stream flows, a suite of statistical metrics were computed. These include key metrics 
used by the Chesapeake Bay Program to assess new calibrations, the two metrics described 
above which measure model error for the lowest 5th percentile of observed flows, and additional 
metrics for base flow computed using base flow separation methods. Some of the Bay Programs 
metrics, such as peak flow and peak flow volume, assess the model’s ability to simulate high 
flows, and are not expected to be sensitive to changes in the simulation of groundwater 
discharge. Others, such as the Nash-Sutcliffe efficiency for daily and monthly mean flows, are 
also to a large degree influenced by high flow events. However, values of these metrics are 
included in the discussion below because changes in the simulation of base flow may affect the 
calibration of parameters governing other processes, so it is important to verify that the high flow 
performance of the model was not degraded by the presence of the altered groundwater discharge 
algorithm.
Calibrated parameters

Phase 5’s automated calibration routine was run to adjust values of the key hydrology input parameters listed in Table 1, including the redefined parameters which govern groundwater discharge to streams, $\text{AGWRC} = k$ and $\text{KVARY} = \beta$. Phase 5’s simulation period is 1984 through 2005 with 1984 serving as a “spin up” period and 1985 through 2005 used as the calibration period. A new calibration run was also done for the linear case, $\beta = 1$, for comparison purposes, since the definition of AGWRC had been altered. Because of uncertainty concerning the effectiveness of the calibration strategy devised for the nonlinear model, the parameter, $\beta$, the exponent in the groundwater storage-discharge relationship, equation (4), was constrained to lie within the range, 0.85 to 3.00. The parameter, $k$, was limited to the range, 0.0001 to 0.1000, to maintain consistency with values of $k$ determined in the linear case, $\beta = 1$.

Calibrated HSPF parameter values for the altered model in linear ($\beta = 1$) and nonlinear cases for forested landuse are given in Table 2. Parameter values for other landuses are related to forest landuse values by fixed ratios, and the value of the parameter representing the upper zone soil moisture index, UZSN, is set at a fixed ratio to LZSN, with the ratio depending on month (see USEPA, 2010). Other HSPF parameters are set to fixed values in Phase 5 based on GIS analyses or literature recommendations. After model calibration, values of $\beta$ ranged from 0.85 to 3.00, with a median of 2.18. This indicates, according to equation (7), that recession rates in most upper Potomac basin streams tend to decrease with decreasing flow. Calibrated values of $k$ ranged from 0.009 to 0.100, with both a median and mean of 0.051 (in units of inches$^{(1-\beta)}$ per day; conversion from inches, the units used in the actual calibration, to millimeters depends on the calibrated value of $\beta$, which varies by land segment).
Figure 4 is a map of Phase 5’s upper Potomac River basin land segments, showing the spatial pattern of the values of $\beta$ produced by the automated calibration routine. The upper Potomac basin has 28 land segments with 90% or more area lying in the Valley and Ridge, and in these segments, the median and mean values of $\beta$ are 2.41 and 2.23, respectively, indicating that simulated recession rates in Valley and Ridge streams tend to decrease with decreasing flow. Conversely, there are six land segments in the upper basin with 90% or more of their area lying in the Piedmont province. The median and mean values of $\beta$ in these segments are 1.05 and 1.15, respectively, indicating that, on average, simulated recessions rates in Piedmont streams only decrease slightly with decreasing flows. These results are reasonably consistent with past observations of Rutledge and Mesko (1996), who found that all Valley and Ridge streams in their study had recession curves with recession rates that decrease with decreasing flow, while more than half of the recession curves for Piedmont streams had recession rates that increase with decreasing flow. Similarly, Trainer and Watkins (1975) found that upper Potomac basin streams in the Valley and Ridge tend to have recession curves with slopes which decrease with decreasing flow, often consisting of a first, steep segment and a later, distinct, less steep segment.

Streamflow simulations

Two sets of stream flow simulation results from the altered version of Phase 5 are compared: the linear case with the simple linear storage-discharge relationship ($\beta = 1$), which leads to the standard exponential decay model of recessions, and the nonlinear case, incorporating the general power law groundwater storage-discharge relationship given by equation (4). Model performance is evaluated using a variety of flow statistics computed at the 45 observation points shown in Figure 3, with drainage areas ranging from 126 to 29,950 square kilometers. These
observation points are locations of USGS stream gage stations with daily flow data available in the calibration period, 1985 to 2005. (Results from one 28 square kilometer drainage area, which were very poor for both versions of the model, were discarded.) In addition, model performance for the six major upper Potomac River tributaries shown in Figure 2 and for the upper Potomac River itself, is evaluated using Nash-Sutcliffe efficiencies for flow and base flow and also using the low flow metrics defined in equations (14) and (15): low flow MAPE and low flow percent bias.

A summary of key flow statistics used by the Chesapeake Bay Program to evaluate calibration results is given in Table 3. These were computed using observed and simulated daily stream flows at the 45 model observation points. Many of these statistics would be expected to be primarily determined by high flow events, and results in Table 3 show that these are very similar in the linear and nonlinear cases, indicating no degradation of the model’s high flow performance from implementation of the power law storage-discharge relationship. The Nash-Sutcliffe efficiency coefficients for daily flows range from 0.34 to 0.85 for the linear version of the model and 0.35 to 0.84 for the nonlinear version, with a median of 0.66 in the linear case and 0.67 in the nonlinear case. Efficiencies of monthly mean flows range from 0.71 to 0.94 in the linear case and 0.70 to 0.94 in the nonlinear case, with medians of 0.84 for both versions of the model. Values for total bias, base flow bias, peak flow bias, and peak flow volume bias changed only slightly in the nonlinear case, and almost all changes that did occur showed improvement. There were significant improvements in efficiencies for the natural logarithm of daily flow, which would be expected to be more sensitive to changes in low flow results. These range from 0.37 to 0.87 with a median value of 0.72 for the linear model, and range from 0.42 to 0.88 with a median of 0.75 for the nonlinear model.
Because of the somewhat coarse spatial discretization of the Phase 5 land areas relative to the scale of the upper Potomac basin, the performance of the power law storage-discharge model can be more appropriately assessed using results for larger drainage areas. A review of statistics for larger streams also allows a comparison of model performance by hydrogeomorphic region. Table 4 presents low flow, flow, and base flow statistics for the six largest tributaries of the upper Potomac River, the Cacapon River, Conococheague Creek, the Monocacy River, the North Branch of the Potomac River, the South Branch of the Potomac River, and the Shenandoah River (see Figure 2), and also for the upper Potomac River itself, at Point of Rocks, Maryland. The low flow MAPE is a measure of the model’s absolute error under low flow conditions. The low flow percent bias is defined as the model’s mean error, expressed as a percent of the mean observed flow, under low flow conditions. In both cases low flow conditions are defined as days in which observed flow is in the lowest 5th flow percentile. Graphs are also provided to compare model simulations in the six large tributaries. Figure 5 shows observed and simulated daily flows over a multi-year period of drought in the Potomac basin that extended from 1998 to 2002.

Results in Table 4 for the linear and nonlinear versions of the model show that the power law groundwater storage discharge relationship reduced low flow MAPE in five out of six of the major upper Potomac River tributaries by 1 to 12 percentage points, and left it unchanged in one tributary. Low flow MAPE dropped in the Potomac River itself by 4 percentage points. Low flow percent bias improved in four out of the six tributaries, by 8 to 18 percentage points, and improved in the Potomac River by 8 percentage points, changing from -11% in the linear case to -3% in the nonlinear case. The four tributaries where low flow results improved were located in the Valley and Ridge and Appalachian Plateau physiographic provinces, where the standard version of Phase 5 tends to under-predict low flows, as is evident in the graphs in Figure 5. Low
flow percent bias did not significantly change in one of the six tributaries, the North Branch Potomac River, which is highly regulated by reservoirs, and increased by 6 percentage points in another tributary, the Monocacy River, a stream located primarily in the Piedmont province, where low flows tend to be over-simulated by Phase 5. The results in Table 4, similar to those in Table 3, show little change in values of efficiencies of daily and monthly flows, confirming that no degradation has occurred in the simulation of high flow events. Efficiencies of log daily flows, which would be expected to be more sensitive to changes in low flow errors, improve for the four Valley and Ridge and Appalachian Plateau tributaries and are unchanged for the North Branch Potomac and Monocacy tributaries and for the Potomac River.

Since the implementation of the power law groundwater storage-discharge relationship alters the model’s simulation of base flow, Table 4 also provides Nash- Sutcliffe efficiencies for daily, monthly mean, and log daily base flows, and Figure 6 shows graphs of base flow duration curves (Kunkle, 1962) for base flows in the low to medium percentile ranges (30 to 100 percent exceedance probabilities). The base flow duration curves indicate that the nonlinear version of the model significantly improves simulation of low and medium-ranged base flows in the same four tributaries where improvements in low flow statistics occurred: Cacapon River, Conococheague Creek, Shenandoah River at Millville, and South Branch of the Potomac River. Little change in base flows are evident in the duration curve for the North Branch of the Potomac River, where it should again be noted that flows are highly regulated by the Jennings Randolph and Savage dams. There is arguably a minor improvement visible in the duration curve for the Monocacy River, except at the very far right-hand end of the curve (lowest flows). Values of efficiencies for daily and mean monthly base flows are disappointing, decreasing in some cases in the nonlinear case. Because Figure 6 indicates improvements in low and medium-ranged base
flows, these efficiency values are apparently being largely determined by errors in high-ranged base flow values, which may be poorly estimated by base flow separation routines. This is supported by results for efficiencies of log daily base flows, which did improve significantly. As discussed above, base flows were estimated using the USGS’s hydrograph separation program, PART, for consistency with Phase 5’s automated calibration routine. To investigate whether base flow efficiency results might change with use of another base flow separation routine, observed and simulated base flows were also estimated using the Local Minimum Method of the online WHAT - Web-based Hydrographic Analysis Tool (Lim et al., 2005). However, there was no qualitative change in efficiency results computed using the base flows from the WHAT.

DISCUSSION

The objective of this study was to show that implementation of the power law groundwater storage-discharge relationship in HSPF would improve low flow simulations. In addition it was important to demonstrate that the altered groundwater discharge algorithm did not degrade model performance at higher flows. The altered version of HSPF was implemented in the Chesapeake Bay Program’s Phase 5 watershed model and was calibrated, using Phase 5’s automated multi-objective function routine, with streamflow data from the upper Potomac River basin. It was found that the power law storage-discharge relationship increased minimum and median Nash-Sutcliffe efficiencies for log daily flows at the model’s 45 calibration points, and reduced low flow errors in four out of six of the major upper Potomac River tributaries, and in the Potomac River itself. In these tributaries, improvements were seen in Nash-Sutcliffe efficiencies for log daily flows, in base flow duration curves, and in two measures devised for this study to assess
model performance in low flow periods: low flow MAPE and low flow percent bias, where low flow periods were defined as days when observed flows were in the lowest 5\textsuperscript{th} percentile range. The improvement in the model’s ability to simulate dry weather flows enhances its usefulness for water supply planning and operations purposes, and also for other purposes which require accuracy during low flow periods, such as studies of ecological flow requirements or NPDES permitting impacts. The Potomac River at Point of Rocks, Maryland, is located just upstream of the Washington, DC water supply intakes and is of considerable importance to water supply planners. A number of the large upper Potomac tributaries are relied upon for water supply by upstream municipalities.

Models can only provide an imperfect representation of complex physical systems. The Chesapeake Bay Program’s Phase 5 model, which is based on sets of meteorological, land use, channel geometry, surface water withdrawal and discharge, reservoir operations, and other input data that are the result of a large amount of time and effort by many organizations and individuals, is a powerful tool for computing nutrient and sediment loads to the Bay and an invaluable resource for hydrologic and water quality modelers in the Bay watershed. However, users must exercise caution in applications of this model at smaller spatial scales and for more restricted purposes. In the case of this study, Phase 5’s land segmentation, based largely on county boundaries, undoubtedly limits the model’s ability to simulate flows in smaller streams, and likely affects calibration results. In addition, Phase 5, like HSPF, does not simulate groundwater withdrawals, which may have a significant impact on flows in some of the basin’s smaller watersheds during dry weather periods. As discussed above, extractions from groundwater storage tend to distort base flow recession curves. Thus, unaccounted for
groundwater withdrawals would change the flow-dependence of the recession rate for the lowest flows, and might artificially reduce calibrated values of the parameter, $\beta$.

HSPF uses a relatively large number of input parameters to represent the physical characteristics of each modeled land segment, and the determination of appropriate values for these parameters, via the model calibration process, poses a challenge in the case of large and complex spatial domains. Phase 5’s automated multi-objective function calibration strategy successfully meets this challenge, providing a calibration with consistent and high quality results throughout the Chesapeake Bay watershed. In the current study, Phase 5’s automated routine was altered by the addition of a new base flow recession statistic which was a measure of the difference between recession rates at average flows and at low flows. This statistic was used to adjust the power law storage-discharge relationship exponent, $\beta$. A condition for use of Phase 5’s calibration strategy, which makes independent adjustments to each model parameter in each iteration of the calibration routine, is that the various calibrated parameters are reasonably uncorrelated. Model test runs indicated that the parameters of the groundwater discharge algorithm, $\beta$ and $k$, are somewhat correlated. None-the-less, good agreement was found between observed recession rates and recession rates simulated by the enhanced model, which both exhibit pronounced flow dependence, indicating that the new version of the calibration routine successfully calibrated the new parameter, $\beta$, as well as the parameter, $k$.

The current study shows that use of the power law groundwater storage-discharge relationship, (4), in HSPF can improve predictions of dry weather flows. Based on promising results in the upper Potomac basin, a preliminary version of the model, making use of equation (3) and restricted to the case, $\beta > 0$, has been implemented in the latest version of the Chesapeake Bay Program’s Phase 5 model, version 5.3.2. Other potential changes to existing watershed
models may provide equal or greater benefits for simulation performance during low flow periods. In particular, more detailed data on surface water withdrawals and discharges, representation of groundwater withdrawals, and a better understanding of the spatial and temporal variations of riparian evapotranspiration would improve dry weather flows predictions and enhance the usefulness of watershed models.

APPENDIX

This appendix contains the details of changes made to HSPF to implement the groundwater discharge algorithm with a power law storage-discharge relationship. Code for the groundwater discharge algorithm appears in HSPF’s PERLND module, in the PPWATR and GWATER subroutines, which are contained in the FORTRAN file, hperwat.f.

HSPF’s standard groundwater discharge algorithm

HSPF’s standard algorithm for groundwater discharge can be related to the lumped storage model, equations (4) and (5), with $\beta = 1$. The algorithm, given in the user’s manual (Bicknell et al., 2001) and used in the GWATER subroutine, is

$$\text{AGWO} = KGW \times (1.0 + KVARY \times GWVS) \times AGWS$$  \hspace{1cm} (A-1)$$

where

$\text{AGWO} = \text{active groundwater outflow (length per simulation time interval)}$

$\text{KGW} = \text{groundwater outflow recession parameter (per simulation time interval)}$
KVARY = parameter which can make active groundwater storage to outflow relation nonlinear (per length)

GWVS = index to groundwater slope (length)

AGWS = active groundwater storage at the start of the interval (length)

and where KGW is calculated as

\[ KGW = 1.0 - (AGWRC)^{DEL60/24.0} \]  \hspace{1cm} (A-2)

with

\[ AGWRC = \text{daily recession constant of groundwater flow if KVARY or GWVS = 0; i.e., the ratio of current groundwater discharge to groundwater discharge 24-hr earlier} \]

\[ DELT60 = \text{number of hours per simulation time interval} \]

In the nomenclature used in the current study, the daily recession constant, AGWRC, is represented by \( \kappa \), which appears in equation (1b), with \( \kappa \) related to the parameter \( k \) by equation (2). The quantity, \( DELT60/24.0 \), represents the simulation time interval, \( (t_{i+1} - t_i) = \Delta t \), in units of days. Active groundwater storage is represented by \( S \), which appears in equations (4) and (5). Substituting this notation into equation (A-2), KGW can be written as

\[ KGW = (1 - \kappa^{\Delta t}) \]  \hspace{1cm} (A-3)

and from equation (A-1) the expression for groundwater outflow at simulation time, \( t_i \), in terms of \( S \), \( \kappa \) and \( \Delta t \), is
where it has been assumed that the option of modifying outflow based on the index to groundwater slope has been removed from the equation.

HSPF’s standard algorithm for groundwater outflow, equation (A-1), can be derived from the linear storage model, equations (4) and (5) with $\beta = 1$, as follows. A discretized version of equation (4) provides the expression for the volume of groundwater outflow over the time interval, $(t_{i+1} - t_i) = \Delta t$, that is,

$$\text{outflow} = Q(t_i) \Delta t = S(t_i) - S(t_{i+1}) \quad (A-5)$$

Using equation (4) to re-write $S$ in terms of $Q$, and substituting for $Q$ using equation (1b), this becomes

$$\text{outflow} = \frac{Q_0 \kappa^{(t_i - t_0)}}{k} - \frac{Q_0 \kappa^{(t_{i+1} - t_0)}}{k} \quad (A-6)$$

Substituting $t_{i+1} = t_i + \Delta t$ and rearranging, this becomes

$$\text{outflow} = (1 - \kappa^{\Delta t}) \frac{Q_0 \kappa^{(t_i - t_0)}}{k} \quad (A-7)$$
Finally, using equation (1b) and the linear relationship between Q and S, this reduces to

\[
\text{outflow} = (1 - \kappa^{\Delta t}) \frac{Q(t_i)}{k}
\]  

(A-8)

which, since \(S = Q/k\) by equation (4), is identical to equation (A-4).

**Implementation of the nonlinear storage-discharge algorithm**

HSPF’s standard groundwater discharge algorithm was replaced with an algorithm based on the storage-discharge relationship given by equation (4). To do this, HSPF’s input parameters, AGWRC and KVARY, were redefined to store the parameters of equation (4), \(k\) and \(\beta\), respectively. In addition, changes were made to equations (A-1) and (A-2), which appear in the code of the PPWATR and GWATER subroutines. The intermediary variable, KGW, is redefined as \(KGW = k \Delta t\), and groundwater outflow, AGWO, is computed directly from equation (4), that is, \(AGWO = k S^\beta\).

These changes were implemented by the following modifications to the original HSPF code, given by equations (A-1) and (A-2):

\[
AGWO = KGW \times AGWS^\text{KVARY}
\]  

(A-9)

where

- \(AGWO\) = active groundwater outflow (length per simulation time interval)
- \(KGW = k \times (DELT60/24.0)\), where \(k\) is a parameter in equation (4) (length\(^{(1-\beta)}\) per unit time)
KVARY = β, a parameter appearing in equation (4) (unitless)

AGWS = active groundwater storage at the start of the interval (length)

and where KGW is calculated as

\[ KGW = AGWRC \times \left( \frac{DELT60}{24.0} \right) \] (A-10)

where

AGWRC = the parameter, k, appearing in equation (4) (length^{1-β} per simulation time interval)

DELT60= number of hours per simulation time interval

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TABLE 1. HSPF input parameters for Phase 5’s hydrology calibration, and statistics used in model calibration.

<table>
<thead>
<tr>
<th>HSPF Parameter Type</th>
<th>Description</th>
<th>Corresponding Sensitive Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original Phase 5 HSPF parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LAND_EVAP</td>
<td>Potential evapotranspiration adjustment (similar to pan evaporation coefficient)</td>
<td>Bias in simulated versus observed flows</td>
</tr>
<tr>
<td>INFILT</td>
<td>Infiltration rate</td>
<td>Bias in simulated versus observed baseflows</td>
</tr>
<tr>
<td>LZSN</td>
<td>Lower zone soil moisture storage index</td>
<td>Bias in simulated versus observed summer flows versus bias in simulated versus observed winter flows</td>
</tr>
<tr>
<td>INTFW</td>
<td>Ratio of interflow to surface runoff</td>
<td>Bias in simulated versus observed values of paired peak flows, and bias in simulated versus observed values of paired peak volumes</td>
</tr>
<tr>
<td>IRC</td>
<td>Interflow recession constant</td>
<td>Ratio of simulated to observed quickflow recession constants</td>
</tr>
<tr>
<td>AGWETP</td>
<td>Evapotranspiration from groundwater storage</td>
<td>Bias in simulated versus observed summer flows</td>
</tr>
<tr>
<td>AGWRC</td>
<td>Baseflow recession constant, $\kappa = e^k$</td>
<td>Ratio of simulated to observed baseflow recession constants</td>
</tr>
<tr>
<td>KVARY</td>
<td>Represents variable recession rates based on recent recharge</td>
<td>Not calibrated in earlier version of Phase 5</td>
</tr>
<tr>
<td><strong>Altered model parameters:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGWRC</td>
<td>Represents the multiplicative factor, $k$, in the power law groundwater storage-discharge relationship, equation (4)</td>
<td>Ratio of simulated to observed average baseflow recession rates</td>
</tr>
<tr>
<td>KVARY</td>
<td>Represents the exponent, $\beta$, in the power law groundwater storage-discharge relationship, equation (4)</td>
<td>Bias in difference between recession rates for average baseflows and for low baseflows</td>
</tr>
</tbody>
</table>
TABLE 2. Comparison of calibrated HSPF parameter values for 53 upper Potomac basin land segments in linear (β = 1) and nonlinear cases - for forested landuse.

<table>
<thead>
<tr>
<th>HSPF parameter</th>
<th>Allowed range</th>
<th>Min/median/max values – linear case</th>
<th>Min/median/max values – nonlinear case</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>INFILT</td>
<td>0.32 – 6.35</td>
<td>0.73/1.63/6.35</td>
<td>0.74/1.65/6.35</td>
<td>mm/hr</td>
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<tr>
<td>LZSN</td>
<td>203 – 305</td>
<td>203/282/305</td>
<td>203/292/305</td>
<td>mm</td>
</tr>
<tr>
<td>INTFW</td>
<td>1.00 – 5.00</td>
<td>1.07/2.50/6.41</td>
<td>0.60/2.50/5.00</td>
<td>None</td>
</tr>
<tr>
<td>IRC</td>
<td>0.30 – 0.85</td>
<td>0.30/0.34/0.85</td>
<td>0.30/0.33/0.85</td>
<td>1/day</td>
</tr>
<tr>
<td>AGWETP</td>
<td>0.0001 – 0.3000</td>
<td>0.006/0.015/0.300</td>
<td>0.006/0.012/0.300</td>
<td>None</td>
</tr>
<tr>
<td>AGWRC = k^β</td>
<td>0.0001 – 0.1000</td>
<td>0.023/0.053/0.100</td>
<td>0.011/0.054/0.100</td>
<td>inches^{1-β}/day</td>
</tr>
<tr>
<td>KVARY = β</td>
<td>0.85 – 3.00</td>
<td>1.0/1.0/1.0</td>
<td>0.85/2.18/3.00</td>
<td>None</td>
</tr>
</tbody>
</table>

\(^1\)Units are those actually used in the calibration, inches, since conversion to millimeters depends on calibrated value of β, which varies by land segment.
TABLE 3. Summary of statistics computed from simulated and observed flows at 45 observation points.

<table>
<thead>
<tr>
<th></th>
<th>minimum</th>
<th>5th percentile</th>
<th>median</th>
<th>95th percentile</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Efficiency – daily flows</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.34</td>
<td>0.40</td>
<td>0.66</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>0.35</td>
<td>0.40</td>
<td>0.67</td>
<td>0.82</td>
<td>0.84</td>
</tr>
<tr>
<td><strong>Efficiency - monthly mean flows</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.71</td>
<td>0.76</td>
<td>0.84</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>0.70</td>
<td>0.77</td>
<td>0.84</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td><strong>Efficiency - log daily flows</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.37</td>
<td>0.43</td>
<td>0.72</td>
<td>0.85</td>
<td>0.87</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>0.42</td>
<td>0.49</td>
<td>0.75</td>
<td>0.86</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Total flow bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>-0.11</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.13</td>
<td>0.20</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>-0.11</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.12</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>Baseflow bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>-0.08</td>
<td>-0.04</td>
<td>0.02</td>
<td>0.13</td>
<td>0.38</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>-0.08</td>
<td>-0.04</td>
<td>0.01</td>
<td>0.13</td>
<td>0.37</td>
</tr>
<tr>
<td><strong>Peak flow bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>-0.28</td>
<td>-0.25</td>
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<td>0.21</td>
<td>0.27</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>-0.27</td>
<td>-0.25</td>
<td>-0.02</td>
<td>0.19</td>
<td>0.24</td>
</tr>
<tr>
<td><strong>Peak flow volume bias</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
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<td>-0.26</td>
<td>-0.03</td>
<td>0.18</td>
<td>0.43</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>-0.28</td>
<td>-0.25</td>
<td>-0.02</td>
<td>0.19</td>
<td>0.43</td>
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</table>
TABLE 4. Low flow and other statistics for the Potomac River at Point of Rocks and for major tributaries.

<table>
<thead>
<tr>
<th></th>
<th>Cacapon River near Great Cacapon WV</th>
<th>Conococheague Creek at Fairview MD</th>
<th>Monocacy River at Jug Bridge near Frederick MD</th>
<th>North Branch Potomac River near Cumberland MD</th>
<th>South Branch Potomac River near Springfield WV</th>
<th>Shenandoah River at Millville WV</th>
<th>Potomac River at Point of Rocks MD</th>
</tr>
</thead>
<tbody>
<tr>
<td>USGS station ID</td>
<td>01611500</td>
<td>01614500</td>
<td>01643000</td>
<td>01603000</td>
<td>01608500</td>
<td>01636500</td>
<td>01638500</td>
</tr>
<tr>
<td>Drainage area, km²</td>
<td>1749</td>
<td>1279</td>
<td>2117</td>
<td>2272</td>
<td>3785</td>
<td>7878</td>
<td>25003</td>
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<tr>
<td><strong>Low flow mean absolute percent error (MAPE)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>84%</td>
<td>33%</td>
<td>65%</td>
<td>24%</td>
<td>57%</td>
<td>43%</td>
<td>32%</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>78%</td>
<td>28%</td>
<td>65%</td>
<td>23%</td>
<td>45%</td>
<td>35%</td>
<td>28%</td>
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<tr>
<td><strong>Low flow percent bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>-56%</td>
<td>-15%</td>
<td>35%</td>
<td>6%</td>
<td>-42%</td>
<td>-28%</td>
<td>-11%</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>-38%</td>
<td>-7%</td>
<td>41%</td>
<td>7%</td>
<td>-32%</td>
<td>-14%</td>
<td>-3%</td>
</tr>
<tr>
<td><strong>Efficiency – daily flows</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.58</td>
<td>0.76</td>
<td>0.79</td>
<td>0.72</td>
<td>0.76</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>0.57</td>
<td>0.77</td>
<td>0.79</td>
<td>0.73</td>
<td>0.75</td>
<td>0.82</td>
<td>0.84</td>
</tr>
<tr>
<td><strong>Efficiency – monthly mean flows</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.81</td>
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<tr>
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<td><strong>Efficiency – log daily flows</strong></td>
<td></td>
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<tr>
<td>Linear</td>
<td>0.56</td>
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<td>0.86</td>
<td>0.76</td>
<td>0.73</td>
<td>0.77</td>
<td>0.87</td>
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<tr>
<td>Nonlinear</td>
<td>0.70</td>
<td>0.86</td>
<td>0.86</td>
<td>0.76</td>
<td>0.80</td>
<td>0.81</td>
<td>0.87</td>
</tr>
<tr>
<td><strong>Efficiency – daily baseflows</strong></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Linear</td>
<td>0.74</td>
<td>0.85</td>
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<td>0.74</td>
<td>0.72</td>
<td>0.85</td>
<td>0.81</td>
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<tr>
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<td>0.81</td>
<td>0.71</td>
<td>0.71</td>
<td>0.80</td>
<td>0.80</td>
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<td><strong>Efficiency – monthly baseflows</strong></td>
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<td>0.88</td>
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<td>0.91</td>
<td>0.87</td>
<td>0.80</td>
<td>0.77</td>
<td>0.87</td>
<td>0.86</td>
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<tr>
<td><strong>Efficiency – log daily baseflows</strong></td>
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<td>0.85</td>
<td>0.75</td>
<td>0.63</td>
<td>0.76</td>
<td>0.84</td>
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<tr>
<td>Nonlinear</td>
<td>0.66</td>
<td>0.86</td>
<td>0.85</td>
<td>0.74</td>
<td>0.76</td>
<td>0.80</td>
<td>0.85</td>
</tr>
<tr>
<td><strong>Total bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.03</td>
<td>0.04</td>
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<td>0.03</td>
</tr>
<tr>
<td>Nonlinear</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.03</td>
<td>0.04</td>
<td>-0.01</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Baseflow bias</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
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<td>0.09</td>
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<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td>0.08</td>
</tr>
</tbody>
</table>
FIGURE 1. Location map showing upper Potomac River basin and Chesapeake Bay watershed.

FIGURE 2. Upper Potomac River basin streams and physiographic provinces.

FIGURE 3. Phase 5 model’s representation of upper Potomac River basin river reaches and land segments, along with model calibration points.

FIGURE 4. Model calibration results for groundwater storage-discharge relationship exponent, β.

FIGURE 5. Comparisons of flow predictions, over a multi-year period of drought, from models with linear versus nonlinear groundwater storage-discharge relationship.

FIGURE 6. Comparison of baseflow duration curves for observed flows and for simulated flows from models with linear versus nonlinear groundwater storage-discharge relationships.
Location map showing upper Potomac River basin and Chesapeake Bay watershed.
Upper Potomac River basin streams and physiographic provinces.
88x88mm (300 x 300 DPI)
Phase 5 model’s representation of upper Potomac River basin river reaches and land segments, along with model calibration points.

88x88mm (300 x 300 DPI)
Model calibration results for groundwater storage-discharge relationship exponent, $\beta$. 

88x88mm (300 x 300 DPI)