Multiobjective Reservoir Operation Using Forecasts Of Water Supply and Water Use

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MULTIOBJECTIVE RESERVOIR OPERATION USING FORECASTS OF WATER SUPPLY AND WATER USE

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FOREWORD

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CHAPTER 1

DAILY MUNICIPAL WATER USE FORECASTING:
A STUDY OF THE
WASHINGTON, D.C METROPOLITAN AREA

1.1 INTRODUCTION

As severity of drought increases, so does a municipal water system's risk of not fulfilling its water obligations. Drought is generally defined as a deficiency of water over a given geographic area during a certain period of time. A more specific viewpoint supported by water management is that of drought as a function of both water supply and demand. When incoming supply is greater than outgoing demand, barring flooding, a favorable surplus of water will exist. When the opposite is true, a deficit will result that can strain the reliability of a system.

A better understanding of the stochastic nature of both water supply and demand can greatly facilitate the efficient management of water resources. On the supply side, much work has been devoted to identifying the probabilistic nature of precipitation and stream flow. Results have not only been intended to predict water supply, but also for other purposes such as assessing flood frequency. Historically, the probabilistic analysis of water demand has received less attention. It, however, has attracted increasing concern with the recognition that only by simultaneously analyzing the probabilities of supply and demand can the vulnerability to system failure be accurately viewed.

This chapter deals with daily municipal water use forecasting based on probabilistic analysis of past experience. The benefit of using a relatively short time resolution (days) is twofold.

1) Critical Situations. When a water system is in danger of delivery failure or of loss of pressure, management's analysis of potential options can be enhanced by short-term daily water use forecasts. Forewarning is useful in decisions to enact water use restrictions, such as temporary peak water use charges (Fairfax County, Griffith [1982]), percentage curtailment from historical use (Los Angeles, Emergency Water Conservation Plan [1977]), and lawn sprinkling restrictions (Fort Worth, Anderson et al. [1981]), or even when to trigger public relations campaigns on conservation (California, Berk et al. [1981]). Forecasts can be used to decide when to utilize marginal supplies and when to obtain outside sources. Daily water use models might even assist in a water system's scaling and sequencing of infrastructure capacity to more effectively handle potentially critical situations. A water system, in a manner similar to the electric industry, sizes its operations to meet peak use. Facilities designed on the basis of only the maximum day use in a given year (Howe and Linaweaver [1967], Hughes [1980], Weeks and McMahon [1973) may conceal important dynamic implications. Capacity sized for a single maximum day may fail if surrounded by other nearly maximum days. The forecast pattern derived from a daily model may well convey useful information regarding frequency and duration of such critical situations.

2) Operations. More efficient allocation of existing resources can be brought about by using daily water use

forecasts. Water that is stored a distance away from where it is demanded has an inherent transit time. In the absence of significant buffers, the farther the releases deviate from actual use, the more error and potential waste in the system. Better forecasts of water use can decrease these errors.

The objective of the chapter is to establish an understanding of the basic factors driving water use in the Washington, D.C. Metropolitan Area. A modeling framework is constructed to this means. The ultimate goal in undertaking this task is to create a short-term forecasting model to help in an operational setting; short-term being approximately defined as up to 7 days. A simplified forecast model is developed in Chapter 2.

Before starting the modeling procedure, a review is made of the publications on water demand analysis. Learning from the past sets the foundation on which to build.

1.2 LITERATURE REVIEW

In a review of the literature, over 50 empirical water demand studies are identified (Figure 1.2). Citations are obtained both from a search through the Selected Water Resources Abstracts data base and by inspection of bibliographies as papers are found. Each study is examined and classified with respect to three primary categories: time resolution, explanatory variables, and model estimation.

1.2.1 Time Resolution

The studies reviewed differed in their time horizons. The dependent variable of water use included annual, seasonal, monthly, and daily time resolutions. The vast majority of the studies are either annual or monthly, and it is striking that only Anderson [1980], Maidment et al. [1984a, 1986], and Steiner [1984] constructed daily models.

1.2.2 Explanatory Variables

Water use is dependent on a vast number of individual decisions embedded in a complex underlying process, where both physical and psychological time-dependent variables are involved. To better understand variations in water use, a functional relationship between water use and its more significant and quantifiable explanatory variables needs to be identified. The literature suggests both weather and socioeconomic forces.

I. Weather. Air temperature, precipitation, evaporation, and soil moisture deficit have been linked to water use.

All the studies using air temperature find it to have a positive relationship with water use. As temperature increases, communities demand more water for cooling, lawn and shrub sprinkling, and other related purposes. Most studies base their observations on average daily temperature. Only Camp [1978], Maidment et al. [1984a, 1984b, 1986], and Steiner [1984] use maximum daily temperature.

Precipitation has a negative or inverse relationship with water use. The more it rains, the less water is demanded. A variant of the quantity of precipitation is to treat rainfall as a binary variable (rain/no rain). This implies that there is a threshold level beyond which water use is relatively insensitive to precipitation magnitude. Danielson [1979] uses the number of dry days in a month as an explanatory variable for water use, but finds this variable to be less effective than accumulated rainfall. Steiner [1984] finds the number of antecedent dry days (rain < .1 inch) to have significantly greater explanatory power than same day precipitation in a daily model.

Evaporation has a positive relationship with water use; Grunwald et al. [1976] and Maidment and Parzen [1984a, 1984b] find evaporation to be a significant factor. Other studies attempt to capture the potential soil moisture loss due to the combination of evaporation and plant transpiration, creating a moisture deficit variable defined as evapotranspiration minus effective rainfall. Evapotranspiration is computed using the Thronthwaite method in all but the Anderson [1980] study where the Jensen-Haise equation is used. Effective rainfall is defined alternatively as actual rainfall (Agthe and Billings [1980], Morgan and Smolen [1976], a set percentage of actual rainfall (Berry and Bonem [1974], Howe and Linaweaver [1967], and as a nonlinear function of actual rainfall (Carver and Boland [1980]).

Some studies attempt to address the dynamic forces connecting water use with weather. Anderson et al. [1980] distribute the impact of precipitation over a period of several days in their soil moisture deficit variable instead of relating it to a single day's use. They find an impressive improvement. Maidment et al. [1985, 1986] conclude that the occurrence of rainfall causes an initial step change reduction in water use that diminishes over time. They also find a dynamic response to temperature, but to a lesser degree. Hansen and Narayanan [1981] lag both precipitation and temperature by one period in a monthly model and find both to be insignificant. They suggest that a monthly time frame is too to broad to capture dynamic responses.

For the studies with intra-year analysis, seasonal effects are commonly identified. The winter is often associated with base use, comprised of domestic or indoor use that is relatively insensitive to weather. Summer is characterized as base use added to weather sensitive outdoor, or sprinkling demand. Some studies deal with this cycle by analyzing the seasons separately (Ben-Zvi [1980], Carver and Boland [1980], Howe and Linaweaver [1967], Morgan and Smolen [1976], while others use a single model with binary variables to distinguish different sub-periods (Gibbs [1978], Morgan [1974], Young et al. [1983]. Hansen and Narayanan [1981] used the percentage of daylight hours (logarithmic transformation) to account for seasonal variations.

Other meteorologic variables such as cloudiness and wind speed might also have important effects on water use. Cloudiness, although difficult to quantify, is measured in a categorical classification by the National Weather Service (e.g.

sunny, partly cloudy, cloudy). The Weather Service also commonly measures wind by the total wind movement in miles through a continuous anemometer. No attempt to use these explanatory variables is found.

II. Socioeconomic. A positive correlation between water use and socioeconomic factors such as population, employment, number of water connections, and income is indicated. One common characteristic these variables share is their relatively slow change over time; all display a relatively gradual influence on water use. Another factor is the price of water. Most of the literature regarding water demand is centered around its relationship with price; in particular, an attempt to identify price elasticity. This type of information is of particular interest because price is a policy variable, subject to change by water managers.

1.2.3 Model Estimation

All of the studies use a single equation format to relate water use to its explanatory variables. However, because of the diverseness of the type of data collected, the data's time resolution, and the possible model specifications, a variety of techniques are employed.

Models with cross-sectional or pooled cross-sectional time series data, or time series data with a relatively long time resolution are often estimated using ordinary least squares (OLS) regression. Although many functional forms are possible, linear models or models made linear by conversions such as a long-transformation (Gibbs [1978], Foster and Beattie [1979], and Billings and Agthe [1980]) are necessary. The linear requirement along with the assumptions on the error term of being normally distributed, unbiased, homoskedastic, and nonautocorrelated are quite restrictive. Nevertheless, the vast majority of the literature incorporates the OLS method.

When analyzing data collected at equally spaced intervals over time, it is common to find that observations are autocorrelated. This violates one of the basic assumptions of OLS regression leading to inefficient estimates and inconsistent standard errors. In yearly models the degree of autocorrelation is often insignificant and OLS regression is most applicable, however, the shorter the periods of individual observations, the greater the likelihood of encountering autocorrelation. Daily, weekly, and even monthly time series of water use are likely candidates for significant autocorrelation each having inherent trends and cycles that can provide useful information in modeling. When autocorrelation is present the sequence of observations can be utilized.

Statistical transformation procedures, such as those of Cochrane-Orcutt and Prais-Winsten, are available to improve the efficiency of models with a first-order autoregressive structure (Kmenta [1986]. In a monthly model, Hansen and Narayanan [1981] apply the Cochrane-Orcutt transformation. This class of estimation falls under generalized least squares.

Another way to approach time series is with regression in a dynamic model using lagged water use as an explanatory variable.

Both Steiner [1984] and Kher and Sorooshian [1986] apply first-order (one period lag) models to their data, while Agthe and Billings [1980] and Carver and Boland [1980] incorporate a Koyck transformation where lagged effects are assumed to decline according to a geometric distribution. These models better utilize the data, but one has to be wary of the residuals still meeting all of the basic assumptions of OLS and in particular that the model specified leaves a nonautocorrelated residual series.

The last general approach involves the use of autoregressive and moving average processes. An autoregressive process is similar to the dynamic model stated above in that the current observation is related to a weighted set of past observations. The moving average process relates the current observation to a weighted set of past disturbance terms. The use of either or both of these processes has the favorable property of describing a complex underlying structure with relative ease and simplicity. Salas-La Cruz and Yevjevich [1972], and Maidment et al. [1985, 1986] after preliminary detrending and deseasonalizing, fit an autoregressive process to water use, and then remove weather influences on the residuals by OLS. Steiner [1984] uses a similar approach, removing weather influences by OLS, and then fitting an autoregressive moving average process (ARMA(1,1)) to the residuals. In these last four studies, the coefficients for the parameters are not estimated simultaneously, but sequentially at each stage (cascade approach). The estimators are not guaranteed to be efficient under such circumstances, although they should be relatively accurate. A more inclusive approach involves the use of transfer function models (Box and Jenkins [1976]). Here the output series (i.e. water use) is linked by linear filters to input series (e.g. weather, white noise) simultaneously. Berk et al. [1981], and Maidment et al. [1984a, 1984b] take advantage of this flexible and encompassing approach.

1.3 DESCRIPTION OF THE WMA WATER SUPPLY SYSTEM

The area under investigation includes the three largest municipal water utilities in the Washington, D. C. Metropolitan Area (WMA). The Washington Suburban Sanitary Commission (WSSC) serves the counties of Montgomery (except Rockville) and Prince Georges in Maryland; the Fairfax County Water Authority (FCWA) serves Fairfax County and Alexandria City in Virginia, and the Washington Aqueduct Division (WAD) serves the city of Washington, D.C.. The area is shown in Figure 1.1.

Daily water production data, provided by each utility, measures the raw water pumped (MGD) into the treatment facilities. Production data can only serve as a surrogate to consumer water use as system losses and gains (losses) to short term buffers in the distribution system, such as storage tanks and small reservoirs, are included. The alternative of eliciting daily consumer use data is rarely possible as recordings are available only for customer meters read on a monthly, bimonthly, or quarterly billing cycle. Two aggregation processes occur in this analysis: the aggregation of production data over all three utilities, and within each utility the aggregation of all user classes. There is merit in delineating individual utility differences and in making distinctions between residential, commercial, industrial and public categories, however these issues will not be pursued here. Data is obtained for the thirteen year period 1974-1986.

The motivation behind this study is to develop a daily water use model to serve as input to an optimization procedure for obtaining multireservoir operating rules in the basin. Growth in the WMA without major new water supplies has placed a premium on careful utilization of existing water resources (see Chapter 5). Demand plays an important role as there is a lag time between the releases at reservoirs and the intakes from treatment plants. There are four major water supply reservoirs within a 1 day travel time (H.T. Duckett, Little Seneca, Occoquan, and Tridelphia) and two with an approximate 5 day travel time (Jennings Randolph and Savage River). If more water is released than demanded, the excesses will no longer be available for water supply as they are carried into the Chesapeake Bay. Conversely, if insufficient water is released there is a failure in meeting demand. The situation is a delicate balancing act where better management will stretch the yield of existing water supplies.

Increasing yield has two practical ramifications: one, it can extend the life of the existing capacity, and two, the proportion of a reservoir allocated for water supply can be lowered while still retaining the same delivery reliability. This is advantageous in a multiobjective context, where freed reservoir space can then either be allocated to further increase delivery reliability (supply), or for other competing objectives such as flood protection and water quality.

1.4 MODELING APPROACH

It is proposed that water use is constituted of three components:

Water Use = Trend and Cycles + Exogenous Influences
+ Stochastic Residual

A model for each of the first two components is presented in this section while the remaining component, stochastic residual, is dealt with in the subsequent chapter on forecasting. To identify long-term trend and various cycles in the data a calendar model is constructed; the word calendar being indicative of the temporal relationships sought. Exogenous influences are functionally related to water use by a causal model; the word causal pertaining to cause and effect relationships. In this report, weather measures are examined.

The parameters of the calendar and causal models are estimated twice because of interdependence. The estimates derived for the calendar model will have an influence on estimates for the causal model and vice versa. This circular dependence can be handled in a two stage approach: the first stage responsible for identification and initial estimates, and the second for re-estimation given improved starting conditions. The second stage also investigates time-varying properties of the estimates.

1.4.1 STAGE 1.

I Calendar Model

This section identifies temporal trends and patterns in the time series following a multiplicative method suggested by Maidment et al. [1984a, 1984b], but more closely resembling techniques developed in Chapter 2. Trend and cycles are comprised of four components.

Trend and Cycles = Yearly Trend + Seasonal Cycle + Day-of-Week Cycle + Holiday Cycle

A. Yearly Trend

In this model, a year is defined as the 21-week period (147 days) beginning with either the first or second Sunday in May, and ending on a Saturday in late September. The choice of two Sundays is needed to consistently line up holidays in the same week as discussed later in the holiday cycle section. This period corresponds to the season of peak water use when forecast information is of the most importance.

$D_i(t)$ denotes water use in year i on day t. Mean daily water use in year i, Y_i, can be constructed from:

$$Y_i = 147^{-1} \sum_{t=1}^{147} D_i(t) \qquad i=1...13$$

For the peak seasons from 1974 to 1986 in the WMA, Figure 1.3a shows the original daily data ($D_i(t)$) and Figure 1.3b the average daily water use (Y_i) for each season. A clear increasing trend is displayed. The factors changing Y_i from year to year are thought to be predominantly socioeconomic. The increasing tend in Y_i being primarily related to increasing population in the service area. Since the socioeconomic factors are characteristically gradual in their change or at least their change has only a gradual impact on water use (e.g. price), they are not included as explanatory influences in inter-peak season analysis. They are allowed to change from year to year, as captured by Y_i, but are assumed to have a relatively constant impact throughout the peak seasons themselves.

Another factor possibly causing fluctuations in Y_i is weather. Water use rises for peak seasons characterized as hot and dry and drops for those cool and wet. Deviations from normal weather conditions is the subject matter of the causal model and its impacts will be accessed there. The calendar model is based on the average response to weather. In the first stage, all peak seasons are assumed to receive average weather conditions. This overly optimistic assumption is relaxed after the results of the causal model allow for the removal of weather-related deviations.

B. Seasonal and Day-of-Week Cycle

The intra-year cycles are patterned and deterministically expressed by a unit demand factor, $m(t)$, that displays the underlying calendar structure of the time series. The objective is to capture the consistent behavioral patterns of water users under average weather conditions. (For details see Sections 2.2 and 2.3 of Chapter 2.)

In this study, $m(t)$ is comprised of both weekly and day-of-week demand factors.

$$m(t) = q_j \cdot p_k \qquad (j-1)\cdot 7+k=t, \quad j=1....147, \quad k=1...7$$

The weekly demand factor, q_j, and the day-of-week demand factor, p_k, can be estimated by:

$$\hat{q}_j = (7\cdot 13)^{-1} \sum_{i=1}^{13} \sum_{k=1}^{7} [D_i(7\cdot(j-1)+k) \;/\; Y_i] \qquad j= 1...21$$

$$\hat{p}_k = (21\cdot 13)^{-1} \sum_{i=1}^{13} \sum_{j=1}^{21} (D_i(7\cdot(j-1)+k) \;/\; (7^{-1} \sum_{s=1}^{7} D_i(7\cdot(j-1)+s)))$$

$$k=1...7$$

The estimator $\hat{q}_j$ is a measure of the average fractional ratio of week j to its corresponding Y_i over all years. Figure 1.4 graphs the weekly demand factors for all weeks; q_j with coefficients greater than 1 have weekly averages over the mean while coefficients less than 1 are under the mean. A definite bell-shaped seasonal pattern is depicted.

The estimator p_k portrays the day-of-week differences in a similar manner. The average fractional ratio of day-of-week k to its corresponding weekly average is constructed. The day-of-week pattern displayed in Figure 1.5 shows weekdays to have higher use than weekends, which is probably caused by the influx of workers to the WMA employment centers.

The appropriateness of this type of multiplicative detrending model depends on two structural assumptions: namely that the weekly and day-of-week patterns remain constant over time. These properties along with an investigation on holiday cycles are analyzed in depth with the calendar model in state 2. At that point weather-related deviations are removed to better highlight the underlying calendar structure.

A new time series, $Z_i(t)$, is created by subtracting the original series from its calendar based predictions.

$$Z_i(t) = D_i(t) - Y_i \cdot m(t)$$

The result of the calendar model's decalendarizing is a 57 percent reduction in variance. The new series, as graphed in Figure 1.6a, appears to be fairly stable in its structure over time. Because of the way the calendar model is designed, the series fluctuates about its mean of zero. The standard deviation for each peak season is graphed in Figure 1.6b. Although relatively high in 1986, no distinctive alteration in the standard deviation is uncovered over time. This supports the notion that the series is reasonably stationary, at least in the wide sense of the first two moments.

II. Causal Model

The objective of this section is to develop a model describing the cause and effect relationship between the decalendarized water use and explanatory influences. A requirement of the approach undertaken is that it be explicitly probabilistic; this will allow confidence statements about forecasts to be made. The type of variable sought should explain deviations from the stated normalization given from the calendar model. They should vary from day to day in a way that explains the day to day changes in water use. Physical alterations in the environment relating to weather factors are considered both because of intuitive appeal and strong empirical evidence in the literature. A single equation model is investigated with decalendarized water use as the dependent variable, weather variables as the independent, and a random disturbance component modeled in an autoregressive moving average (ARMA) framework. This approach is well suited to the autocorrelation exhibited in time series data unlike other techniques, such as ordinary least squares, where assumptions on the error term are more restrictive.

To quantify weather, the two most common and widely observed atmospheric elements are used: temperature and precipitation at the earth's surface. Maximum daily temperature (^{O}F) is selected over average daily temperature as water use, and in particular sprinkling demand, is postulated to be more responsive to maximum than average recordings where extremes are attenuated. The maximum daily temperature observations are subtracted from their corresponding long-term weekly averages as compiled from 1974 through 1986; putting data in the form of deviations is necessary because inferences will be made about deviations from the calendar normalization which is based on average responses. Since precipitation follows a rather uniform distribution throughout the period involved, the transformation into deviations is not undertaken. The temperature and precipitation recordings utilized in the models are calculated from the average of observations at the weather stations at Beltsville and Upper Marlboro (Figure 1.1); two sites are deemed more representative than one. The data are obtained from publications of the National Weather Service, National Oceanic and Atmospheric Administration (NOAA), with occasional missed observations estimated by interpolation and correlation with the other station to give a continuous record for the 1974-1986 period.

Another descriptive element, daily evaporation, is also available from NOAA as calculated by the standard Weather Service-type pan of 4-foot diameter. Evaporation is not pursued here for three reasons. First, evaporation has collinearity problems with temperature that can lead to unstable parameter estimates. Second, Steiner [1984] finds inferior results in a similar study on the same area, and lastly, because forecasts of evaporation are not readily available.

The underlying process linking water use to temperature and precipitation is thought to be fairly intricate. Both nonlinear and dynamic responses are investigated.

A. Nonlinear

The common assumption of a simple linear relationship between variables is often based more on a computational than a theoretical justification. It is possible, and most likely, that some type of nonlinear model better replicates the true interaction between water use and weather. The dilemma encountered in constructing nonlinear models, other than pragmatic reasons, is in its proper specification. There is an infinite number of logarithmic, exponential, and multiplicative combinations that can be tested.

One strain of nonlinear model is tested here; the relationship between water use and weather variables is still assumed to be additive, but the response over the entire range of temperature and precipitation recordings is no longer assumed to be constant. It is allowed to have a changing slope approximated by piecewise linearization.

Temperature is segmented into three separate sections with break points at 75 and 85 degrees. Since temperature deviations from long-term averages are being implemented instead of the actual observed values, a modification to the usual piecewise linear model is needed. If both the long-term average and the observed recording are contained on the same segment, say between 75 and 85, then the deviation is allocated completely to that segment. If the expected and the observed recordings, however, are on different segments, then the deviation needs to be divided and assigned to the appropriate segments.

Three variables are constructed to adhere proper representation of the deviations to their corresponding segments. T_1 represents the part of the deviation occurring below 75 degrees, T_2 captures the part of the deviation resulting between 75 but less than 85 degrees, and T_3 for deviations from 85 degrees and higher. To illustrate the formulation, if the expected temperature is 80 degrees and the observed 95, then $T_1=0$, $T_2=5$, and $T_3=10$.

Precipitation is also broken into three segments, but can be handled in a more straightforward manner as the actual precipitation recordings are used. The break points are set at 0.1 and 0.5 inches. P_1 represents the first 0.1 inches in a day if any, P_2 stands for the precipitation over 0.1 inches and up to 0.5 inches, while P_3 records the remainder over 0.5 inches. For example, if precipitation is 0.7 inches, then $P_1=0.1$, $P_2=0.4$, and $P_3=0.2$.

The selection of the number of piecewise segments and the determination of where the break points reside are based partly on empirical results from previous studies (Maidment [1985], [1986] and Steiner [1984]) and partly on the discretion of the modeler from examining biplots.

B. Dynamic

The weather information available for water use decisions on a given day can be divided into three sources: past, present, and forecasted. While correlation with past and present weather variables has been previously examined, to the author's knowledge the use of future values has not been attempted. This neglect may reflect the indirect connection between the two. Individuals do not know the actual weather conditions in the future, but forecasts by the National Weather Service are widely disseminated by the media and may have an instrumental influence. For example, if rain is predicted for tomorrow, it may well affect some individual's decisions on water use today (e.g. delay lawn watering).

This model uses the observed realizations of the weather variables for future leads instead of forecasts. Ideally, forecasts should be employed since they are the information available to individuals for today's water use decisions. Using future observed values is only a surrogate measure which should be fairly accurate for the 1 day forecast and decline thereafter because of declining skill in weather forecasting.

C. Model Description

The general form of the causal model is as follows:

$$Z_i(t) = a + a_1(B)T_{1,i}(t) + a_2(B)T_{2,i}(t) + a_3(B)T_{3,i}(t)$$
$$+ b_1(B)P_{1,i}(t) + b_2(B)P_{2,i}(t) + b_3(B)P_{3,i}(t)$$
$$+\phi^{-1}(B)\theta(B)a_i(t)$$

where

$Z_i(t)$ = decalendarized water use in year i day t

a = intercept term

F1,..,F6 = number of lead days

l1,..,L6 = number of lag days

$T_{1,i}(t)$ = temperature deviation < 75 in year i day t

$T_{2,i}(t)$ = 75 < temperature deviation < 85 in year i day t

$T_{3,i}(t)$ = 85 < temperature deviation in year i day t

$P_{1,i}(t)$ = precipitation deviation <0.1 in year i day t

$P_{2,i}(t)$ = 0.1 < precipitation deviation <0.5 in year i day t

$P_{3,i}(t)$ = 0.5 < precipitation deviation in year i day t

$$a_1(B) = a_{1,F1}B^{-F1} + a_{1,F1-1}B^{-F1+1} + \dots + a_{1,L1}B^{L1}$$

$$a_2(B) = a_{2,F2}B^{-F2} + a_{2,F2-1}B^{-F2+1} + \dots + a_{2,L2}B^{L2}$$

$$a_3(B) = a_{3,F3}B^{-F3} + a_{3,F3-1}B^{-F3+1} + \dots + a_{3,L3}B^{L3}$$

$$b_1(B) = b_{1,F4}B^{-F4} + b_{1,F4-1}B^{-F4+1} + \dots + b_{1,L4}B^{L4}$$

$$b_2(B) = b_{2,F5}B^{-F5} + b_{2,F5-1}B^{-F5+1} + \dots + b_{2,L5}B^{L5}$$

$$b_3(B) = b_{3,F6}B^{-F6} + b_{3,F6-1}B^{-F6+1} + \dots + b_{3,L6}B^{L6}$$

$\phi(B)$ = autoregressive operator of order p: $1 - \phi_1 B^1 - \dots - \phi_p B^p$

$\Theta(B)$ = moving average operator of order q: $1 - \Theta_1 B^1 - \dots - \Theta_q B^q$

B = backward (forward) shift operator (e.g. $Bx(t)=x(t-1)$ and $B^{-1}x(t)=x(t+1)$)

$a_i(t)$ = white noise residual

D. Identification

$a_1(B)$, $a_2(B)$, $a_3(B)$, $b_1(B)$, $b_2(B)$, $b_3(B)$, and $\phi^{-1}(B)\Theta(B)$ are commonly referred to as transfer functions. They represent a linear aggregation of inputs at various points in time which correspond to an output at a particular time. The particular impacts from a given input in relation to time is often called an impulse response function with the coefficients in front of the backward shift operator in a transfer function constituting its makeup.

The identification process pertaining to the structural variables deals with adequately approximating the finite length of the lead (F1,...,F6) and lag (L1,...,L6) periods of the impulse response. When do future values (forecasts) become insignificant in predicting today's water use? How far back in the past is useful information drawn?

The initial ranges for the weather variables are set both from appeal to physical considerations and statistical analysis. The length of a lead period is limited by the availability of accurate weather forecasts. Since forecasts are generally considered to have little credibility after 5 days, if that, the lead variables are naturally bound to be less. Concerning the lags, as heat dissipates into the atmosphere in a relatively short order, the latent effects for temperature are surmised to not extend past a few days. Precipitation has more storage properties (water held by plants and soil) which can lead to a longer lag. Statistically, the cross-correlation functions between the input series and water use give a rough approximation to the impulse responses. The application of prewhitening filters are employed to cancel correlation dynamics of the input series which are known to corrupt the estimates. The cross-correlation functions coincide with a priori beliefs based on physical considerations and provide initial specification. To maintain a clear interpretation of future results, estimates of the coefficients in the transfer functions are generated from the decalendarized data and not the transformed (prewhitened) data. Prewhitening is an artifact of identification only.

The identification of the disturbance term's transfer function is handled differently. The residual $a_i(t)$ is dynamically filtered through the ratio of two polynomial backward shift operators: the numerator known as a moving average operator and the denominator as an autoregressive. The identification of these operators is facilitated by analyzing the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the residuals of the structural model (i.e. the model without the ARMA process). These two devices are complementary and make up the backbone of time domain identification. From them the degree of the autoregressive process (p) and the moving average process (q) can be approximated. In short, an autoregressive process tails off in the ACF, and has a cut off after lag p in the PACF. A moving average process exhibits the reverse by having a cut off after lag q in the ACF, and a gradual tailing off in the PACF. Mixed processes tail off in both the ACF and PACF (Box and Jenkins [1976]).

E. Estimation

It is possible to estimate each coefficient individually in the causal model, however, by placing some restrictions on the coefficients the number of parameters can be substantially reduced. The advantages to such an approach include not only a more parsimonious model, but also a better way to handle the moderate multicollinearity inherent in the time-distributed variables. Multicollinearity increases the standard errors of the coefficients due to the lower information content derived from correlated data. The basic desirable properties of the estimators (unbiased, consistent, and asymptotically efficient), however, do remain intact. The best way to work with multicollinearity is to have an abundance of data; fortunately this is the case in this analysis. It is also beneficial to minimize the number of parameters as individual estimation of lengthy distributed responses uses up a large number of the degrees of freedom. A number of techniques are available to deal with this common dilemma in time series applications. One technique forces the coefficients to lie on some low order polynomial (e.g. Almon lag). Another, which is used here, forces the coefficients to lie on a ratio of two polynomials (i.e. rational lag) which is the same type of structure imposed on the disturbance term.

Estimation of the parameters is based on the principle of maximum likelihood (ML). The OLS approach is not suited for models with the type of lagged disturbance term presented, so ML is employed (Harvey [1981a]). The estimators derived are those values that would generate the observed sample of decalendarized water use most often. They are statistically efficient. The maximization of the likelihood functions is done via nonlinear least squares using Marquant's method implemented on SAS software.

The data is initially utilized by concatenating the 21 week period under consideration from each year into one time series. Since observations are going to be related to past observations, a problem with wrapping surfaces; the first days from one year are drawing information from the last days of the previous year. This connection delivers spurious results as the dynamic structure between the end of a year and the start of the next is not the assumed one day, but eight months. The magnitude of the problem depends on the length of the lag. Since there are 12 discontinuities in the 13 years of data, every additional lag creates 12 more spurious results. For lower order processes, the concatenation should have little effect. A five period lag, for example, has about 3 percent of its observations influenced by the wrapping of years.

F. Diagnostic Checking

Once a model has been identified and estimated, it is analyzed for goodness of fit. For the structural parameters, this concerns the length of the lead and lag periods as well as the order of relevant rational lags. It is desirable to maintain only variables that provide a significant contribution to the model. T and F tests are useful in the selection process.

Regarding the disturbance term, if the complete model successfully captures systematic movements in the data, the residuals should essentially be uncorrelated random variables with constant mean and variance, commonly referred to as "white noise". Two different types of significance tests are used in checking if a white noise series results from the selection of p and q in the ARMA process. One type examines the sample autocorrelations and partial autocorrelations of the ACF and PACF of the residual to see if they lie within a stated theoretical confidence interval of a white noise process. The second type involves a chi square test on the first s autocorrelations computed using the formula suggested by Ljung and Box [1978]

$$\chi^2_{s-p-q} = n(n+2) \sum_{k=1}^{s} r^2{}_k \,/\, (n-k)$$

where n equals the total number of observations, p the number of autoregressive terms, q the number of moving average terms, and $r^2{}_k$ the kth squared sample autocorrelation. This device gives the probability of different intervals of autocorrelations coming from a white noise process.

Since it is common that different combinations of p and q can pass the white noise tests, some added criterion is needed in the selection stage. Selecting the ARMA that displays the lowest standard deviation of the residual is appealing, however, just by adding more p and q terms to any process this can usually be achieved. A more prudent approach involves using the Akaike's Information Criterion (AIC) (Akaike [1974]). The AIC is computed from

$$AIC = -2\log(L) + 2k$$

where L is the likelihood function and k is the number of free parameters. A common decision rule is to select the model that minimizes the AIC; the first term seeks to maximize the likelihood and the second to penalize models not suitably parsimonious.

G. Results

Repetition of the identification, estimation, and diagnostic checking procedures leads to a suitable model whose results are examined here.

Temperature. The impulse response functions of the temperature variables are shown in Figure 1.7a. The coefficients lie between a 2 day lead and a 1 day lag; the 1 day lag variables are not significantly different from zero, but are included to illustrate this surprising point. Because of this relatively short time span involved, the coefficients are estimated individually.

The responses are somewhat symmetric and centered on the 1 day ahead observation. This is interesting as it implies tomorrow's temperature has more influence on water use today than today's temperature. Water use decisions are being primarily based on anticipation of upcoming conditions. To illustrate, people are watering lawns before they have a chance to dry out, not after.

A distinctive nonlinear response from different temperature ranges also stands out. Water use becomes progressively more sensitive to temperature as temperature increases. For the 1 day lead variables, a 65 percent increase in sensitivity is seen between T_1 and T_2 and a 130 percent increase between T_2 and T_3.

Precipitation. The impulse response functions of the precipitation variables are shown in Figure 1.7b. P_1 is found to be significant between a 2 day lead and a 14 day lag. The transfer function for P_1 is a rational lag type as follows:

$$b_1(B) = (n_1B^{-2} - n_2B^{-1} - n_3B^0 - n_1B^1) \ / \ (1 - dB),$$

where n_1, n_2, n_3, n_4, and d are parameters estimated. By rewriting the denominator as an infinite geometric series and grouping the backward shift operators, a description of the b_1 coefficients distributed over time results.

$$B_1(B) = (n_1B^{-2} - n_2B^{-1} - n_3B^0 - n_1B^1) \cdot (1 + dB + d^2B^2 + \ldots)$$
$$= (n_1)B^{-2} + (n_1d-n_2)B^{-1} + (n_1d^2-n_2d-n_3)B^0 + (n_1d^3-n_2d^2-n_3d-n_4)B^1 + \ldots$$

where

$$b_{1,2} = n_1$$
$$b_{1,1} = N_1d - n_2$$
$$b_{1,0} = n_1d^2 - n_2d - n_3$$
$$b_{1,-1} = n_1d^3 - n_2d^2 - n_3d - n_4$$
$$\ldots$$

The distributed response for P_1 is skewed right and centered on the contemporaneous recording; today's precipitation has the most influence on today's water use. The first two lead variables are significant which again supports the belief that water use decisions are in part functionally related to future weather. As for the past, the impact from P_1 is felt for the entire 14 days.

Precipitation over 0.1 inches has almost no influence on water use. P_2 and P_3 are most prominent at the 1 and 2 day lags, but P_2 is still insignificant at both lags while P_3 is only significant (.10 level) for the 1 day lag and has the opposite sign expected.

Intercept. The intercept term counterbalances the temperature and precipitation influences in the long-run. For temperature, the sum of the deviations from normal equals zero, but since the positive deviations are generally weighted higher because of the nonlinear response an overall positive impact results on the data. To neutralize this effect so as to stay consistent with the average weather response criteria of the calendar model, a portion of the intercept team is committed. In this case, -5.89 neutralizes the temperature effects over the course of the model to zero.

As precipitation takes on only positive values and is dominated from the negative response of the first 0.1 inch, the event of rainfall drops water use. The average reduction for a given day derived from the 2 day lead to the 14 day lag responses equals 24.73 MGD. The interpretation of this number is that it is what is added to decalendarized water use in the event of all days in this scope being dry. If wet days are recorded but their influences amount to less than 24.73, the day is classified as drier than normal. If their influence exceeds 24.73 the day is classified as wetter than normal.

The overall impact of the intercept term keeps the causal model in balance with the calendar model. The model's intercept term comprises the summation of both the temperature and precipitation intercepts (i.e. -5.89 + 24.73 = 18.84).

Disturbance. The disturbance term is set up in the form of a random shock model where past error terms are related to current decalendarized water use. The impulse response function of the disturbance term for the first 7 lags is shown in Figure 1.7c. They are calculated from an ARMA(1,3) specification in a manner similar to that used with the P_1 variable. The 1 day lag error has the most influence with over 50 percent of yesterday's under or over prediction being forwarded to today's prediction. The lag errors monotonically decline in influence over time.

The reliance on past errors stems from three general sources. One, incorrect specification of the weather model both in structural form and in omission of other relevant meteorological variables. Two, possible short-term shocks not explicitly addressed such as short-term conservation programs. And three, misspecification of the calendar model. Since these shocks can have persistence over time, incorporating past errors as information into the current prediction allows for partial adjustment for these unincorporated factors.

The model summary in Figure 1.8a lists the coefficients with their standard errors along with analysis of the residuals. The causal model explains an additional 33 percent of the variance over the calendar's 57 percent for a total of 90 percent of the original data's variance explained. The chi square test and the plots of the ACF and PACF shown in Figure 1.8b. indicate the continued existence of some autocorrelation in the data, but it is reasonably small and the more complex transfer functions that are needed to ensure white noise are found to add no power to the model based on the AIC. The residuals and the standard deviation of the residuals by peak season are plotted over time in Figure 1.9a and 1.9b respectively. Comparing both measures with the time series prior to the causal model (Figure 1.6a and 1.6b) illustrates the noticeable reduction in variation. The residuals appear stable and the differences in standard deviation between seasons have been reduced.

1.4.1 STAGE 2

I. Calendar Model

The input series into the calendar model in stage 2 is the original water use series minus weather-related influences as identified in the causal model from stage 1; this new series is termed deweatherized. Deweatherizing has the advantage of removing a sizable proportion of noise in the data which allows for a more precise focus when determining underlying calendar structure. The assumption made in stage 1 of weather not affecting calendar estimates is relaxed.

A. Re-estimates of Parameters

New estimates for Y_i, q_j, and p_k are computed using the decalendarized data and compared to their original values.

Figure 1.10a plots both the original Y_i and the deweatherized Y_i over time while Figure 1.10b lists the breakdown of weather influences causing their departure. The new Y_i are smoother having less variation year to year. The most notable difference between the original Y_i, from 1979 to 1980, can be partially explained by weather. The year 1979 is not only wet but the coolest season in the analysis, while 1980 is the warmest. Over 40 percent of the variation between the two years is attributable to weather.

The re-estimated q_j plotted in Figure 1.11 show little difference from the originals with the exception of a slight decrease in weeks 17 to 20. Further investigation reveals that this period receives slightly lower precipitation than the seasonal average. The Calendar model captures this effect.

The p_k are plotted in Figure 1.12. As no particular day-of-week is likely to receive significantly different temperature or precipitation recordings from the other days-of-week over the long-run, there should be no change in the estimates after removing weather influences. No change appears.

B. Holiday Cycle.

There are three federal holidays in the period considered: Memorial Day, Independence Day, and Labor Day. The hypothesis is that there is a significant alteration in water use due to a holiday. To check for this deviation, a separate set of p_k for each holiday week is examined. Memorial and Labor Day are consistent in that they always fall on a Monday, in week 4 and 18 respectively. Figure 1.13 shows the new set of p_k for Memorial and Labor Day in comparison with the overall standard. A significant drop is seen in the Monday demand factor in both cases. Independence Day is in the 9th week, but is not restricted to fall on any particular day-of-week. As only thirteen occurrences of week 9 are available in the data set, creating seven different sets of p_k, one for each day-of-week concurring with July 4th, stretches inference. July 4th fell on Tuesday and Saturday only once from 1974 through 1986. Because of this, no special exceptions are made for week 9. A significant alteration in the behavioral pattern of water use is still suggested for July 4th, however, by the distinct drop in the weekly demand factor in week 9 as shown in Figure 1.11.

C. Time-Varing Properties

The underlying process establishing the calendar structure is not necessarily static, and may in fact evolve over time. There are two principal factors stimulating a structural change:

1) The reaction by water consumers to calendar orientation changes due to changes in individual tastes and preferences, available technology, land use patterns, and the like.

2) The WMA base of water users shifts in composition. Relative changes in the user categories may affect the calendar estimates for WMA as a whole, especially if the estimates differ among the user categories. Since 1974, Fairfax County has grown much faster relative to the other areas. As this takes place, the estimates are expected to more closely resemble those of Fairfax.

To check for stability, subsets of q_j and p_k are analyzed for significant differences. Figure 1.14 superimposes the weekly averages of the first 6 years, the last 7 years, and their composite average over the entire time period. The series appear stable and in no week is there a significant deviation (.10 level). Figure 1.15 superimposes day-of-week factors for the first 5, middle 4, and last 4 years of data. A steady flattening of the day-of-week pattern is detected, with the weekends taking on a more prominent role at the expense of the midweek days. The probable cause of this change lies in the relative growth of the FCWA, which has higher use on the weekends (Whitcomb and Boland [1986].

The calendar model in stage 2 is based on the updated Y_i, q_j, three subsets of p_k, and the two p_k associated with the holidays. The estimates are listed in Figure 1.16. Decalendarizing leads to a 55 percent reduction in the original data's variance; the modest lost in explanatory power over stage 1 (2 percent) resulting from the abstention of the estimates to respond to weather.

II. Causal Model

A. Re-estimation of Parameters

The causal model developed in stage 1 is re-estimated using the updated decalendarized data. The temperature responses show little change (Figure 17a) while the precipitation responses have become slightly more sensitive (Figure 17b). The information drawn from past errors in the disturbance term has declined (Figure 17c), inevitably due to better specification of the calendar model. The model summary in Figure 20 shows a slight improvement in variance explained over stage 1. The chi square test and the plots of the ACF and PACF indicate the residual more closely resembles white noise.

Four different variations of the causal model are explored in the quest of substantiating hypotheses on how certain factors affect water use. The variations include: 1) changing the number of weather stations from which weather information is gathered, 1) omitting lead variables to evaluate the value of forecasts, and 3) checking the stability of the parameters over time.

B. Weather Stations

The prior models utilized the average of weather observations from two weather stations. Ideally, the number of stations should be increased until achieving an adequate geographic representation. Each station should further be weighted for the purpose of constructing a weighted average; the station representing the area with highest water use getting the most weight and so on.

To gauge the sensitivity in the accuracy of modeling the WMA water use to the number of weather stations weather data is extracted, the causal model is estimated with the use of data collected from only one station, Beltsville. This results in a modest increase of 2.5 percent in variance due to inferior representation. Two additional models are estimated for the sake of revealing which variable, temperature or precipitation, is more dependent on the number of stations. One model incorporates temperature observations from the average of the two stations and precipitation observations from only Beltsville. Utilizing two stations for temperature is shown to have little value as the variance of the residual is unchanged. The other model uses Beltsville for temperature observations and the average of the two stations for precipitation. The results of this model are very similar to the original model which uses the two stations for both variables. This indicates that in going from one station to two, the precipitation variable gains in explanatory power while temperature is unchanged. This seems logical as precipitation in the summers of the WMA is characterized as a sporadic point process of localized thundershowers. Significant rainfall in the WMA may elude certain stations or a certain station may receive noticeable rainfall while the rest of the WMA remains dry. This potential problem is alleviated by utilizing more sample stations to obtain a better overall measure of precipitation. Temperature, on the other hand, is characterized as being more uniformly distributed. The WMA is fairly homogeneous with respect to elevation and other natural geographic attributes from which temperature differentials usually arise. Statistically speaking, the correlation coefficient (Pearson product-moment correlation) between the two stations is .76 for precipitation and .96 for temperature.

C. Value of Leading Variables

To evaluate the importance of future weather observations in modeling today's water use, the causal model is estimated without the lead variables. The results in Figure 1.20 portray an increase in variance of over 35 percent. This verifies the integral nature of leading variables as an explanatory influence which should not be omitted from consideration.

D. Time-Varying Properties

The estimates of the causal model may be time-dependent for the same reasons as those listed for the calendar model estimates: namely changes in behavioral response and in composition of the WMA base of water users. To check for stability, the estimates of the causal model are mapped over time. The time series is split into four subgroups consisting of the first four years of the series in the first group, and sequential three year groupings for the remainder. Any finer division lacks the number of observations needed to give suitable results. Figure 1.21a and 1.21b graph the estimates of the structural variables over the four subgroups. As a whole the estimates appear relatively stable with the possible exception of the 1985-1986 period. This most recent period displays an increased sensitivity to weather; this is especially true in the 1 day lead variable for T_3 and all of the P_1 variables for precipitation.

1.4.3 Summary of Models

This section summarizes the results of the approach taken to model water use. Figure 1.22 contrasts the original data with the residuals from the calendar and causal models in both stages by listing their standard deviations, variances, and coefficients of determination (R^2). Although the second iteration did not remarkably out preform the first, it is believed that the components comprising water use became more accurately represented. The main beneficiaries in this particular study turn out to be the Y_i. Further iterations are indeed possible and although no mathematical theorem on convergence is presented, the improved accuracy of one model should lead to the improvement of the other.

1.5 FORECASTING

The calendar and causal models describe the patterns and interactions underlying water use. The description bases its foundation on statistical analysis of historical data resulting in estimates that fit the past. Explaining the past is interesting in itself, but the major practical benefit lies in the use of this knowledge in predicting the future. The requirement for short-term forecasts motivates the investigations reported here.

A forecast is defined as a prediction of a future state conditional on certain assumptions holding true. In forecasting water use by the modeling scheme presented here, a number of assumptions are built into the models. A clear understanding of these assumptions enhances the base on which opinions are formed on the degree of credibility to be given for any inferences rendered.

1.5.1 Assumptions

This section lists a number of the premises from which conclusions are drawn. Although each one is unlikely to be strictly valid, it is hoped that they are substantially correct.

1. The average daily water use for the peak season, Y_i, in which forecasts are being made is assumed to be known before all of the observations for that season are realized. This necessitates the prior estimation of Y_i. A separate model based on long-term trend and some of the more prominent socioeconomic variables is suggested. The vast amount of the literature cited on water demand analysis addresses this type of issue. Changes

in employment, population, income, and price, to name a few, from the previous peak period may well help in prediction of the average use in the current period. The compatibility between long-range studies and the daily modeling procedure presented here makes the natural divisions within the scope of demand forecasting more manageable.

Uncertainty in the estimates of Y_i should add error to the results. The residuals derived from the combination of the calendar and casual models on the historical data should have less disturbance than the residuals when the value of Y_i is uncertain. A negligible difference would be seen, however, if Y_i could be accurately modeled.

2. Estimates derived from the calendar and causal models are assumed to remain constant or predictable in the future. Now that the interactions of the past have been highlighted, some reasonable projection of the estimates is needed. Since the seasonal pattern by week displays remarkable stability throughout the thirteen year period, it seems logical to assume it will persist. On the other hand, the flattening of the day-of-week cycle over time suggests the use of a set of p_k derived from the more recent data. The estimates from the causal models are relatively stable and should be good estimators for the future. A forecaster might, however, postulate that water use has become more sensitive to weather based on the results from the 1984-1986 subset of data and hedge forecasts accordingly.

3. Using observed recordings of temperature and precipitation for the lead variables is assumed to provide the same information as the forecasts themselves. Ideally, the forecasts are better information as they convey the current status of what water users perceive as future weather. To illustrate their difference, a forecast of rain might affect today's water use even if rain does not end up occurring. In any case, with the causal model only up to a two day lead for temperature and precipitation is of concern. The weather service has had good experience with such short-term forecasting.

1.5.2 Residuals

Conclusions dealing with the precision and confidence of forecasts are based on analysis of the residuals of the calendar and causal models from stage 2.

Descriptive statistics of the residuals of the calendar model are listed in Figure 23a. The residuals appear unbiased with a mean of -0.1 (rounding error), skewed towards positive deviation with a 0.77 skewness coefficient, and more peaked (leptokurtic) in character than the normal distribution indicated by the 0.55 kurtosis coefficient (3 characterizes normal) and the Kolomogorov D statistic.

For forecasting purposes, the distribution of the residuals is integral to any confidence statement made. The all too common assumption of normality would, in this instance, led to a misrepresentation in the confidence of a forecast. To help describe the characteristics of the distribution at hand, a set of quantiles is included. These provide useful markers for assessing the probability of water use being under or over a selected amount. For example, there is a 90 percent probability that a forecast conceived from the calendar model will not underestimate water use by more than 42.4 MGD. This type of information provides valuable input to the management of water supplies. Looking at how models perform at their worst moments is also informative; the residuals' extremes are included in Figure 1.23a.

The residuals of the causal model are statistically described in Figure 1.23b. They are unbiased, symmetric, and leptokurtic. Since the presence of a normal distribution is again refuted, a histogram, quantiles, and extreme values are presented to provide insight into the distribution.

These residuals are based on a one-step ahead forecast. Forecasts of greater duration will be expected to provide less precision because of two factors. The first is that since the expected value of the errors of future forecasts is zero, the information derived from the disturbance term is only based on errors prior to the first forecasting period. Recalling that the impulse response function of the disturbance term monotonically declines over time (Figure 1.17c), these past errors play a less significant role as the forecasting horizon increases. This decreases the power of the model.

The second factor arises from the uncertainty involved with future weather. The confidence placed on future water use will largely depend on the accuracy of weather forecasts. This accuracy, as demonstrated by the National Weather Service, declines over time. At the present state of forecasting, temperature and precipitation can be predicted with some skill up to five days.

There are a variety of measures used by the National Weather Service in conveying the probabilistic nature of forecasts. Point estimates and intervals with stated confidence bands are common with temperature while quantitative precipitation forecasts (QPFs) show the likelihood of occurrence of various amounts of precipitation. Since the relationship between water use and precipitation is fairly well represented by a step-function, using the subset of the QPFs associated with what is commonly referred to as the probability of precipitation (PoP) of noticeable rainfall (>.01 inch) can simplify the process.

When concern is focused on obtaining predictions for future water use, using expected values of future temperature and precipitation observations provides the best measure; expected value gives both unbiased and efficient results. (It can be shown that the mean is the point that minimizes squared dispersion of any distribution (von Mises [1964]). Point forecasts of maximum daily temperature and PoP forecasts for precipitation can be substituted into the causal model to get predictions.

Although these proceedings can lead to a multi-step forecast for water use, the confidence in that prediction is not so easily addressed. The distribution of the residual should increase in variance with increasing steps; the one step being the best and declining thereafter until the weather information available today (past, present, forecasted) loses value. At that juncture, the calendar model alone becomes the best available forecasting tool.

Two conditional cases are examined to bound the uncertainty involved with future forecasts; the best case bases its predictions on the assumption of perfect weather forecasting while the worst case utilizes only past (already observed) weather. Monte Carlo simulations are conducted to establish the nature of probability density functions of residuals from forecasts extending out to 14 days. The mean square errors (MSE) of the two conditional cases are shown to increase over time as plotted in Figure 1.24. The perfect forecast assumption starts with a 14.2 MSE for the one day lead and monotonically increases thereafter to approximately 17.8; the cause of the increase due to decreasing information derived from the disturbance term. The more conservative assumption of only using past weather starts with a MSE of 16.6 which asymptotically grows to 30.4 over time; the cause of the increase due to decreasing information drawn from both the disturbance term and past weather observations. It appears that after 7 days the causal model no longer noticeably out performs the calendar model under these circumstances.

Although the MSE is an informative measure in the description of a distribution, other statistics can enhance the characterization. This is especially true when the distribution at hand deviates from the classical distributions (e.g. Normal). Since the 5 day forecast is of particular concern in making releases from two upstream reservoirs, an example of a more exhaustive description of these forecasts under both conditional cases is presented in Figures 1.25a and 1.25b. They are seen to be unbiased, positively skewed, and leptokurtic. This type of information is fundamental to probabilistic analysis of operations. It conveys the stochastic nature of water demand in a way that can be integrated into the decision making process concerning reliability.

1.6 CONCLUSIONS

The approach taken to discern the forces driving water use in the peak seasons of the WMA relies on the use of two harmonious models: calendar and causal. A number of important conclusions are drawn from their application.

1. The pattern of use by water consumers is at least partially dependent on temporal orientation; yearly trend and week, day-of-week, and holiday cycle have been identified.

2. The relationship between water use and the weather variables is nonlinear. Water use is more sensitive to deviations from temperature norms at higher temperatures. For precipitation, the occurrence and not the magnitude of rainfall is the paramount factor.

3. Water use is dynamically related to the weather. Temperature's influence generally ranges from up to a 2 day lead

to a 1 day lag. Interestingly enough, tomorrow's temperature is the most influential on today's water use. For the first 0.1 inches of precipitation, the significant influence rendered ranges from a 2 day lead to a 14 day lag; the same day precipitation exerting the most explanatory power. Precipitation over 0.1 is found to have little effect.

4. Increasing the number of weather stations from which weather information is gathered improved the quality of the precipitation measure, but not temperature.

Study Area

FIGURE 1.1

EMPIRICAL WATER USE STUDIES		TIME RESOLUTION				EXPLANATORY VARIABLES							C=CROSS-SECTION P=POOLED T=TIME SERIES
AUTHORS(S)	YEAR	DAY	MONTH	SEASON	YEAR	TEMP.	PREC.	EVAPO.	LAG USE	DEMOG.	INCOME	PRICE	
Agthe & Billings	1980		X					X	X		X	X	T
Anderson et al.	1980	X						X					T
Ben-Zui	1980			X	X	X	X			X	X	X	C
Ben-Zui	1980				X					X		X	C
Berk et al.	1981		X			X	X		X			X	T
Berry & Bonem	1974				X			X		X	X	X	C
Billings & Agthe	1980		X					X			X	X	T
Billings	1982		X					X			X	X	T
Camp	1978				X	X				X		X	C
Carver & Boland	1980		X	X				X	X	X	X	X	P
Cassuto	1979		X			X	X			X	X	X	C
Chicoine et al.	1986		X								X	X	P
Clark & Goddard	1977				X						X	X	C
Cochran & Cotton	1985				X	X	X			X	X	X	T
Conley	1967				X							X	C
Csallany & Neill	1972				X					X	X		C
Danielson	1979		X	X		X	X			X		X	P
DeRoony	1974				X					X		X	C
Foster & Beattie	1979				X		X			X	X	X	C
Gibbs	1978			X						X	X	X	P
Gottlieb	1963				X						X	X	C
Grima	1972			X	X					X		X	C
Grunwald et al.	1976				X			X		X	X	X	C
Hanke	1970		X									X	T
Hanke & deMare	1982			X			X			X	X	X	P
Hansen & Narayanan	1981		X	X		X	X					X	T
Howe & linaweaver	1967			X	X		X	X		X		X	C
Howe	1982			X	X			X		X		X	C
Jones & Morris	1984				X					X	X	X	C
Kher & Sorooshian	1986		X			X	X	X	X		X	X	T
Lynne et al.	1978		X							X		X	C
Maidment & Miaou	1986	X				X	X		X				T
Maidment et al.	1985	X				X	X		X				T
Maidment & Parzen	1984a		X			X	X	X	X	X	X	X	T
Maidment & Parzen	1984b		X			X	X	X	X				T
Moncur	1987		X				X		X	X	X	X	P
Morgan	1974		X	X			X					X	T
Morgan & Smolen	1976		X	X		X	X	X			X	X	P
Salas-LaCruz et al.	1972		X*			X	X		X				T
Sewell & Roueche	1974				X	X	X				X	X	T
Steiner	1984	X		X		X	X	X	X				T
Turnovsky	1969				X					X		X	C
Whitford	1972				X					X		X	T
Weeks & McMahon	1973				X	X	X	X					T
Wong	1972				X	X					X	X	T
Yamauchi	1977		X	X									T
Young et al.	1983			X						X			P
Young	1973				X		X					X	T
Ziegler & Bell	1984				X					X		X	C

* = weekly

FIGURE 1.2

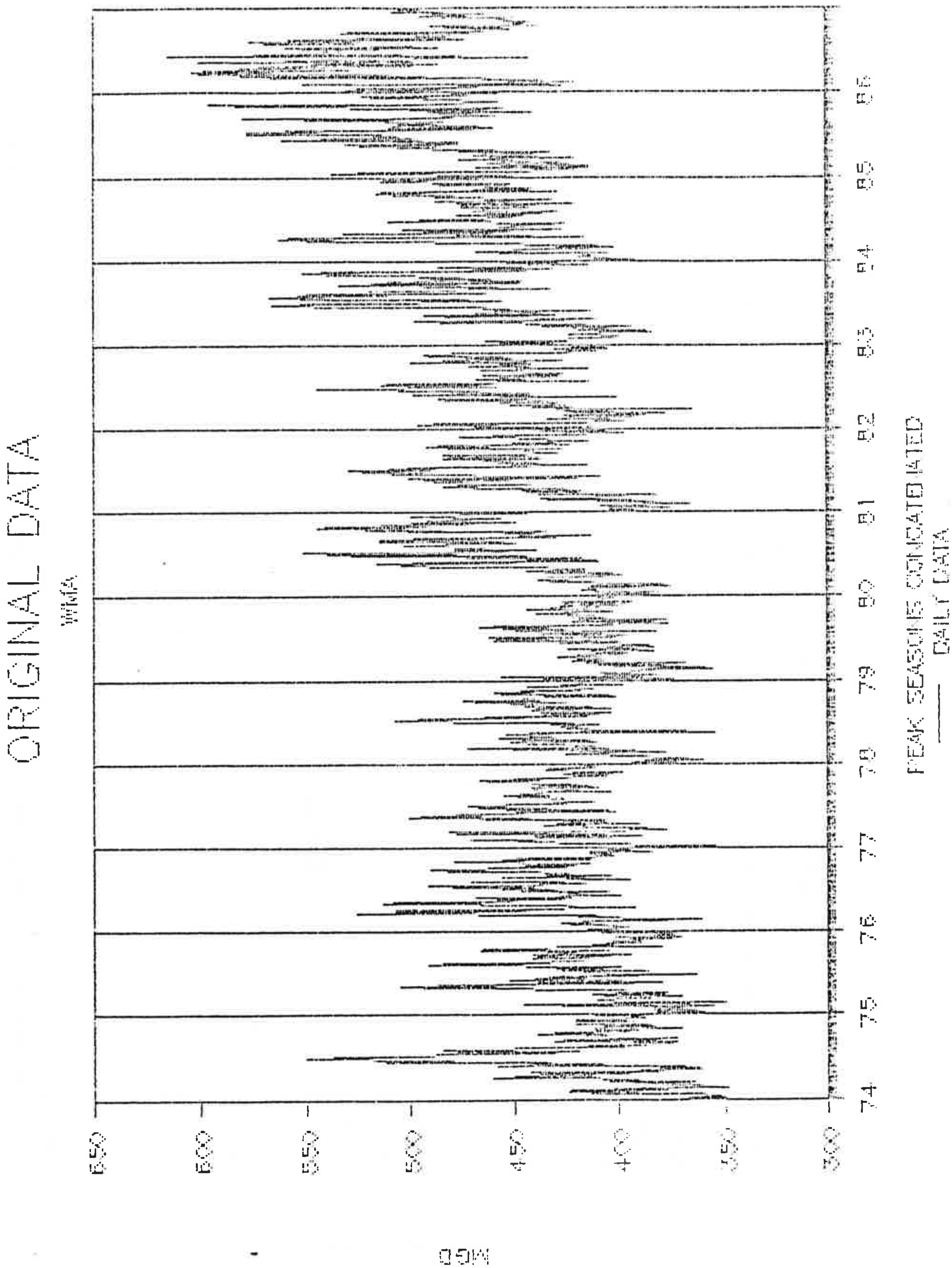

ORIGINAL DATA
WMA
PEAK SEASONS CONCATENATED
DAILY DATA
MGD
650
600
550
500
450
400
350
300
74
75
76
77
78
79
80
81
82
83
84
85
86

FIGURE 1.3a

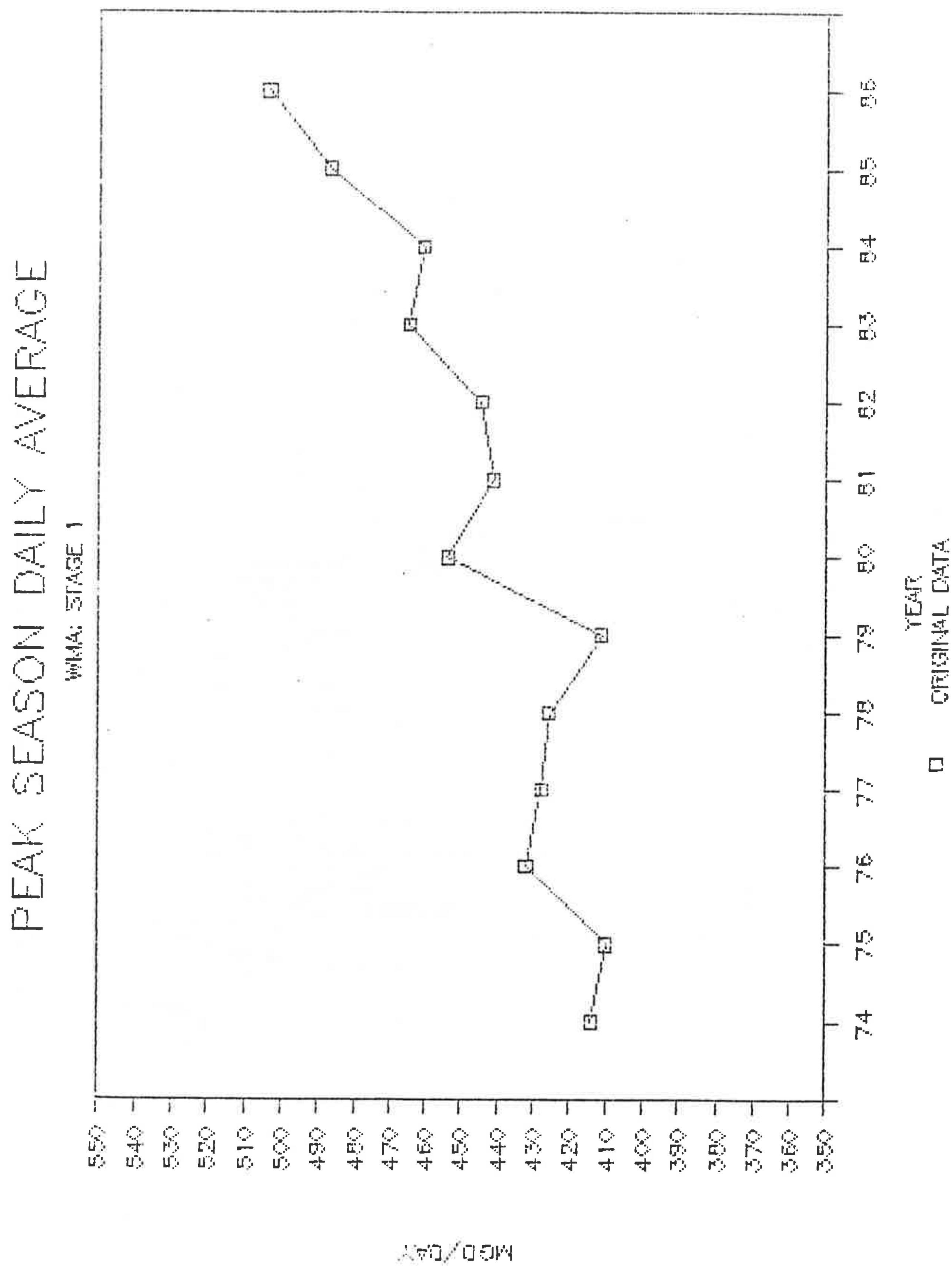

PEAK SEASON DAILY AVERAGE
WMA: STAGE 1
MGD/DAY
550
540
530
520
510
500
490
480
470
460
450
440
430
420
410
400
390
380
370
360
350
74
75
76
77
78
79
80
81
82
83
84
85
86
YEAR
ORIGINAL DATA

FIGURE 1.3b

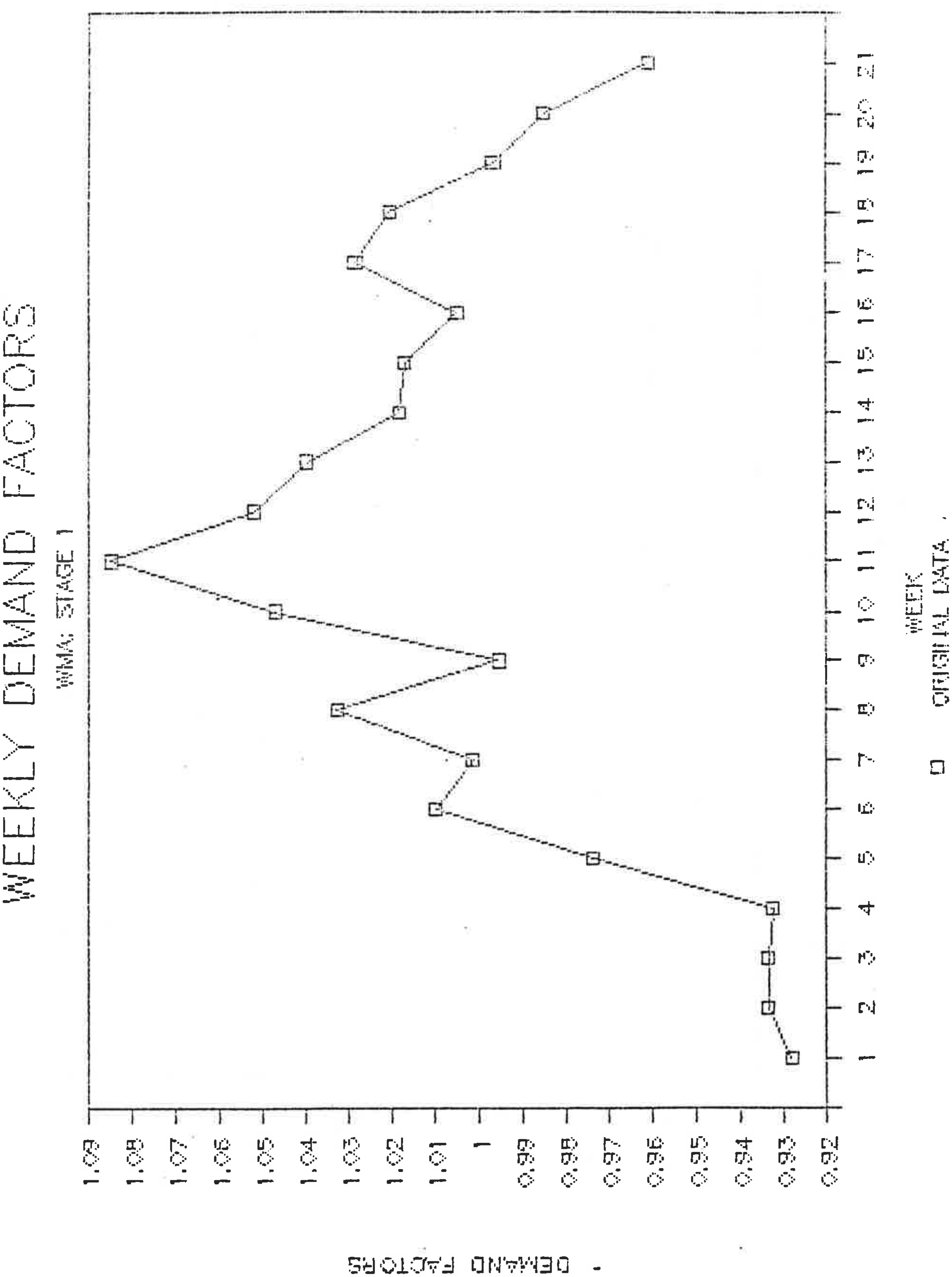

WEEKLY DEMAND FACTORS
WMA; STAGE 1
DEMAND FACTORS
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
WEEK
ORIGINAL DATA

FIGURE 1.

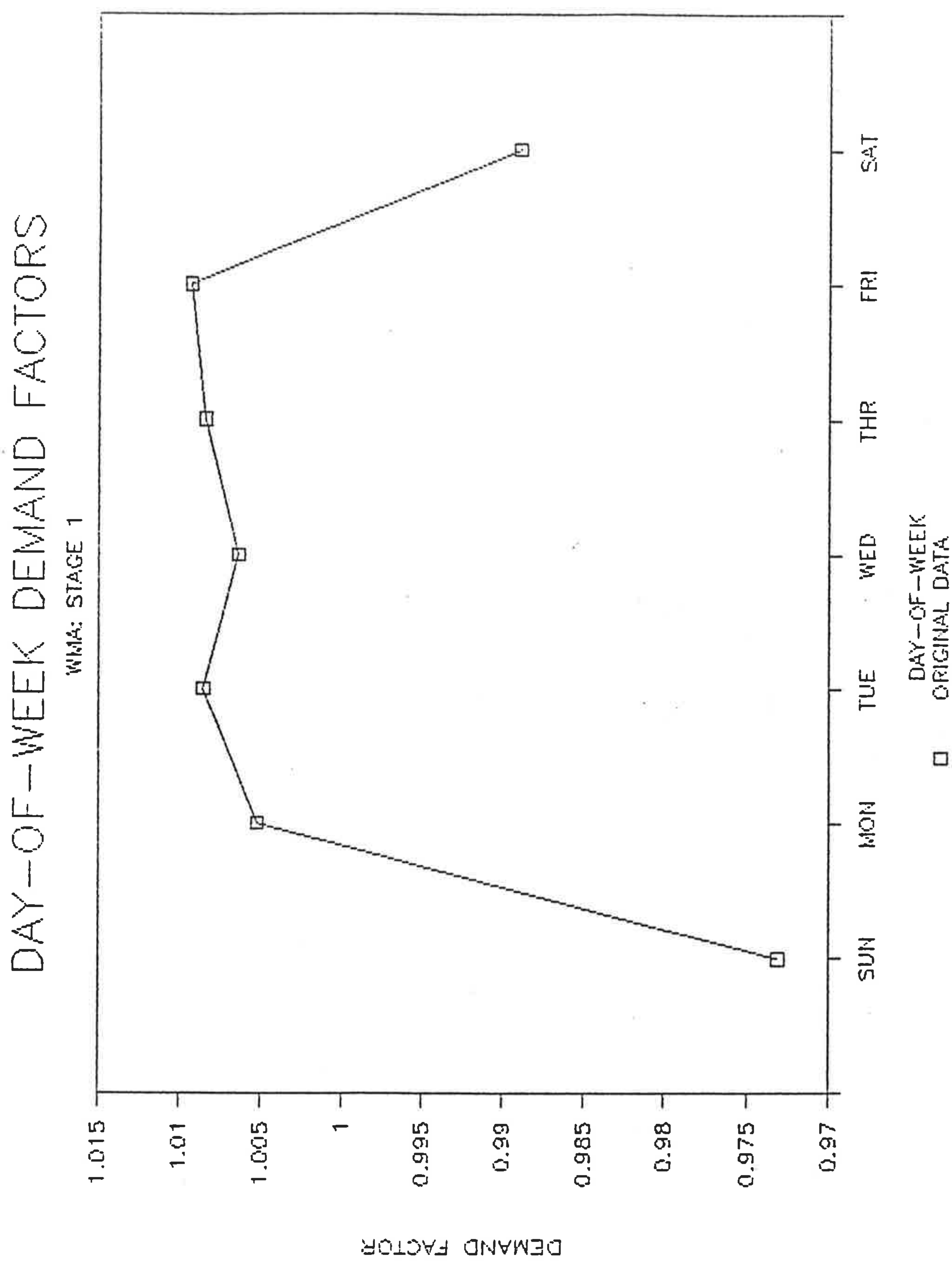

DAY-OF-WEEK DEMAND FACTORS
WMA: STAGE 1
DEMAND FACTOR
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
0.975
0.97
SUN
MON
TUE
WED
THR
FRI
SAT
DAY-OF-WEEK
ORIGINAL DATA

FIGURE 1.5

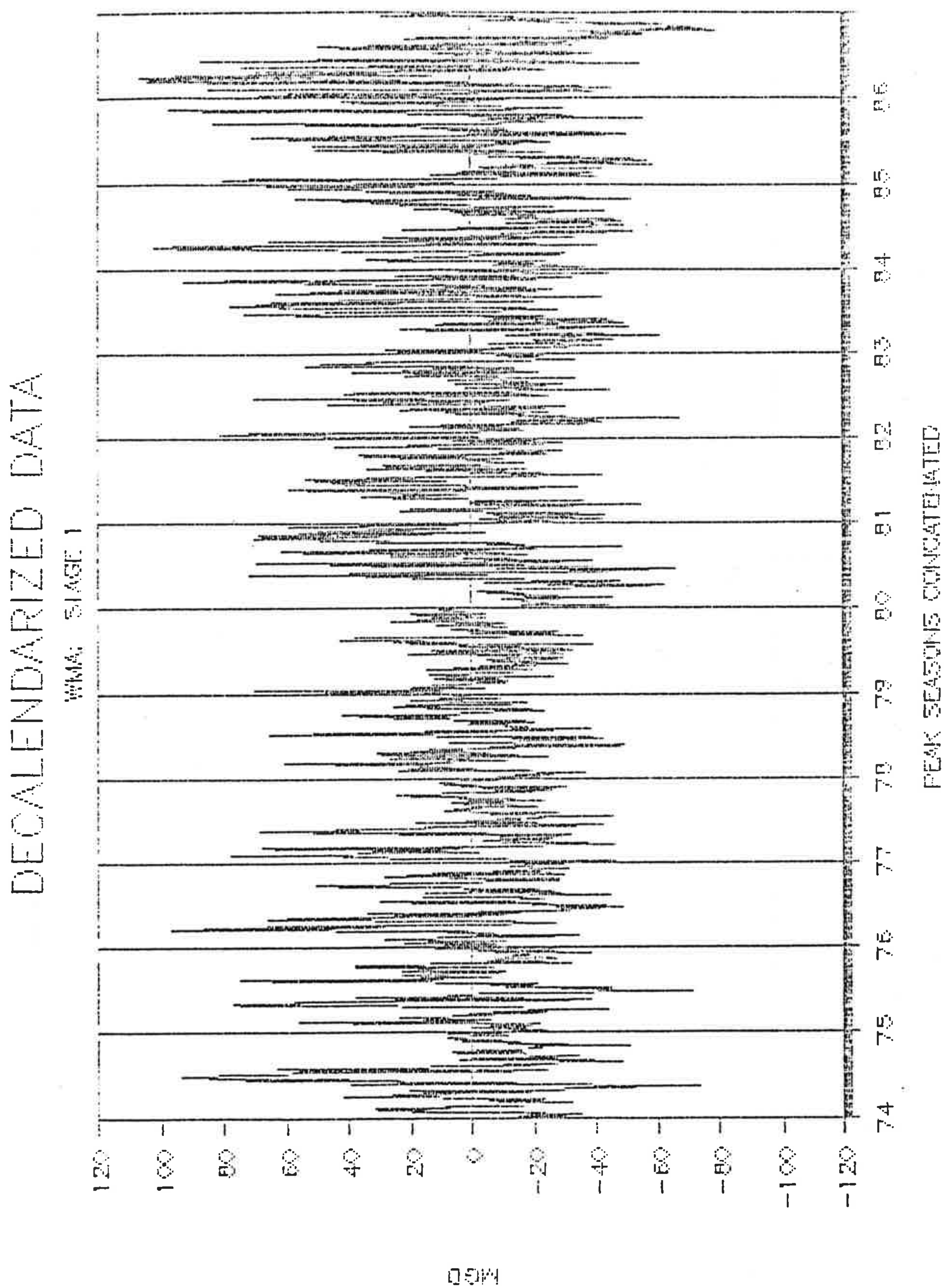

DECALENDARIZED DATA
WMA: STAGE 1
PEAK SEASONS CONCATENATED
MGD
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
74
75
76
77
78
79
80
81
82
83
84
85
86

FIGURE 1.6

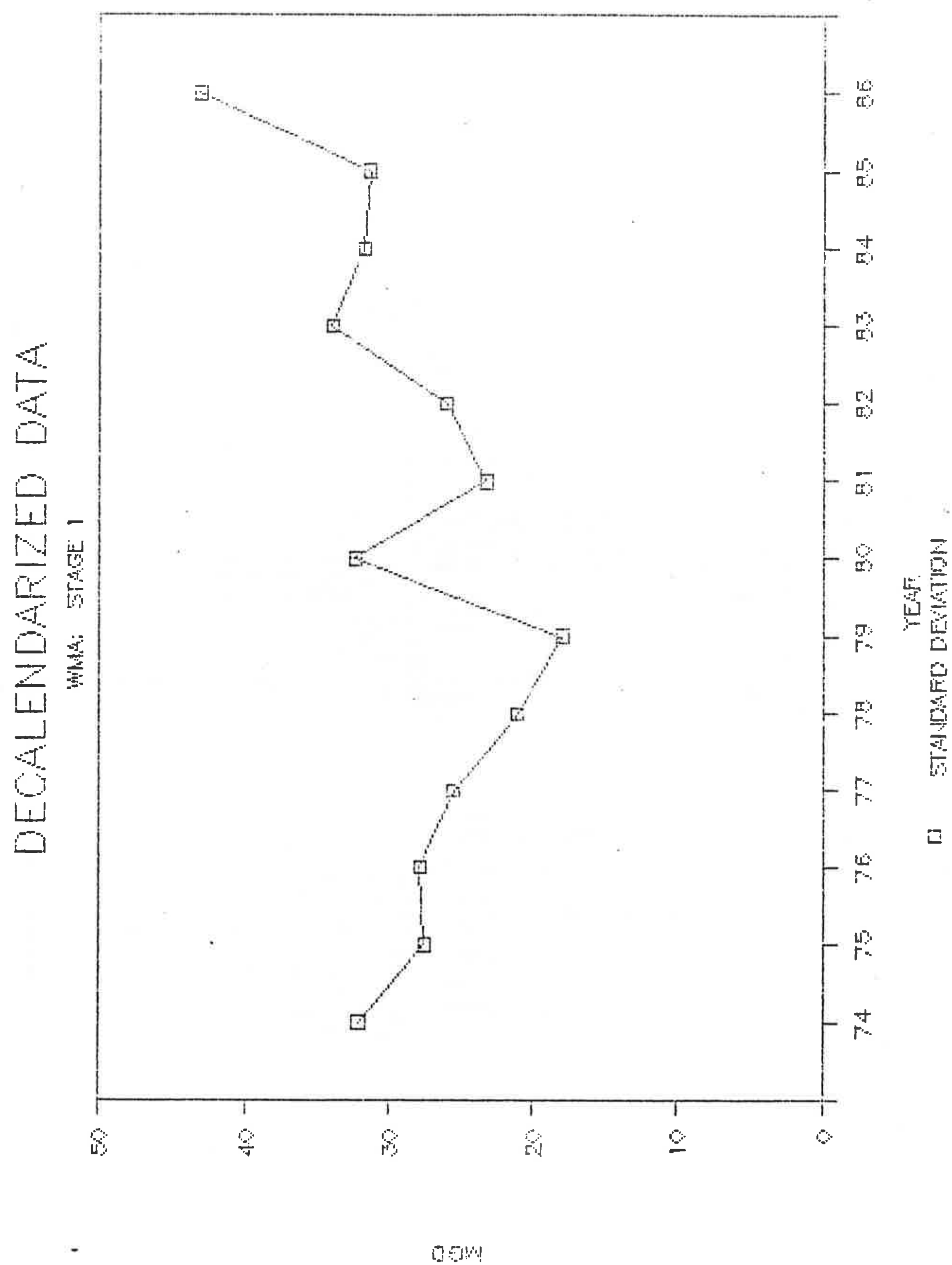

DECALENDARIZED DATA
WMA: STAGE 1
YEAR
STANDARD DEVIATION
MGD
0
10
20
30
40
50
74
75
76
77
78
79
80
81
82
83
84
85
86

FIGURE 1

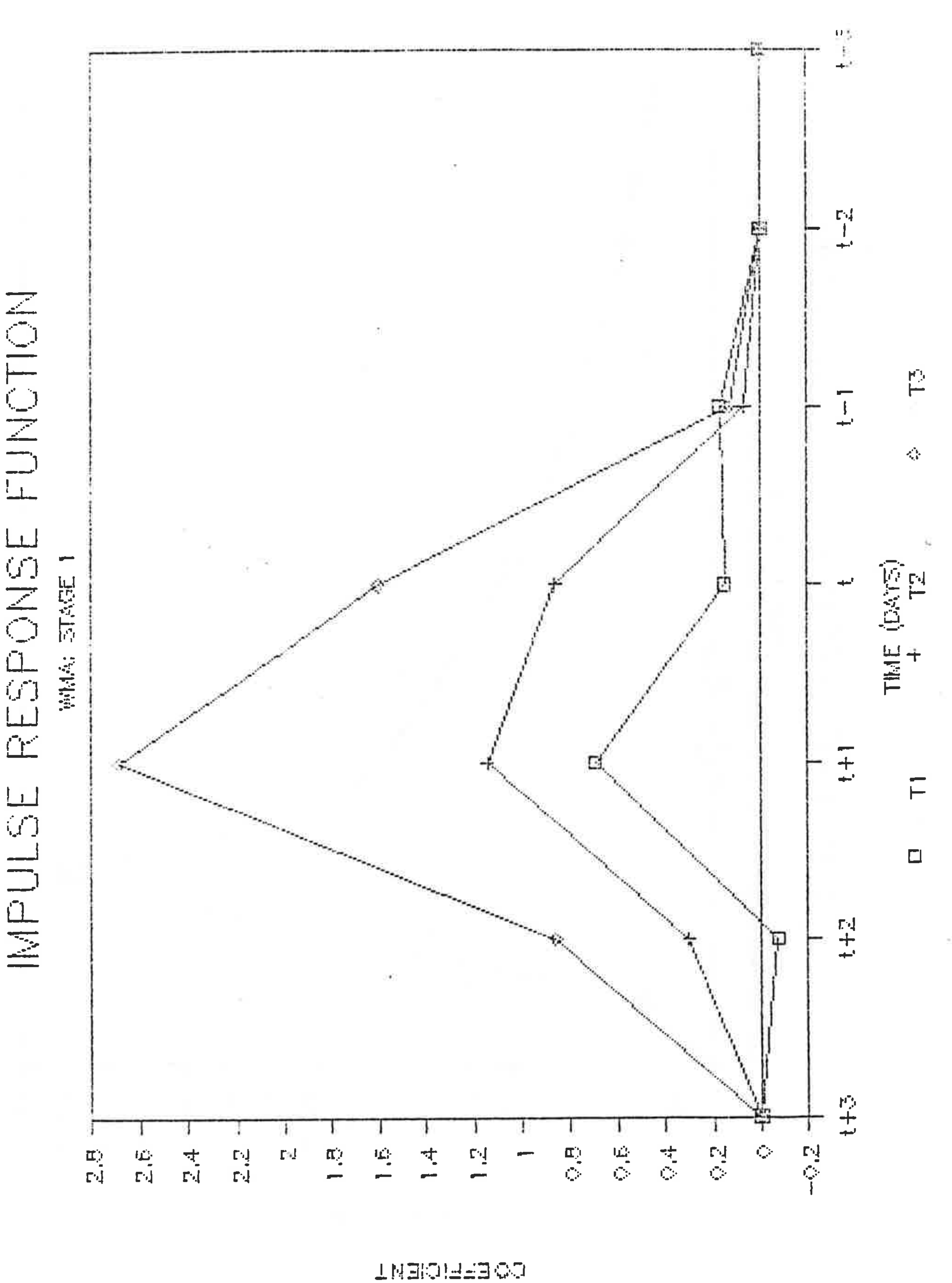

IMPULSE RESPONSE FUNCTION
WMA: STAGE 1
COEFFICIENT
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
t+3
t+2
t+1
t
t-1
t-2
t-3
TIME (DAYS)
T1
T2
T3

FIGURE 1.7

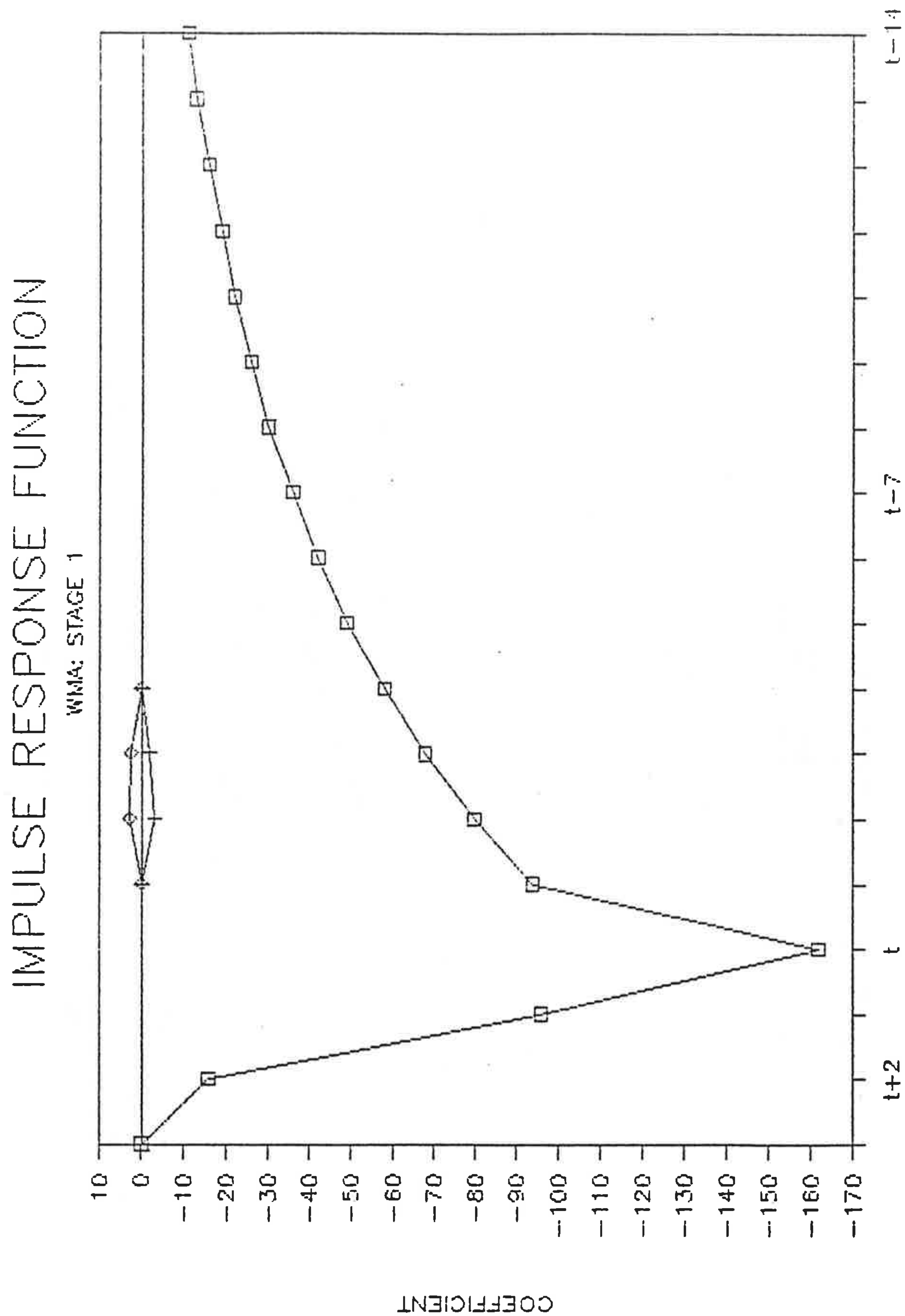

IMPULSE RESPONSE FUNCTION
WMA: STAGE 1
COEFFICIENT
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
-160
-170
t+2
t
t-7
t-14
TIME (DAYS)
□ P1
+ P2
◇ P3

FIGURE 1.7

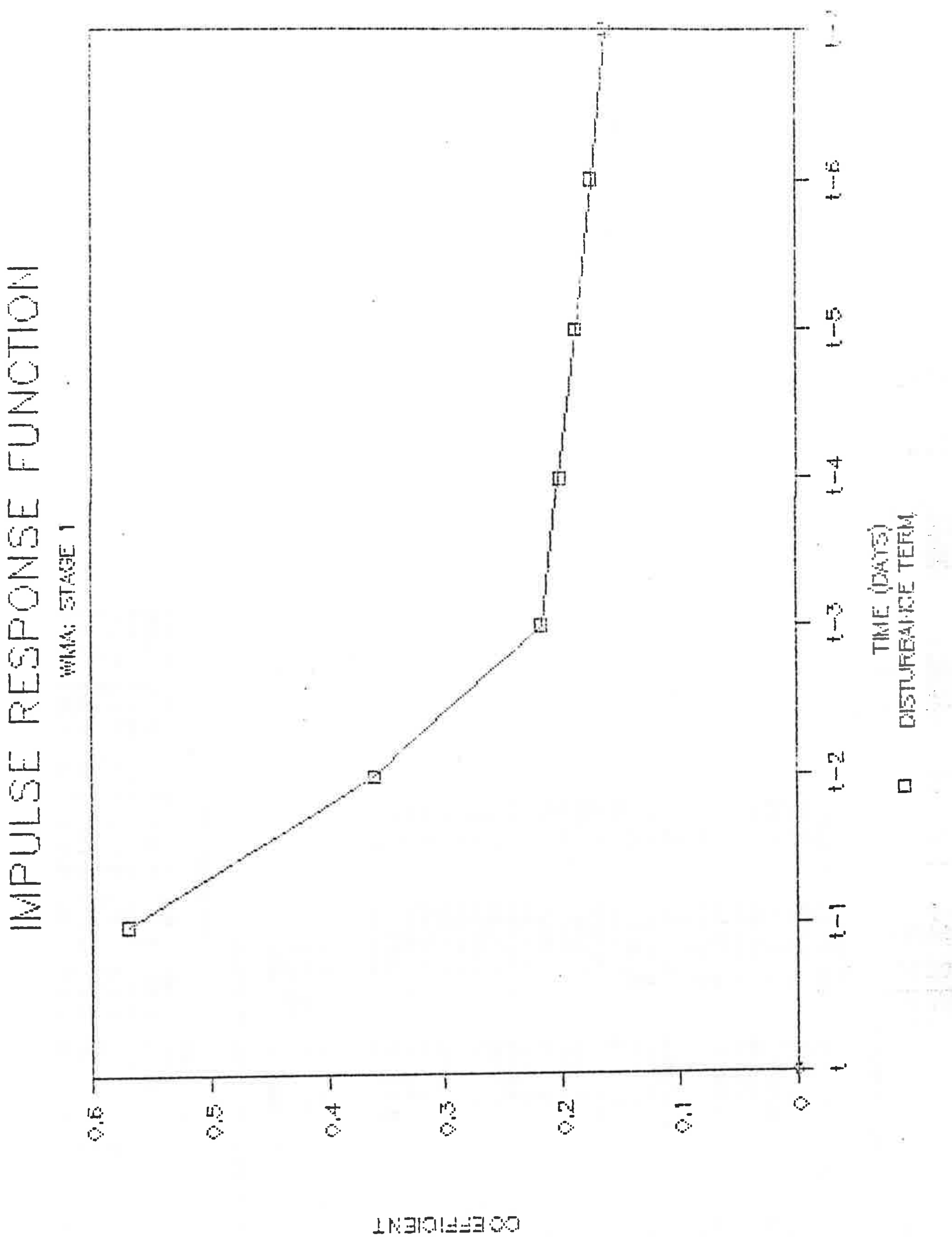

IMPULSE RESPONSE FUNCTION
TIME (DAYS)
DISTURBANCE TERM
COEFFICIENT
0.6
0.5
0.4
0.3
0.2
0.1
0
t
t-1
t-2
t-3
t-4
t-5
t-6

FIGURE 1.7

SAS

CAUSAL MODEL: STAGE 1

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO
MU	18.84301	3.86478	5.49
a1,2	-0.0716461	0.143438	-0.50
a1,1	0.686517	0.144904	4.74
a1,0	0.152488	0.145424	1.05
a1,-1	0.174464	0.142077	1.23
a2,2	0.296582	0.120049	2.47
a2,1	1.13637	0.121274	9.37
a2,0	0.857457	0.121067	7.08
a2,-1	0.0654944	0.11967	0.55
a3,2	0.863561	0.168307	5.13
a3,1	2.68785	0.168836	15.92
a3,0	1.60316	0.16902	9.49
a3,-1	0.125477	0.1668	0.75
n1	-15.8352	7.91997	-2.00
n2	82.8893	8.81833	9.40
n3	79.6489	9.24505	8.62
n3	-42.9704	12.8586	-3.34
d	0.854768	0.0263601	32.43
b2,-1	-3.11828	3.71556	-0.84
b2,-2	-1.85637	3.23256	-0.57
b3,-1	2.55633	1.46328	1.75
b3,-1	2.26958	1.45736	1.56
MA1	0.363038	0.0287288	12.64
MA2	0.169962	0.0268245	6.34
MA3	0.119232	0.0255387	4.67
AR1	0.933546	0.0163702	57.03

VARIANCE ESTIMATE = 212.759
STD ERROR ESTIMATE = 14.5863
AIC = 15659.5
NUMBER OF RESIDUALS= 1907

AUTOCORRELATION CHECK OF RESIDUALS

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	4.51	1	0.034	0.000	0.005	0.009	0.010	-0.043	-0.017
12	13.17	7	0.068	0.028	0.032	-0.047	-0.010	0.007	-0.017
18	22.96	13	0.042	0.002	0.052	0.021	-0.007	-0.039	-0.019
24	31.73	19	0.034	0.039	0.013	0.016	0.027	-0.016	-0.041
30	39.24	25	0.035	-0.028	-0.005	0.030	0.026	0.038	0.007
36	48.49	31	0.024	-0.013	-0.024	-0.045	0.011	0.032	-0.028
42	54.22	37	0.034	-0.012	0.032	-0.032	0.027	0.004	-0.002

FIGURE 1.8c

SAS

CAUSAL MODEL: STAGE 1

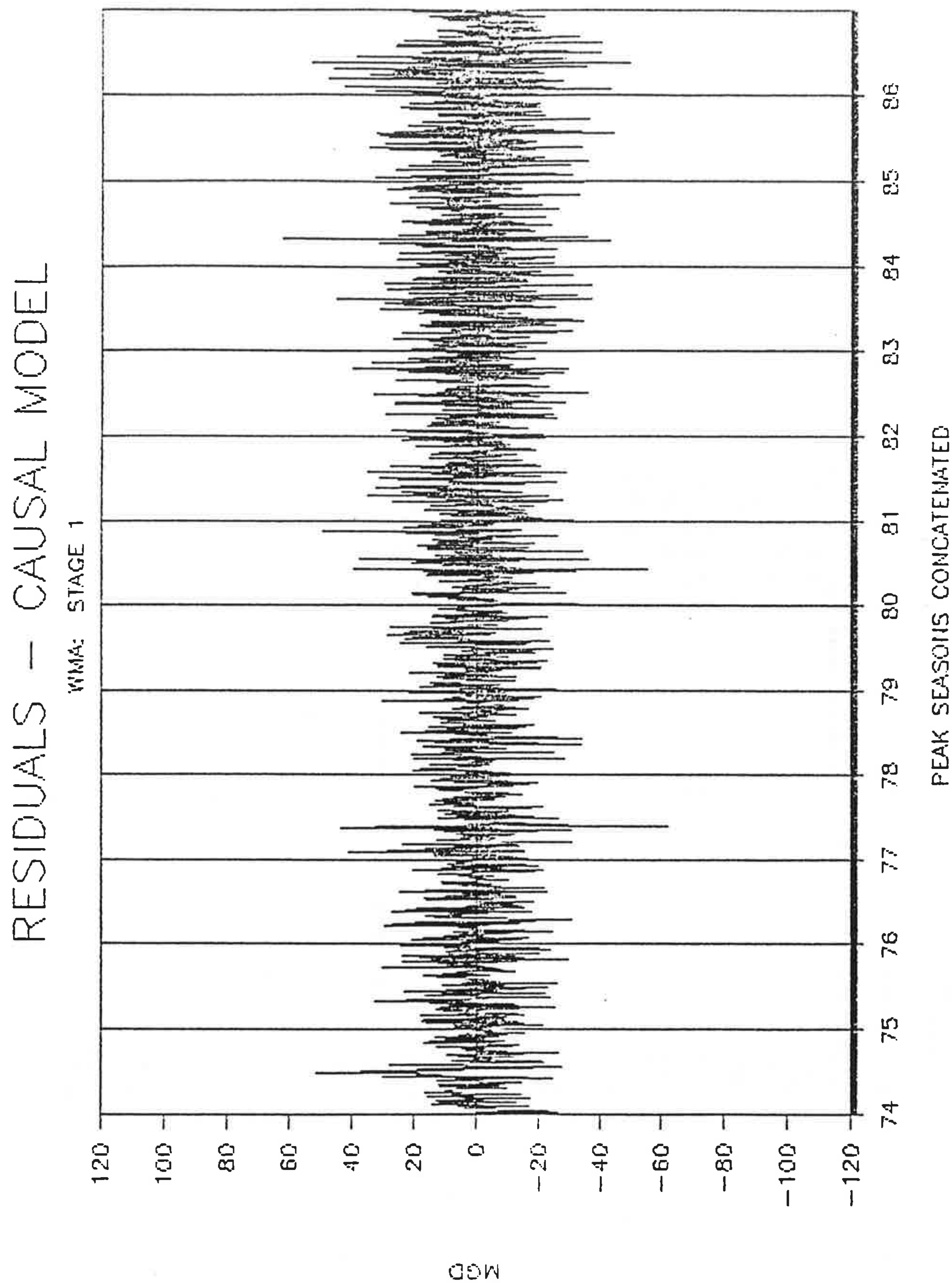
AUTOCORRELATION PLOT OF RESIDUALS

```
LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1      STD
  0    212.759     1.00000 |                    |********************|         0
  1  0.0841177     0.00040 |                   .|.                   | 0.0228994
  2    1.13864     0.00535 |                   .|.                   | 0.0228994
  3    1.94418     0.00914 |                   .|.                   | 0.0229001
  4    2.08173     0.00978 |                   .|.                   |  0.022902
  5   -9.21101    -0.04329 |                   *|.                   | 0.0229042
  6   -3.51789    -0.01653 |                   .|.                   | 0.0229471
  7    5.87127     0.02760 |                   .|*                   | 0.0229533
  8    6.90583     0.03246 |                   .|*                   | 0.0229707
  9   -10.0951    -0.04745 |                   *|.                   | 0.0229947
 10   -2.21873    -0.01043 |                   .|.                   |  0.023046
 11    1.45034     0.00682 |                   .|.                   | 0.0230485
 12   -3.65302    -0.01717 |                   .|.                   | 0.0230496
 13   0.385543     0.00181 |                   .|.                   | 0.0230563
 14    10.9994     0.05170 |                   .|*                   | 0.0230563
 15    4.53971     0.02134 |                   .|.                   |  0.023117
 16   -1.46256    -0.00687 |                   .|.                   | 0.0231274
 17   -8.32564    -0.03913 |                   *|.                   | 0.0231284
 18   -4.13766    -0.01945 |                   .|.                   | 0.0231631
 19    8.23746     0.03872 |                   .|*                   | 0.0231717
 20    2.70293     0.01270 |                   .|.                   | 0.0232056
 21    3.48548     0.01638 |                   .|.                   | 0.0232092
 22    5.66177     0.02661 |                   .|*                   | 0.0232153
 23   -3.39833    -0.01597 |                   .|.                   | 0.0232313
 24   -8.63238    -0.04057 |                   *|.                   | 0.0232371
                           '.' MARKS TWO STANDARD ERRORS
```

PARTIAL AUTOCORRELATIONS

```
LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.00040 |                   .|.                   |
  2     0.00535 |                   .|.                   |
  3     0.00913 |                   .|.                   |
  4     0.00975 |                   .|.                   |
  5    -0.04341 |                   *|.                   |
  6    -0.01672 |                   .|.                   |
  7     0.02797 |                   .|*                   |
  8     0.03345 |                   .|*                   |
  9    -0.04684 |                   *|.                   |
 10    -0.01305 |                   .|.                   |
 11     0.00501 |                   .|.                   |
 12    -0.01457 |                   .|.                   |
 13     0.00658 |                   .|.                   |
 14     0.04859 |                   .|*                   |
 15     0.01717 |                   .|.                   |
 16    -0.00607 |                   .|.                   |
 17    -0.03832 |                   *|.                   |
 18    -0.02270 |                   .|.                   |
 19     0.04357 |                   .|*                   |
 20     0.01897 |                   .|.                   |
 21     0.01259 |                   .|.                   |
 22     0.01807 |                   .|.                   |
 23    -0.01683 |                   .|.                   |
 24    -0.03456 |                   *|.                   |
```


FIGURE 1.8b



RESIDUALS — CAUSAL MODEL
WMA: STAGE 1
PEAK SEASONS CONCATENATED
MGD
120
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
74
75
76
77
78
79
80
81
82
83
84
85
86

FIGURE 1.9

RESIDUALS — CAUSAL MODEL

WMA: STAGE 1

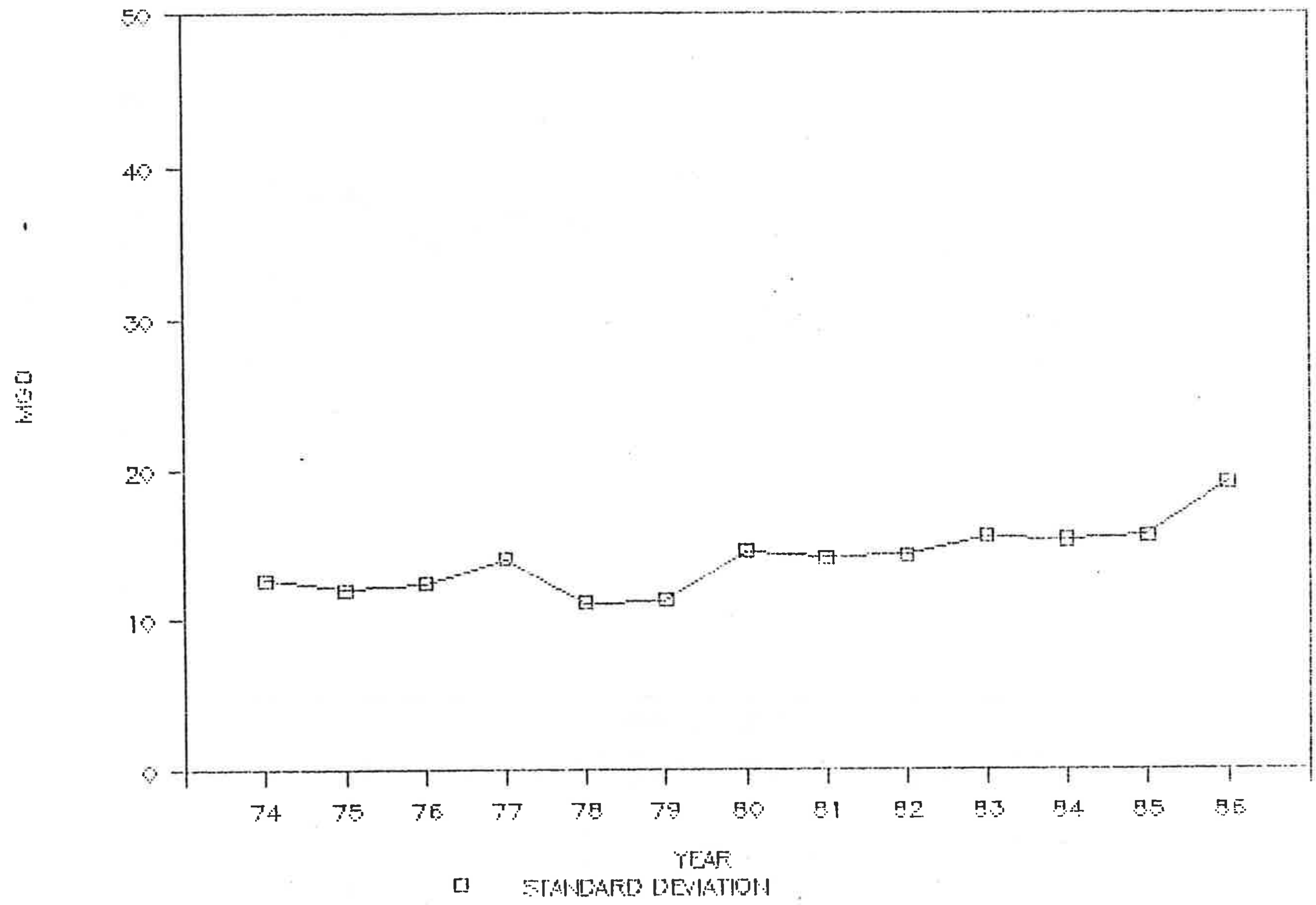
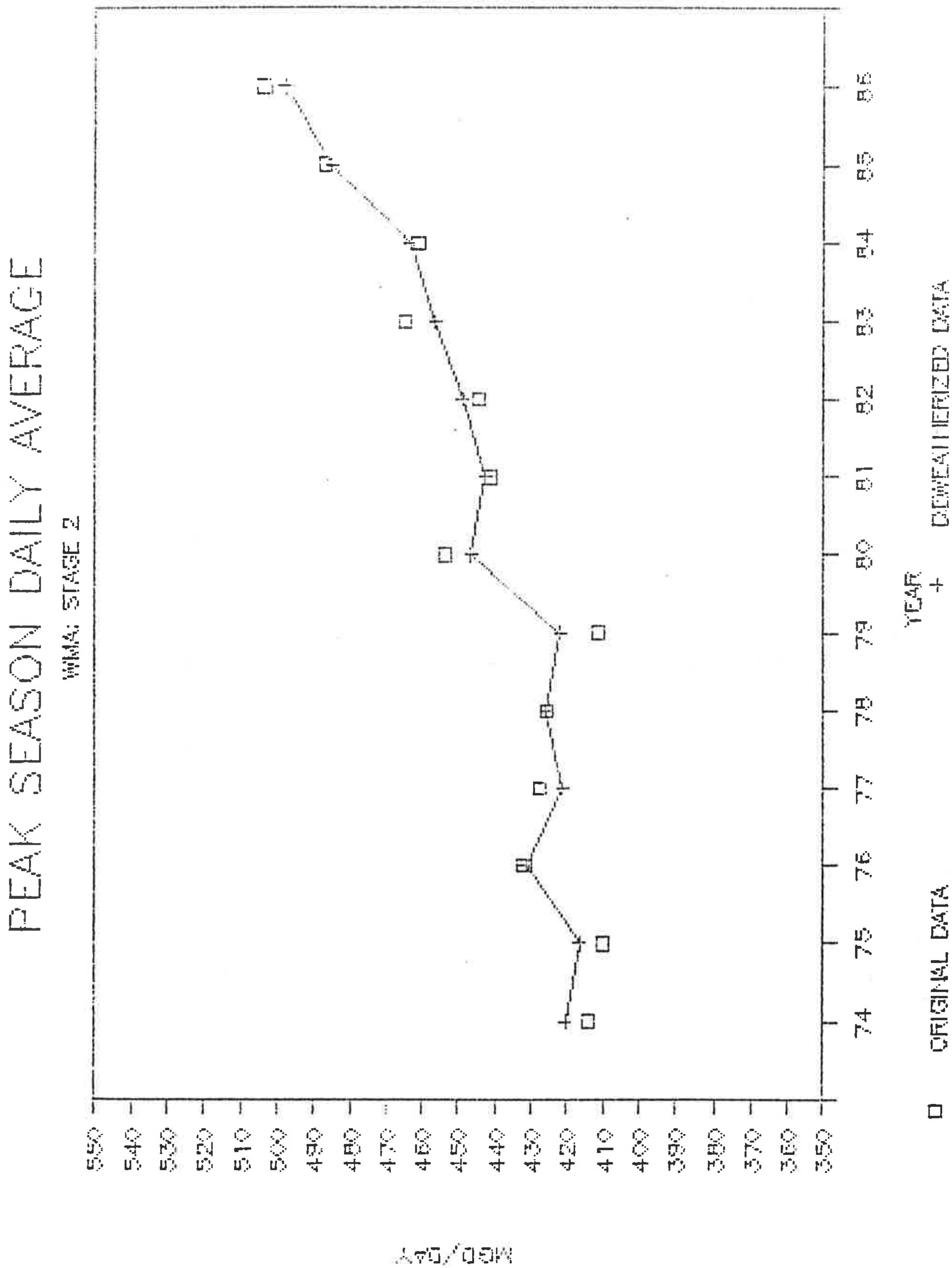


FIGURE 1.9



PEAK SEASON DAILY AVERAGE
WMA: STAGE 2
MGD/DAY
550
540
530
520
510
500
490
480
470
460
450
440
430
420
410
400
390
380
370
360
350
YEAR
74
75
76
77
78
79
80
81
82
83
84
85
86
ORIGINAL DATA
DEWEATHERIZED DATA

FIGURE 1.

WEATHER INFLUENCES ON ORIGINAL PEAK SEASON DAILY AVERAGES: STAGE 1

OBS	YEAR	ORIGINAL YI	TEMP	PREC	INTERCEPT	TOTAL WEATHER	DEWEATHERIED YI
1	74	414.182	1.1624	-26.760	18.843	-6.325	420.506
2	75	410.036	4.7125	-30.426	18.843	-6.863	416.900
3	76	432.045	3.8684	-21.870	18.843	0.848	431.197
4	77	427.733	11.7101	-23.998	18.843	6.562	421.172
5	78	425.651	6.1416	-25.387	18.843	-0.396	426.047
6	79	411.232	-1.9565	-28.253	18.843	-11.359	422.591
7	80	454.109	13.7012	-24.929	18.843	7.622	446.487
8	81	441.224	2.6944	-22.719	18.843	-1.175	442.399
9	82	444.958	2.0583	-24.926	18.843	-4.018	448.976
10	83	465.015	12.7560	-22.478	18.843	9.128	455.887
11	84	461.032	4.2733	-25.888	18.843	-2.765	463.797
12	85	487.046	6.4601	-23.242	18.843	2.068	484.978
13	86	503.937	8.8432	-20.832	18.843	6.870	497.067

FIGURE 1.10b

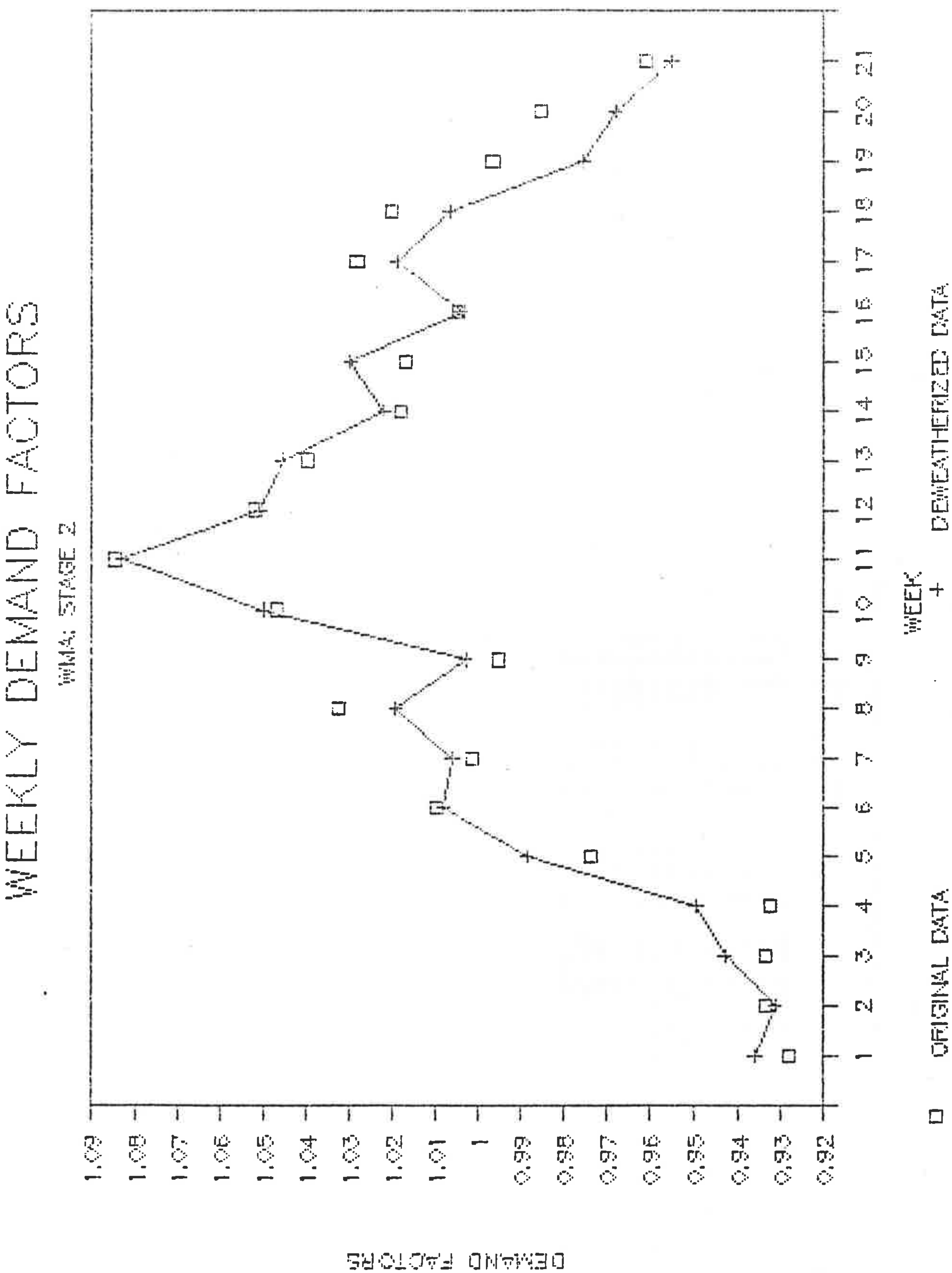

WEEKLY DEMAND FACTORS
WMA: STAGE 2
DEMAND FACTORS
1.09
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
WEEK
ORIGINAL DATA
DEWEATHERIZED DATA

FIGURE 1.11

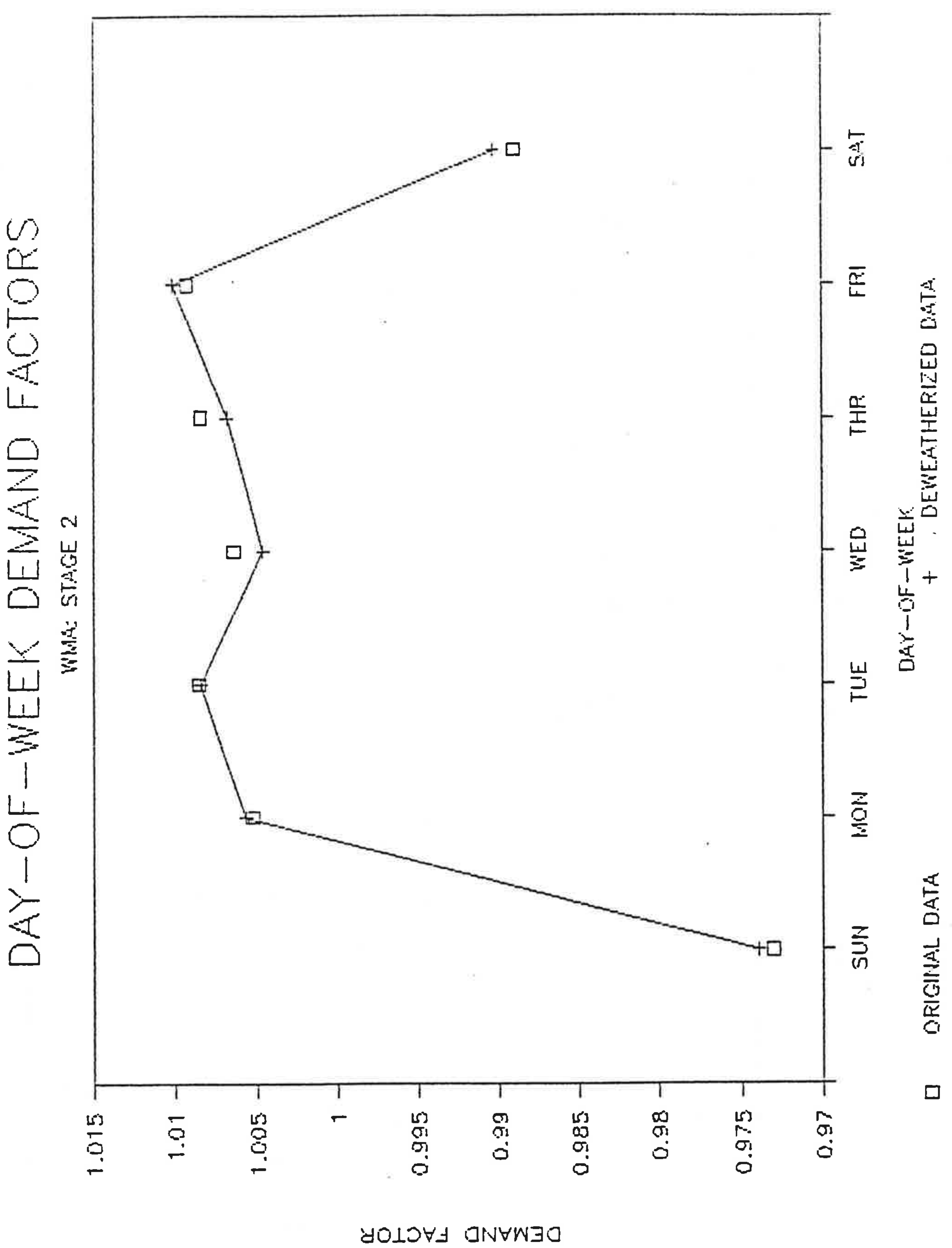

DAY-OF-WEEK DEMAND FACTORS
WMA: STAGE 2
DEMAND FACTOR
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
0.975
0.97
SUN
MON
TUE
WED
THR
FRI
SAT
DAY-OF-WEEK
ORIGINAL DATA
+ DEWEATHERIZED DATA

FIGURE 1.1

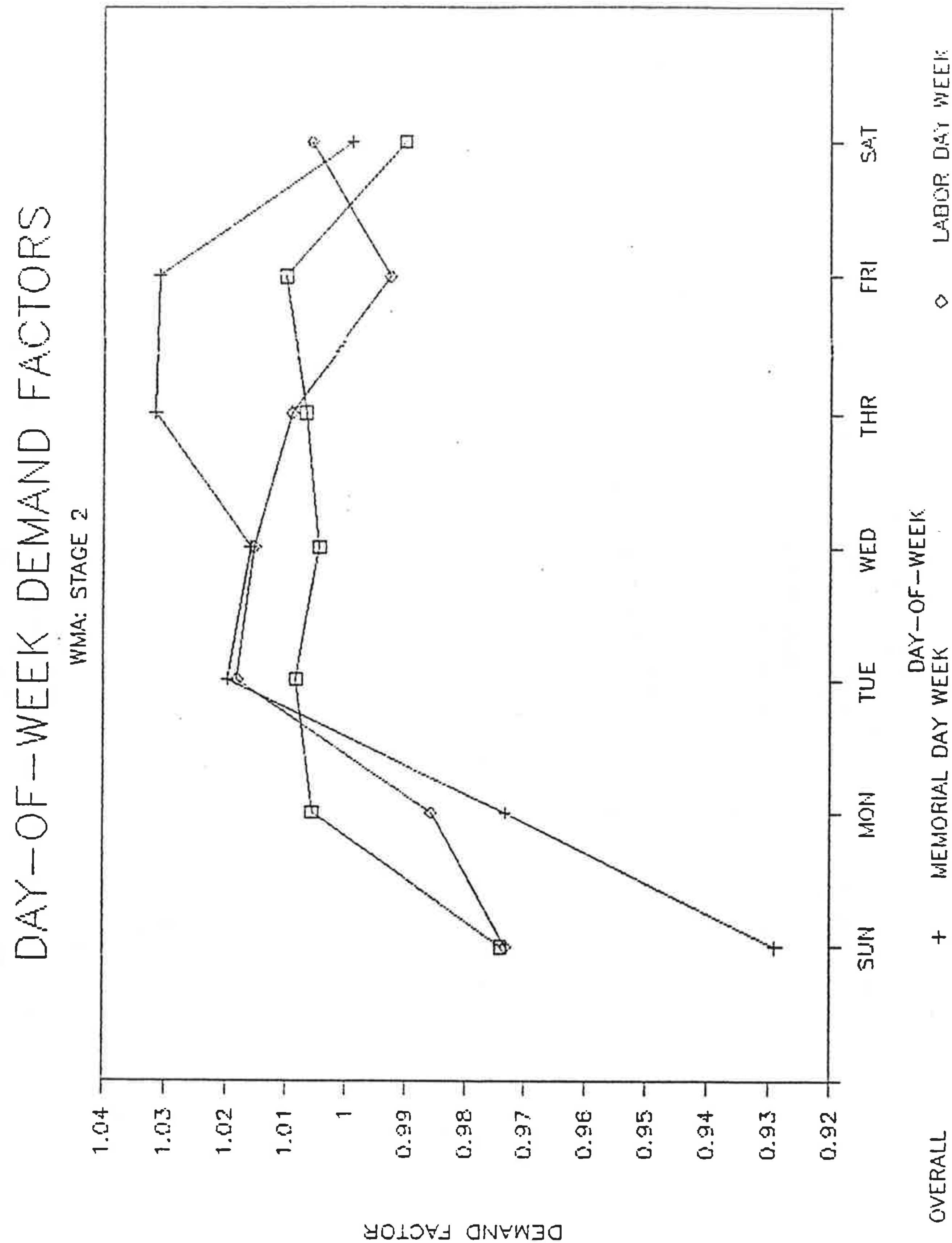
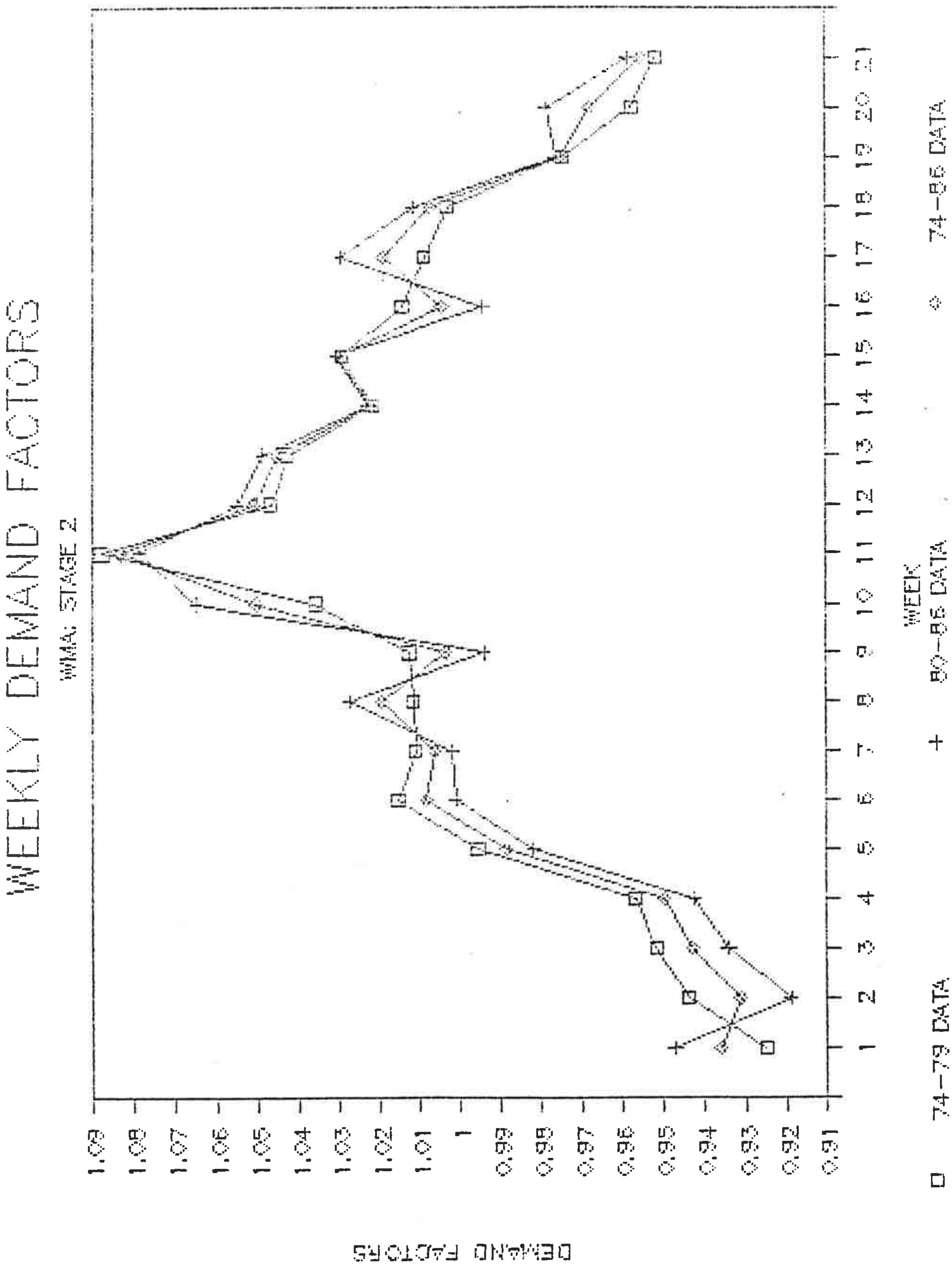

DAY-OF-WEEK DEMAND FACTORS
WMA: STAGE 2
DEMAND FACTOR
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
SUN
MON
TUE
WED
THR
FRI
SAT
DAY-OF-WEEK
OVERALL
+ MEMORIAL DAY WEEK
◇ LABOR DAY WEEK

FIGURE 1.



WEEKLY DEMAND FACTORS
WMA: STAGE 2
DEMAND FACTORS
WEEK
1.09 1.08 1.07 1.06 1.05 1.04 1.03 1.02 1.01 1 0.99 0.98 0.97 0.96 0.95 0.94 0.93 0.92 0.91
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
□ 74-79 DATA
+ 80-86 DATA
◇ 74-86 DATA

FIGURE 1.1

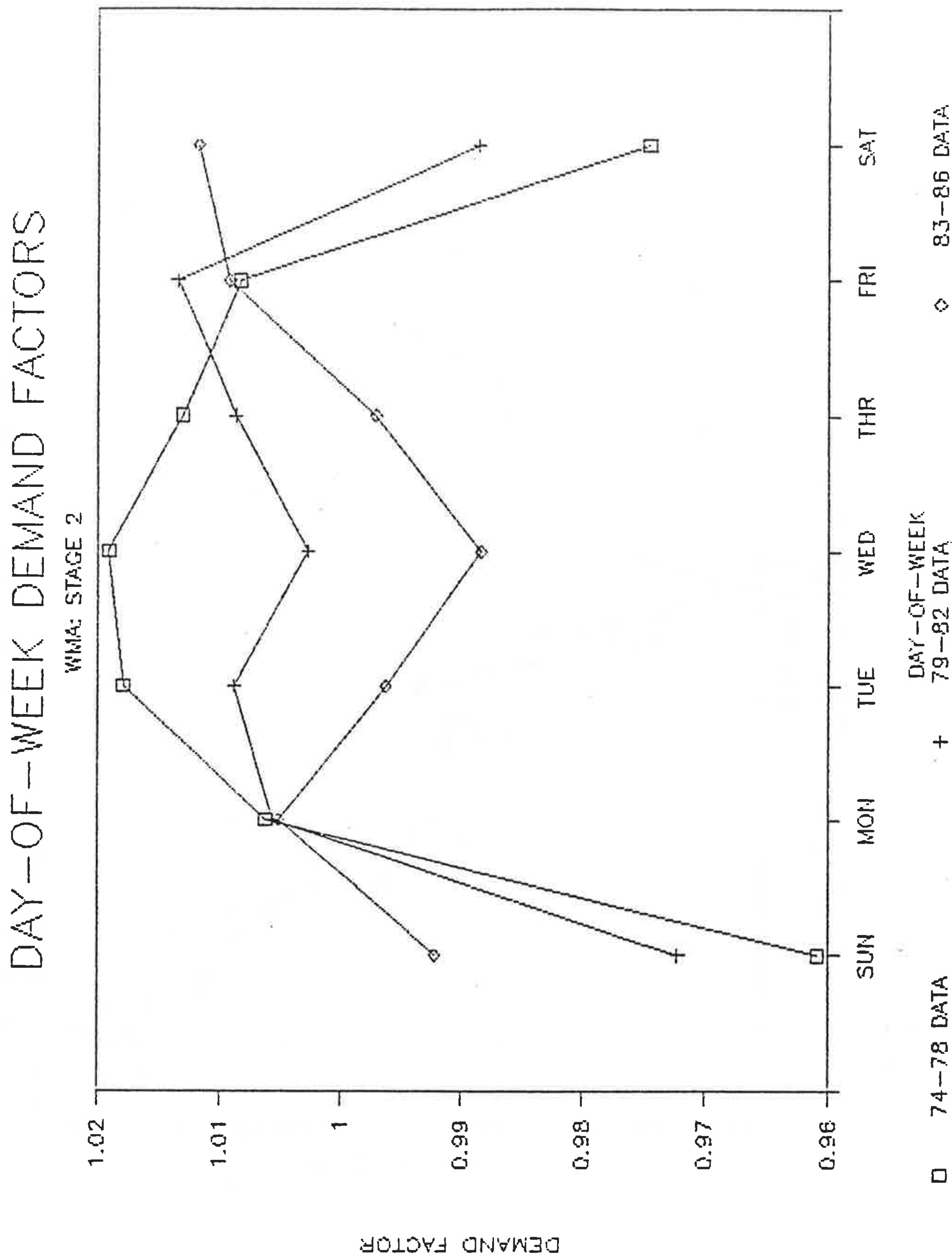

DAY-OF-WEEK DEMAND FACTORS
WMA: STAGE 2
DEMAND FACTOR
1.02
1.01
1
0.99
0.98
0.97
0.96
SUN
MON
TUE
WED
THR
FRI
SAT
DAY-OF-WEEK
74-78 DATA
79-82 DATA
83-86 DATA

FIGURE 1.1

CALENDAR ESTIMATES: STAGE 2

YEAR	YI	WEEK	WJ
74	420.51	1	0.938
75	416.90	2	0.934
76	431.20	3	0.945
77	421.17	4	0.951
78	426.05	5	0.992
79	422.59	6	1.009
80	446.49	7	1.006
81	442.40	8	1.019
82	448.98	9	1.000
83	455.89	10	1.051
84	463.80	11	1.083
85	484.98	12	1.051
86	497.07	13	1.046
		14	1.022
		15	1.031
		16	1.006
		17	1.018
		18	1.005
		19	0.973
		20	0.966
		21	0.954

			Dk		
DAY-OF-WEEK	WEEK4	WEEK18	1974-78	1979-82	1983-86
SUN	0.929	0.973	0.961	0.972	0.992
MON	0.973	0.986	1.006	1.006	1.005
TUE	1.020	1.018	1.018	1.009	0.996
WED	1.016	1.015	1.019	1.003	0.988
THR	1.031	1.009	1.013	1.009	0.997
FRI	1.031	0.993	1.008	1.013	1.009
SAT	0.999	1.005	0.975	0.988	1.012

FIGURE 1.16

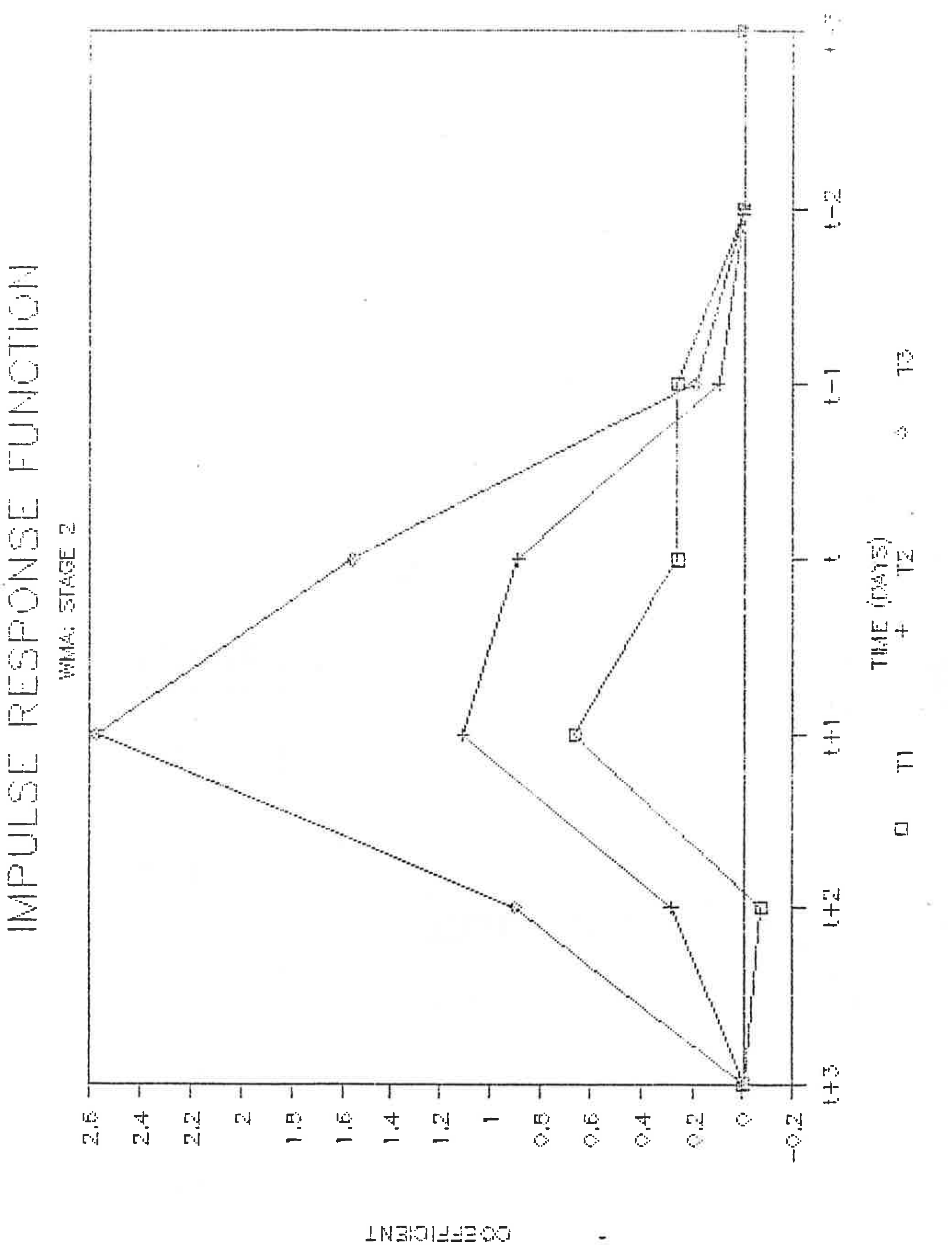

IMPULSE RESPONSE FUNCTION
WMA: STAGE 2
COEFFICIENT
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
t+3
t+2
t+1
t
t-1
t-2
t-3
TIME (DAYS)
T1
T2
T3

FIGURE 1.1

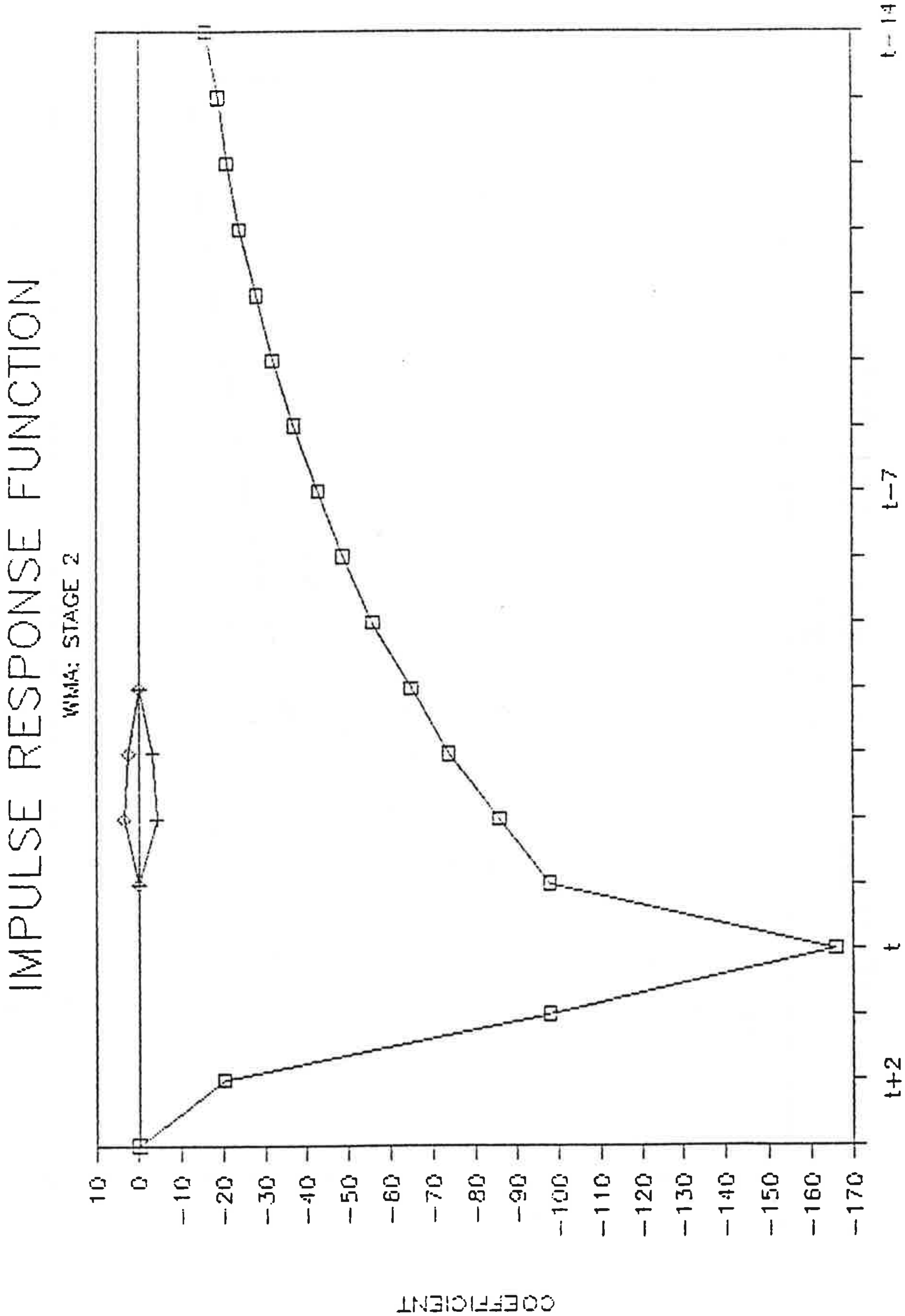

IMPULSE RESPONSE FUNCTION
WMA: STAGE 2
COEFFICIENT
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
-150
-160
-170
t+2
t
t-7
t-14
TIME (DAYS)
P1
P2
P3

FIGURE 1.17b

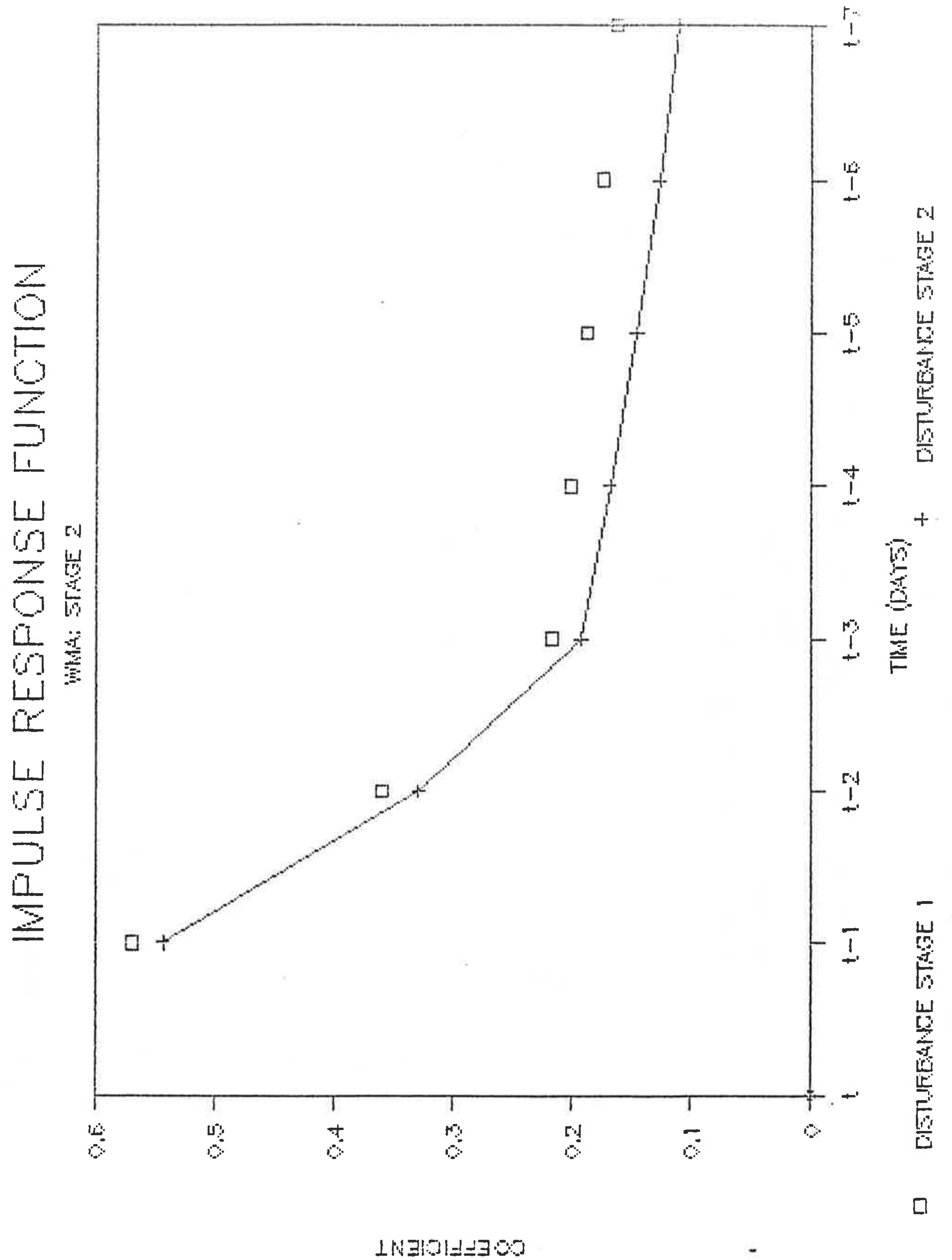

IMPULSE RESPONSE FUNCTION
WMA: STAGE 2
COEFFICIENT
0.6
0.5
0.4
0.3
0.2
0.1
0
t
t-1
t-2
t-3
t-4
t-5
t-6
t-7
TIME (DAYS)
DISTURBANCE STAGE 1
DISTURBANCE STAGE 2

FIGURE 1.17c

SAS

CAUSAL MODEL: STAGE 2

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO
MU	26.0749	3.49391	7.46
a1,2	-0.0741964	0.139746	-0.53
a1,1	0.658304	0.14037	4.69
a1,0	0.257834	0.140782	1.83
a1,-1	0.258662	0.138335	1.87
a2,2	0.275693	0.116343	2.37
a2,1	1.11411	0.116679	9.55
a2,0	0.89452	0.116419	7.68
a2,-1	0.102244	0.116012	0.88
a3,2	0.900552	0.16424	5.48
a3,1	2.58086	0.164585	15.68
a3,0	1.56113	0.164955	9.46
a3,-1	0.188694	0.163051	1.16
n1	-20.1813	7.73615	-2.61
n2	80.2853	8.76636	9.16
n3	80.2681	9.04685	8.87
n4	-45.5912	11.7701	-3.87
d	0.875217	0.0191	45.82
b2,-1	-4.40005	3.57555	-1.23
b2,-2	-3.16966	3.16762	-1.00
b3,-1	3.37802	1.44185	2.34
b3,-2	2.29669	1.43648	1.60
MA1	0.325669	0.0399711	8.15
MA2	0.144429	0.0311672	4.63
MA3	0.0938804	0.0283215	3.31
AR1	0.872233	0.0316812	27.53

VARIANCE ESTIMATE = 202.839
STD ERROR ESTIMATE = 14.2422
AIC = 15568.4
NUMBER OF RESIDUALS= 1907

AUTOCORRELATION CHECK OF RESIDUALS

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	2.01	1	0.156	-0.001	0.001	0.001	0.017	-0.027	-0.005
12	10.56	7	0.159	0.023	0.023	-0.054	-0.013	0.007	-0.017
18	19.43	13	0.110	0.014	0.046	0.027	0.002	-0.038	-0.010
24	27.43	19	0.095	0.037	0.010	-0.001	0.029	-0.017	-0.040
30	33.84	25	0.111	-0.036	0.010	0.027	0.004	0.033	-0.008
36	42.64	31	0.080	-0.028	-0.024	-0.038	-0.011	0.027	-0.030
42	49.31	37	0.085	-0.012	0.034	-0.034	0.026	-0.001	-0.017

FIGURE 1.1

SAS

CAUSAL MODEL: STAGE 2

AUTOCORRELATION PLOT OF RESIDUALS

```
LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1         STD
  0    202.839    1.00000  |                    |********************|          0
  1  -0.188914   -0.00093  |                   .|.                   |  0.0228994
  2   0.182425    0.00090  |                   .|.                   |  0.0228994
  3   0.126028    0.00062  |                   .|.                   |  0.0228995
  4    3.41785    0.01685  |                   .|.                   |  0.0228995
  5   -5.53619   -0.02729  |                   *|.                   |   0.022906
  6  -0.934202   -0.00461  |                   .|.                   |   0.022923
  7    4.64053    0.02288  |                   .|.                   |  0.0229235
  8    4.69929    0.02317  |                   .|.                   |  0.0229355
  9    -10.883   -0.05365  |                   *|.                   |  0.0229477
 10    -2.6139   -0.01289  |                   .|.                   |  0.0230134
 11    1.50891    0.00744  |                   .|.                   |  0.0230172
 12   -3.49599   -0.01724  |                   .|.                   |  0.0230185
 13    2.90187    0.01431  |                   .|.                   |  0.0230252
 14    9.25502    0.04563  |                   .|*                   |  0.0230299
 15    5.57345    0.02748  |                   .|*                   |  0.0230773
 16   0.486282    0.00240  |                   .|.                   |  0.0230944
 17   -7.75336   -0.03822  |                   *|.                   |  0.0230945
 18   -2.00883   -0.00990  |                   .|.                   |  0.0231277
 19    7.50723    0.03701  |                   .|*                   |  0.0231299
 20    1.97048    0.00971  |                   .|.                   |  0.0231609
 21  -0.135488   -0.00067  |                   .|.                   |  0.0231631
 22    5.81273    0.02866  |                   .|*                   |  0.0231631
 23   -3.35711   -0.01655  |                   .|.                   |  0.0231817
 24   -8.07357   -0.03980  |                   *|.                   |  0.0231879
                           '.' MARKS TWO STANDARD ERRORS
```

PARTIAL AUTOCORRELATIONS

```
           LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
             1   -0.00093  |                   .|.                   |
             2    0.00090  |                   .|.                   |
             3    0.00062  |                   .|.                   |
             4    0.01685  |                   .|.                   |
             5   -0.02727  |                   *|.                   |
             6   -0.00468  |                   .|.                   |
             7    0.02292  |                   .|.                   |
             8    0.02299  |                   .|.                   |
             9   -0.05286  |                   *|.                   |
            10   -0.01368  |                   .|.                   |
            11    0.00666  |                   .|.                   |
            12   -0.01669  |                   .|.                   |
            13    0.01751  |                   .|.                   |
            14    0.04321  |                   .|*                   |
            15    0.02514  |                   .|*                   |
            16    0.00486  |                   .|.                   |
            17   -0.03698  |                   *|.                   |
            18   -0.01350  |                   .|.                   |
            19    0.03819  |                   .|*                   |
            20    0.01245  |                   .|.                   |
            21   -0.00361  |                   .|.                   |
            22    0.02521  |                   .|*                   |
            23   -0.01486  |                   .|.                   |
            24   -0.03429  |                   *|                    |
```


FIGURE 1.18b

SAS

CAUSAL MODEL: STAGE 2 - BELTSVILLE WEATHER ONLY

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO
MU	23.2808	3.25385	7.15
MA1	0.320526	0.03905	8.21
MA2	0.154498	0.0309459	4.99
MA3	0.0925817	0.0280984	3.29
AR1	0.87612	0.0305607	28.67
a1,2	-0.0122569	0.136164	-0.09
a1,1	0.676425	0.136743	4.95
a1,0	0.326193	0.137244	2.38
a1,-1	0.226998	0.134771	1.68
a2,2	0.297521	0.11744	2.53
a2,1	1.14482	0.117551	9.74
a2,0	0.895665	0.117212	7.64
a2,-1	0.0963433	0.116696	0.83
a3,2	0.947204	0.164022	5.77
a3,1	2.57575	0.164728	15.64
a3,0	1.38982	0.164998	8.42
a3,-1	0.243091	0.163606	1.49
n1	-28.4576	7.82613	-3.64
n2	65.6632	8.76198	7.49
n3	74.5303	9.04864	8.24
n4	-36.8439	11.6737	-3.16
d	0.876445	0.0193245	45.35
b2,-1	-3.07789	3.63145	-0.85
b2,-2	-2.39847	3.24862	-0.74
b3,-1	2.77215	1.33606	2.07
b2,-2	1.12375	1.33306	0.84

VARIANCE ESTIMATE = 207.824
STD ERROR ESTIMATE = 14.4161
AIC = 15614.7

FIGURE 1.

SAS

CAUSAL MODEL: STAGE 2 - NO LEADING VARIABLES

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO
MU	24.5359	4.10747	5.97
a1,0	0.282493	0.1606	1.76
a1,-1	0.14394	0.160754	0.90
a2,0	0.906499	0.139405	6.50
a2,-1	0.0462948	0.138664	0.33
a3,0	1.30248	0.204679	6.36
a3,-1	-0.207954	0.201674	-1.03
n3	-160.059	9.4637	-16.91
n4	-34.4891	11.8655	-2.91
d	0.856977	0.026814	31.96
MA1	0.188687	0.0449713	4.20
MA2	0.14521	0.0374375	3.88
MA3	0.0376651	0.0292111	1.29
AR1	0.828995	0.0347801	23.84

VARIANCE ESTIMATE = 282.513
STD ERROR ESTIMATE = 16.8081
AIC = 16213.8
NUMBER OF RESIDUALS= 1910

FIGURE 1.20

IMPULSE RESPONSE FUNCTION

WMA: STAGE 2

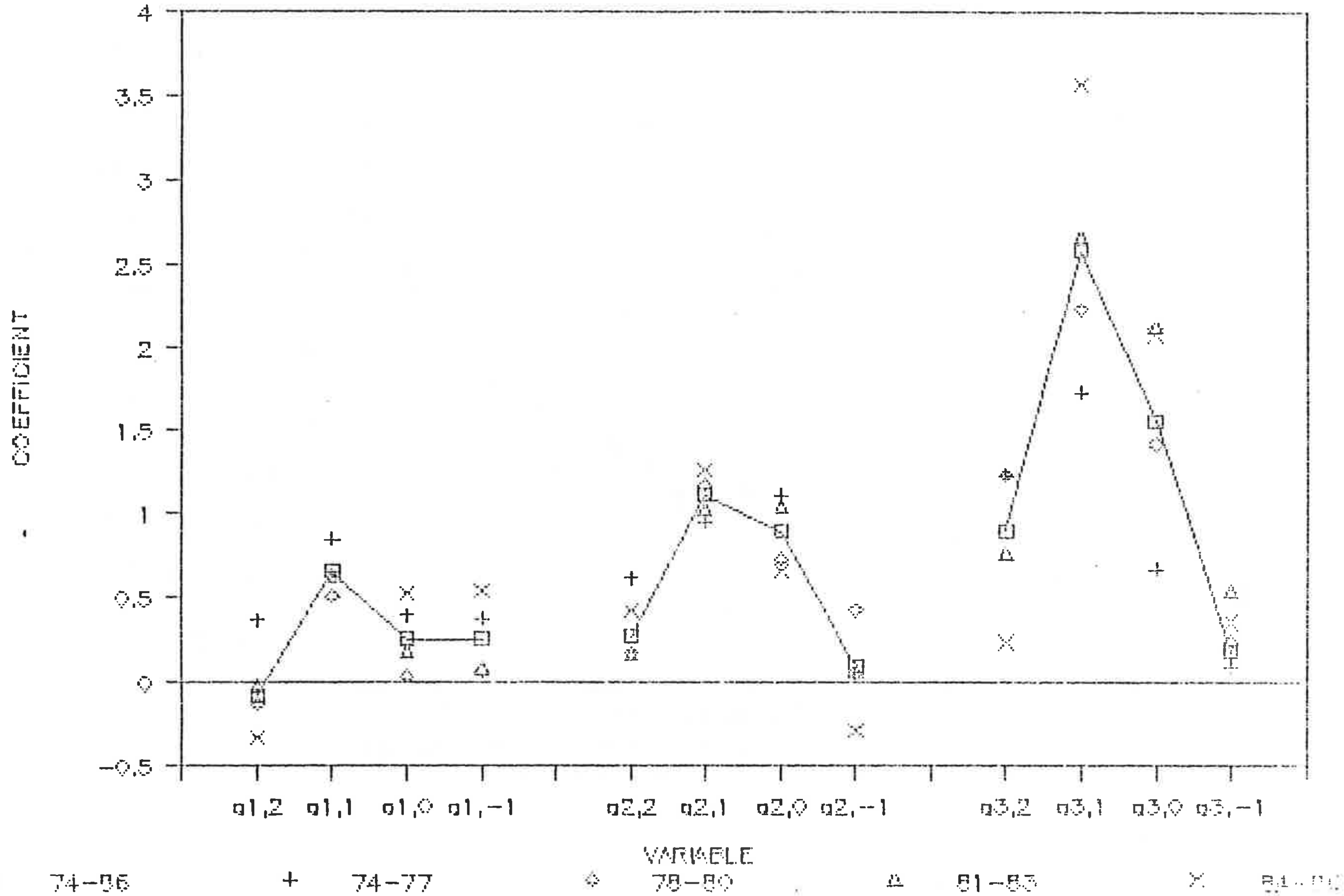


FIGURE 1.2

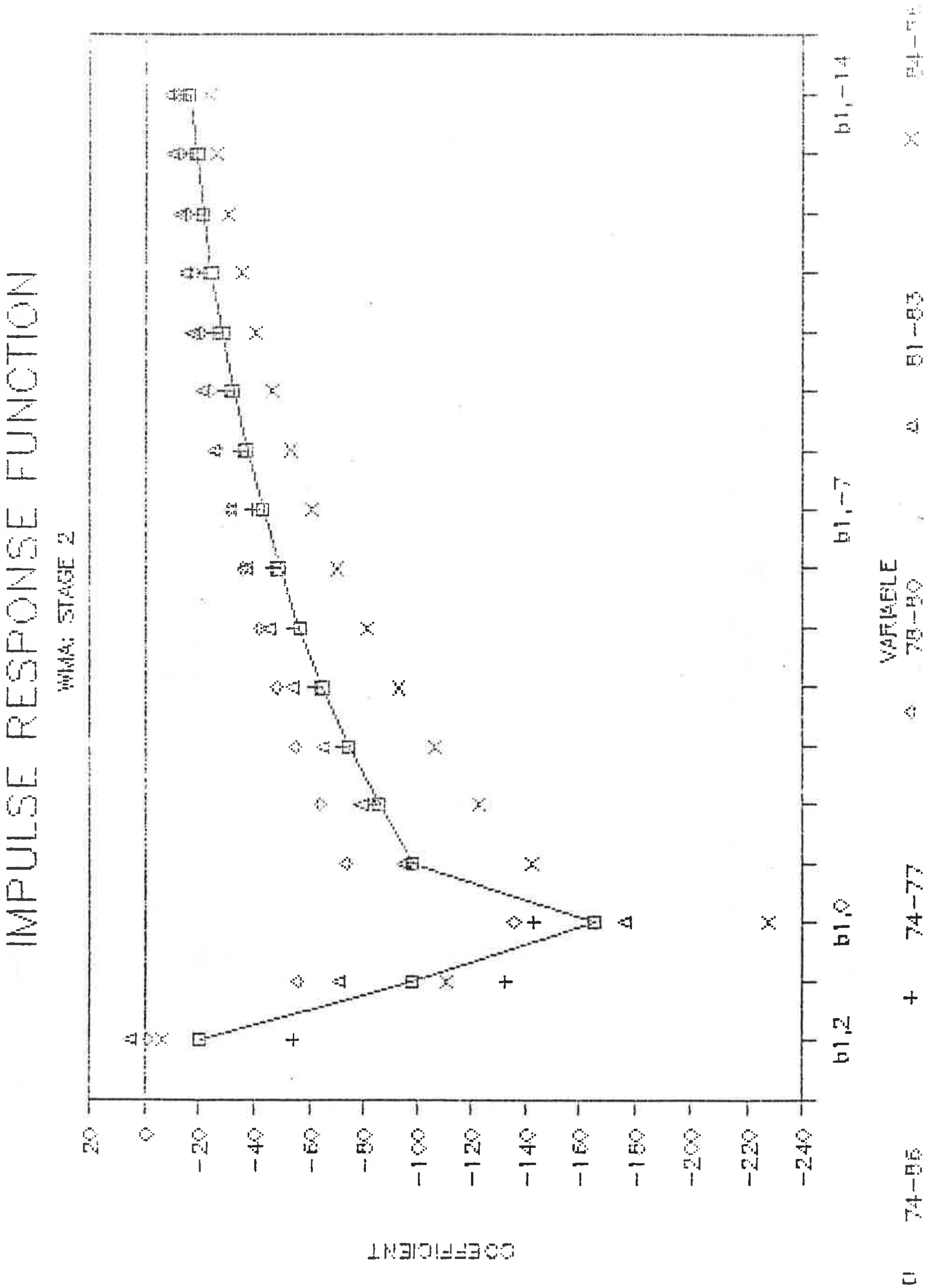

IMPULSE RESPONSE FUNCTION
WMA; STAGE 2
COEFFICIENT
VARIABLE
b1,2
b1,0
b1,-7
b1,-14
74-77
78-80
81-83

FIGURE 1.21b

	STANDARD DEVIATION	VARIANCE	COEFFICIENT OF DETERMINATION (R**2)
ORIGINAL DATA	45.3	2052.1	-
STAGE 1			
CALENDAR MODEL	29.8	881.2	57
CAUSAL MODEL	14.6	212.8	89.6
STAGE 2			
CALENDAR MODEL	30.4	925.6	55
CAUSAL MODEL	14.2	202.8	90.1

FIGURE 1.

```
CALENDAR MODEL: STAGE 2

VARIABLE= CALENDAR RESIDUALS

                  MOMENTS                                     QUANTILES(DEF=4)                        EXTREMES

N              1911  SUM WGTS          1911  100% MAX   108.952      99%     86.1613     LOWEST     HIGHEST
MEAN      -0.103169  SUM           -197.156   75% Q3     16.435      95%     62.4701   -75.8283      101.28
STD DEV     30.4232  VARIANCE       925.569   50% MED   -5.4653      90%     42.4379   -74.8979     103.857
SKEWNESS   0.770813  KURTOSIS      0.547886   25% Q1   -21.0092      10%    -34.2037   -71.0488     105.395
                                               0% MIN  -75.8283       5%    -42.6365   -69.3927      107.23
                                                                      1%    -56.1962   -66.5174     108.952
D:NORMAL  0.0774526  PROB>D            <.01  RANGE      184.781
                                             Q3-Q1      37.4442
                                             MODE      -75.8283

                  HISTOGRAM                          #
   105+*                                             7
      .*                                             5
      .***                                          20
      .****                                         27
      .******                                       42
      .*******                                      47
      .**********                                   65
      .*************                                86
      .******************                          124
    15+************************                    163
      .**********************************          235
      .****************************************    274
      .*********************************************  310
      .***********************************         243
      .*********************                       145
      .************                                 82
      .****                                         26
      .*                                             7
   -75+*                                             3
       ----+----+----+----+----+----+----+----+----+
       * MAY REPRESENT UP TO 7 COUNTS
```


FIGURE 1.23a

CAUSAL MODEL: STAGE 2

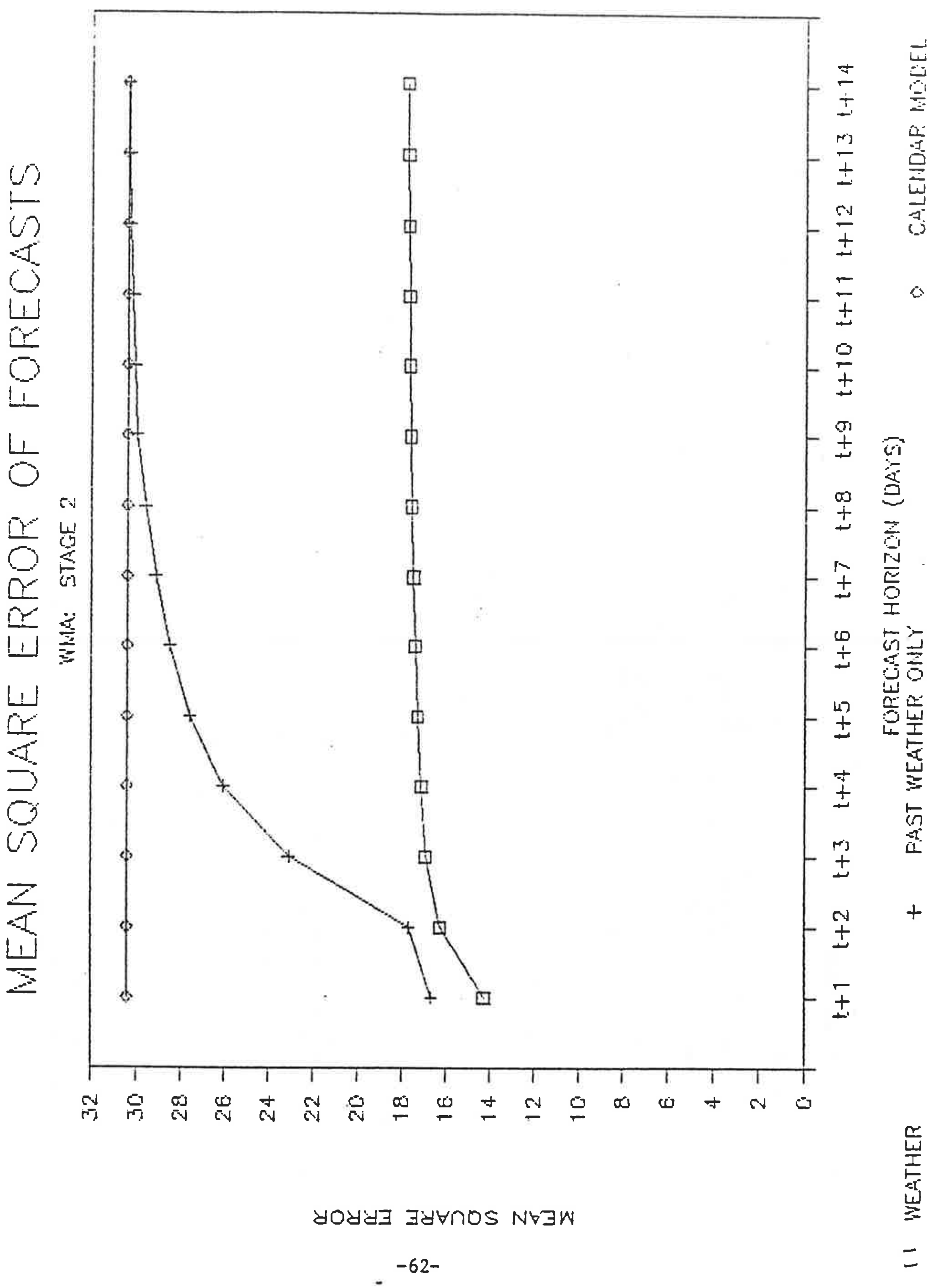
VARIABLE= 1-STEP FORECAST RESIDUALS

```
                   MOMENTS                                     QUANTILES(DEF=4)                          EXTREMES

N               1907  SUM WGTS        1907  100% MAX    62.5327     99%    34.9258       LOWEST    HIGHEST
MEAN       0.0462714  SUM          88.2395   75% Q3     9.00384     95%    23.4089     -62.2037    47.7194
STD DEV      14.1484  VARIANCE     200.177   50% MED    0.39126     90%    16.9207     -55.0147      49.29
SKEWNESS   0.0526566  KURTOSIS    0.736988   25% Q1    -9.39075     10%   -17.7914     -49.4161    51.1877
                                              0% MIN   -62.2037      5%   -22.4469     -44.0696    53.2889
                                                                     1%   -34.0999      -43.412    62.5327
D:NORMAL    0.021495  PROB>D         0.032  RANGE      124.736
                                            Q3-Q1      18.3946
 \                                          MODE      -62.2037


                HISTOGRAM                             #
   62.5+*                                             1
       .
       .*                                             2
       .*                                             4
       .*                                             4
   37.5+*                                             7
       .***                                          20
       .******                                       38
       .**********                                   69
       .**************                               94
   12.5+****************************                191
       .************************************        251
       .******************************************  294
       .**************************************      260
       .********************************            223
  -12.5+****************************                196
       .****************                            112
       .***********                                  71
       .*****                                        34
       .***                                          20
  -37.5+**                                            9
       .*                                             4
       .*                                             1
       .
       .*                                             1
  -62.5+*                                             1
        ----+----+----+----+----+----+----+----+--
        * MAY REPRESENT UP TO 7 COUNTS
```


FIGURE 1.23b



MEAN SQUARE ERROR OF FORECASTS
WMA: STAGE 2
MEAN SQUARE ERROR
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
t+1
t+2
t+3
t+4
t+5
t+6
t+7
t+8
t+9
t+10
t+11
t+12
t+13
t+14
FORECAST HORIZON (DAYS)
WEATHER
+ PAST WEATHER ONLY
◇ CALENDAR MODEL

FIGURE 1.24

CAUSAL MODEL: STAGE 2 - PAST WEATHER ONLY

VARIABLE= 5-STEP FORECAST RESIDUALS

```
                MOMENTS                                    QUANTILES(DEF=4)                         EXTREMES

N               1895  SUM WGTS        1895  100% MAX    112.104     99%     77.3143     LOWEST     HIGHEST
MEAN     -.00384328  SUM        -7.28302   75% Q3      15.2531     95%     53.2447   -73.1144    89.8165
STD DEV     27.5973  VARIANCE    761.612   50% MED     -3.2978     90%     37.9496   -67.6486    94.7102
SKEWNESS   0.642082  KURTOSIS   0.527603   25% Q1     -18.4337     10%    -31.7573   -66.8228    94.9989
                                            0% MIN    -73.1144      5%    -39.7359   -64.8231    105.605
                                                                     1%    -52.9835   -64.2474    112.104
D:NORMAL  0.0609728  PROB>D         <.01  RANGE       185.219
                                           Q3-Q1       33.6868
                                           MODE       -73.1144

            HISTOGRAM                                 #
 115+*                                                1
    .*                                                1
    .*                                                2
    .**                                              13
    .**                                              12
    .*****                                           31
    .*********                                       59
    .********                                        56
    .***********                                     74
    .*********************                          143
    .***************************                    185
    .***************************************        269
    .*********************************************  312
    .*******************************************    301
    .******************************                 208
    .********************                           136
    .*********                                       63
    .****                                            22
    .*                                                6
 -75+*                                                1
     ----+----+----+----+----+----+----+----+----+
     * MAY REPRESENT UP TO 7 COUNTS
```


FIGURE 1.25a

```
CAUSAL MODEL: STAGE 2 - PERFECT WEATHER FORECASTING

VARIABLE= 5-STEP FORECAST RESIDUALS

                MOMENTS                                      QUANTILES(DEF=4)                         EXTREMES

N              1893  SUM WGTS        1893 100% MAX   72.3528      99%    49.5624      LOWEST    HIGHEST
MEAN     0.00205704  SUM          3.89398  75% Q3     10.268      95%    29.6188    -50.8026    63.5561
STD DEV     17.3908  VARIANCE     302.439  50% MED -0.628726      90%    22.5127    -50.5225     65.767
SKEWNESS   0.428606  KURTOSIS    0.750314  25% Q1   -11.5927      10%   -21.4498     -49.711    69.7571
                                            0% MIN  -50.8026       5%   -26.6009    -45.2037    72.2377
                                                                   1%   -37.1503    -44.8299    72.3528
D:NORMAL  0.0318913  PROB>D           <.01 RANGE     123.155
                                           Q3-Q1     21.8606
                                           MODE     -50.8026

                 HISTOGRAM                          #
   72.5+*                                           2
       .*                                           2
       .*                                           2
       .**                                          8
       .*                                           4
   47.5+**                                          7
       .***                                        15
       .****                                       19
       .*****                                      29
       .***********                                62
   22.5+*************                              74
       .******************                        105
       .***************************               158
       .*********************************         193
       .*****************************************  242
   -2.5+*************************************     220
       .**************************************    225
       .****************************              165
       .************************                  140
       .*****************                         100
  -27.5+************                               68
       .****                                       22
       .****                                       20
       .**                                          7
       .*                                           2
  -52.5+*                                           2
        ----+----+----+----+----+----+----+----+-
        * MAY REPRESENT UP TO 6 COUNTS
```


FIGURE 1.25b

CHAPTER 2

WATER USE FORECASTING:
THE RANDOM MEAN MODEL

2.1. INTRODUCTION

In this chapter a forecast system for daily municipal water use is developed. The forecast system is based on a time series model of water use and is used to schedule releases from two water supply reservoirs that serve the Washington D. C. Metropolitan Area (WMA). The model is termed a conditional autoregressive process and can be viewed as an autoregressive process with randomly varying mean. The randomly varying mean accounts for changes in water use that result from the complex interaction over time of "structural features" of the water supply system. These features may include the price of water, total service area connections, plumbing code provisions, and customer income, among many others.

The modeling approach is semi-parametric. The model can be split into a component that is treated in a nonparametric framework and a component that is treated parametrically. The random mean process, which represents long-term trend in mean water use, is treated in a nonparametric framework. Conditional on the random mean water use, the model reduces to a Gaussian autoregressive process with a modest number of parameters. Time series models with randomized parameters have been used in a variety of applications, including economic forecasting (e.g., Nicholls and Quinn [1982] and Swamy [1971]) and hydrologic modeling (Klemes [1974], Potter [1976], and Smith and Karr [1983]).

In the WMA, trend in mean water use (see Figure 2.1) is tied to a number of factors beyond the control of water managers (such as sectors of growth in the local economy and the influence of interest rates on housing development). The determining factors of water use are not all beyond the control of water managers. Numerous publications have appeared assessing the possibility (and subsequent successes and failures) of manipulating the water use values appearing in Figure 2.1 through pricing and conservation measures (see, for example, Howe and Linaweaver [1967], Davis and Hanke [1973], and Carver and Boland [1980]). Carver and Boland [1980] report that seasonal price elasticities of water use for Washington D.C. are "not significantly different from zero". Their results contrast with those of Howe and Linaweaver [1967] in which a seasonal price elasticity of -1.6 is reported. Carver and Boland note that "the elasticity of seasonal water use may have fallen in the WMA during the interval which separates the two studies (1963-1969). Present attitudes toward the environment and resource conservation differ considerably from attitudes of the early 1960's". The complex interaction of changing attitudes, prices, and interest rates all contribute to the random fluctuations over time of mean water use.

Parameter estimation and forecasting procedures developed for the random mean model are non-standard (compare, for example, Salas-LaCruz and Yevjevich [1972] and Maidment and Parzen [1984]). A notable feature of the parameter estimation procedure is inclusion of a "state estimation" step (for a similar estimation procedure, see Smith and Karr [1985]). Model structure dictates that the key step in producing a water use forecast is an updating step in which a revised estimate of current mean water use is produced (updating algorithms for water use forecasting models are also considered in Kher and Sorooshian [1986]).

Contents of the sections are as follows. Model development is the topic of Section 2.2. In Section 2.3 we develop estimators for unknown parameters of the model. State estimation techniques necessary for implementing the forecast system are also developed in Section 2.3. The forecast system is applied to WMA water use in Section 2.4. A summary and conclusions are presented in Section 2.5.

2.2. MODEL DEVELOPMENT

In this section we present a model for daily municipal water use. We denote daily water use on day t of year i by $D_i(t)$. A year consists of $T = 7 \cdot J$ days (the number of weeks in the year is J). To facilitate modeling day-of-week features of water use, the first day of each year is taken to be a Sunday. In Section 2.4, for example, a model for the period May - September is developed; the first day of a given year is taken to be the first Sunday in May.

The random mean water use for day t of year i is assumed to be the product of two terms: Y_i, the random mean daily water use for year i, and m(t), the "unit demand function", which does not vary from year to year (we adopt the notational convention that upper case symbols refer to random processes and random variables, e.g. Y_i, while lower case symbols are used for deterministic functions and parameters, e.g. m(t)). The random variables $\{Y_i\}$ are assumed to represent structural attributes of the service area that vary slowly over time and interact in complex fashion. These attributes may include the price of water (and price history), total service area connections, plumbing code provisions, and customer income, among many others. The actual mean daily water use for year i will differ from Y_i due to the influence of random factors, such as climatological conditions, which operate on a rapidly fluctuating time scale relative to the factors which influence Y_i. The random variables $\{Y_i\}$ representing mean annual water use are not necessarily independent or identically distributed. Indeed, trend and correlation are likely to be important features of the process. Distributional assumptions are not made on the random process $\{Y_i\}$.

The model is specified by

$$D_i(t) = m(t) \cdot Y_i + a \cdot [D_i(t-1) - m(t-1) \cdot Y_i] + Y_i^{1/2} A_i(t) \qquad (2.1)$$

where

$$E[D_i(t)|Y_i] = m(t)\cdot Y_i \quad , \tag{2.2}$$

$$E[(D_i(t)-m(t)Y_i)^2|Y_i] = v\cdot Y_i \quad , \tag{2.3}$$

a is a real-valued parameter, v is a non-negative parameter, and

$$m(t) = \frac{E[D_i(t)]}{T^{-1}\sum_{k=1}^{T} E[D_i(k)]} \tag{2.4}$$

Equation (2.2) states that, conditional on Y_i (that is, if we know Y_i), $m(t)Y_i$ is the mean of $D_i(t)$; equation (2.3) states that vY_i is the conditional variance of $D_i(t)$.

The error process $\{A_i(t)\}$ is assumed to be an independent and identically distributed (i.i.d) sequence of Gaussian random variables, with mean 0 and variance s^2. The error process is also assumed to be independent of $\{D_i(t)\}$ and $\{Y_i\}$.

It follows from equations (2.1) and (2.3) that the conditional variance parameter v must satisfy the equation

$$v = a^2 v + s^2 \quad , \tag{2.5}$$

implying that, for $|a| < 1$,

$$v = \frac{s^2}{1 - a^2} \tag{2.6}$$

Furthermore, it follows from equation (2.1) and the Gaussian assumption on $\{A_i(t)\}$ that, conditional on Y_i, $D_i(t)$ has a Gaussian distribution with mean $m(t)Y_i$ and variance $(s^2/(1-a^2))Y_i$; we will write

$$D_i(t) \overset{D}{\sim} N(m(t)Y_i,[s^2/(1-a^2)]Y_i) \tag{2.7}$$

We include a lagged water use term in our model to reflect persistence in daily water use. Persistence may result from several causes, including limitations of the distribution system and meteorological conditions. To the extent that there is persistence in meteorological conditions that affect water use, this persistence is translated to water use. We do not directly incorporate meteorological variables in our model due to difficulty in forecasting these variables.

The unit demand function at time t, m(t), is the ratio of mean water use on day t to mean daily water use over the course of the year. The unit demand function does not vary from year to year even if long-term trend in mean water use is present. This assumption implies that, although mean water use may exhibit trends over time, seasonal and day-of-week structure of water use do not.

We denote the J weekly average values of the unit demand function by $q_1,\ldots,q_J$, that is,

$$q_j = \sum_{t=1}^{7} m(7(j-1)+t)/7 \qquad j=1,\ldots,J. \qquad (2.8)$$

Structure of the unit demand function is determined by one final assumption. We assume that day-of-week coefficients $p_1,\ldots,p_7$ exist such that

$$m(t) = q_j p_k \qquad \text{for } 7(j-1)+k = t \qquad (2.9)$$

Mean daily water use can vary by day-of-week; day-of-week effects can not, however, vary seasonally or from year to year.

We denote the water use data set available on day t of year n by

$$H_n(t) = \{D_i(k);i=1,\ldots,n-1;k=1,\ldots,T;D_n(1),\ldots,D_n(t)\} \qquad (2.10)$$

The data set $H_n(t)$ contains n-1 consecutive years of complete daily water use data and the first t days of data for year n. For a random variable X, we use the notation $E[X|H_n(t)]$ as shorthand for the conditional expectation of X given $\{D_i(k);i=1,\ldots,n-1,k=1,\ldots,T;D_n(1),\ldots,D_n(t)\}$.

The model is described in the Introduction as a conditional autoregressive process (more specifically, an autoregressive process with random mean). It follows from equation (2.1) that

$$E[D_n(t+1)|H_n(t),Y_n] = E[D_n(t+1)|D_n(t),Y_n] \qquad (2.11)$$

$$= m(t+1)\cdot Y_n + a\cdot[D_n(t) - m(t)\cdot Y_n] \quad ,$$

so that, conditional on Y_n, the process $\{D_n\}$ has autoregressive structure. The model is not, however, an autoregressive process. It is not even a Markov Process; note that

$$E[D_n(t+1)|H_n(t)] \neq E[D_n(t+1)|D_n(t)] \quad . \tag{2.12}$$

Intuitively, the left side of equation (2.12) differs from the right because the data set $H_n(t)$ contains information about the random (and unknown) mean Y_n as well as the correlation information contained in the previous day's observation, $D_n(t)$. This point is further illustrated in deriving the forecast equations below.

The forecast that we will use on day t+1 for water use on day t+k is the conditional expectation of water use on day t+k given observations up to and including day t, $E[D_n(t+k)|H_n(t)]$. The conditional expectation is obviously a function of the data in $H_n(t)$. It is, perhaps most notably, that function of the data which minimizes the expected squared error with $D_n(t+k)$. The following result tells us how to construct our forecasts.

$$E[D_n(t+k)|H_n(t)] = a^k D_n(t) + [m(t+k)-a^k m(t)]E[Y_n|H_n(t)] \tag{2.13}$$

The result can be proven as follows.
For k=1,

$$\begin{aligned} E[D_n(t+1)|H_n(t)] &= m(t+1)E[Y_n|H_n(t)] \\ &+ a\{D_n(t)-m(t)E[Y_n|H_n(t)]\} \\ &+ E[Y_n^{1/2}A_n(t+1)|H_n(t)] \\ &= m(t+1)E[Y_n|H_n(t)] \\ &+ a\{D_n(t) - m(t)E[Y_n|H_n(t)]\} \\ &+ E[Y_n^{1/2}|H_n(t)]E[A_n(t+1)|H_n(t)] \\ &= m(t+1)E[Y_n|H_n(t)] \\ &+ aD_n(t) - am(t)E[Y_n|H_n(t)] \\ &= aD_n(t) \\ &+ [m(t+1)-am(t)]E[Y_n|H_n(t)]. \end{aligned} \tag{2.14}$$

Assume the result is true for k-1.

$$E[D_n(t+k)|H_n(t)] = m(t+k)E[Y_n|H_n(t)] \tag{2.15}$$

$$+ a\{E[D_n(t+k-1)|H_n(t)]-m(t+k-1)E[Y_n|H_n(t)]\}$$

$$+ E[Y_n^{1/2}A_n(t+k)|H_n(t)]$$

$$= m(t+k)E[Y_n|H_n(t)]$$

$$+ a\{a^{k-1}D_n(t) + m(t+k-1)E[Y_n|H_n(t)]$$

$$- a^{k-1}m(t)E[Y_n|H_n(t)] - m(t+k-1)E[Y_n|H_n(t)]\}$$

$$= a^kD_n(t) + [m(t+k)-a^km(t)]E[Y_n|H_n(t)] .$$

The result follows by induction.

Note that

$$E[D_n(t+k)|H_n(t)] \approx m(t+k)E[Y_n|H_n(t)] \tag{2.16}$$

for "large" k. Equation (2.13), and especially equation (2.16), emphasize our interest in accurately modeling mean water use (see also Section 2.4). It follows from equation (2.13) that for short forecast lead times (roughly, 1-3 days), the estimate of mean water use ($m(t+k)E[Y_n|H_n(t)]$) is an important component. It follows from equation (2.16) that for longer lead times the forecast is virtually identical to the estimate of mean water use.

To conclude this section we note that in some situations it may be desirable to allow the parameters a, v, and s to depend on time (as the unit demand function m(t) does). In this case equation (2.6), which relates the parameters a, v, and s, is changed to the recursive equation

$$v(t) = a(t)^2v(t-1) + s(t)^2. \tag{2.17}$$

The forecast equation (2.13) becomes

$$E[D_n(t+k)\,|\,H_n(t)] = [\prod_{j=1}^{k} a(t+j)]D_n(t) \tag{2.18}$$

$$+ [m(t+k) - (\prod_{j=1}^{k} a(t+j))m(t)] \cdot E[Y_n\,|\,H_n(t)]$$

Extensions of the parameter and state estimation procedures developed in the following sections to the extended model are also straightforward, but are not pursued.

It is also conceptually straightforward to extend the model to a "conditional ARMA" or "conditional ARMAX" form. The forecast equation (2.13) , however, is not generalized in a straightforward fashion. Computational tractability is a major reason for restricting consideration to the conditional autoregressive model of equation (2.1).

2.3. PARAMETER AND STATE ESTIMATION

To implement a forecast system based on equation (2.13) we need to estimate the unknown parameters $m(t)$, a, and s and estimate the unknown random mean water use Y_n. The second problem is one of "state estimation", that is, the optimal prediction of an unobserved random variable. Nearly always the optimal predictor is (as in equation (2.13)) the conditional expection of the unobservable random variable given the observations.

Because the random mean process is treated in a nonparametric framework, it is especially difficult to separate the problems of parameter and state estimation. Our approach to parameter estimation is to replace the random mean water use Y_i, for each year i, by the state estimator

$$\hat{Y}_i = T^{-1} \sum_{k=1}^{T} D_i(k) \tag{2.19}$$

and proceed as though no error were involved. In other words, to estimate the parameters $m(t)$, a, and s, we replace the random variables $Y_1,\ldots,Y_{n-1}$ by the sample means $Y_1,\ldots,Y_{n-1}$ and treat the random mean as known. We begin with the unit demand function $m(t)$.

From equation (2.9) it is clear that we need to estimate the weekly coefficients $q_1,\ldots,q_J$ and day-of-week coefficients $p_1,\ldots,p_7$. Based on equations (2.4), (2.8), and (2.19) we choose our estimator of the weekly demand coefficients to be

$$\hat{q}_j = (n-1)^{-1} \sum_{i=1}^{n-1} [7^{-1} \sum_{k=1}^{7} D_i(7(j-1)+k)/\hat{Y}_i] \qquad j = 1,\ldots,J \tag{2.20}$$

The estimator $\hat{q}_j$ is the average value of "scaled" daily demand for week j, with daily values scaled by the average daily demand for the year.

Our estimators of the day-of-week coefficients are given by

$$\hat{p}_k = (n-1)^{-1} \sum_{i=1}^{n-1} [J^{-1} \sum_{j=1}^{J} D_i(7(j-1)+k)/(\hat{q}_j\hat{Y}_i)] \qquad k = 1,\ldots,7 \tag{2.21}$$

The estimator of the Monday day-of-week coefficient, for example, is the average of all Monday values; each value must be scaled by the product of the estimated weekly coefficient q_k and yearly sample mean Y_i. It is straightforward at this point to construct our estimator for the unit demand function. From equation (2.9) we have

$$\hat{m}(t) = \hat{q}_j\hat{p}_k \; ; \qquad 7(j-1) + k = t \; ; \tag{2.22}$$

Centered water use values, $\tilde{D}_i(t)$, are given by

$$\tilde{D}_i(t) = D_i(t) - \hat{m}(t)\hat{Y}_i \tag{2.23}$$

Our estimator for the autoregressive parameter a is given by

$$\hat{a} = \frac{\sum_{i=1}^{n-1} \sum_{t=2}^{T} \tilde{D}_i(t)\tilde{D}_i(t-1)}{\sum_{i=1}^{n-1} \sum_{t=2}^{T} \tilde{D}_i(t)^2} \tag{2.24}$$

The estimator $\hat{a}$ is a standard least-squares estimator except that $m(t)Y_i$ is a state estimator for the unknown random mean.

The estimator for the standard error coefficient s is given by

$$\hat{s}^2 = (n-1)^{-1} \sum_{i=1}^{n-1} (T-1)^{-1} \sum_{t=2}^{T} \left[\frac{\tilde{D}_i(t) - \hat{a}\cdot\tilde{D}_i(t-1)}{\hat{Y}_i} \right]^2 \tag{2.25}$$

We now turn our attention to the state estimation problem of computing $E[Y_n|H_n(t)]$. Because $\{Y_i\}$ is treated in a nonparametric framework, we will not be able to explicitly compute the conditional expectation of Y_n given $H_n(t)$. As is often the case in state estimation problems (see, for example, Karr [1986]) we will retreat to estimators that are linear combinations of our forecast data (or nearly so). The estimator of $E[Y_n|H_n(t)]$ will be denoted by $M_n(t)$.

In constructing the estimator $M_n(t)$, we wish to exploit structure of the data set $H_n(t)$, which divides naturally into two components: data from previous years $H_{n-1}(T)$ and observations from the current year $D_n(1),\ldots,D_n(t)$. To forecast mean water use it is natural to condense information from previous years to the sample means $\hat{Y}_1,\ldots,\hat{Y}_{n-1}$. Similarly, observations for the current year are condensed to the "partial sample means"

$$\hat{Y}_n(t) = \frac{\sum_{k=1}^{t} D_n(k)}{\sum_{k=1}^{t} m(k)} \tag{2.26}$$

Nonparametric trend techniques developed by Hirsch et al. [1979] are used to forecast Y_n from previous years' sample means, $\hat{Y}_1,\ldots,\hat{Y}_{n-1}$. The forecast is of the form $\hat{Y}_{n-1} + \hat{b}_n$ where $\hat{b}_n$ is the Kendall slope estimator obtained from $\hat{Y}_1,\ldots,\hat{Y}_{n-1}$.

Using equations (2.1) and (2.26), it is straightforward to show that

$$\hat{Y}_n(t) = Y_n + \tilde{a}(t)\Big[\sum_{k=1}^{t} m(k)\Big]^{-1}(D_n(0) - m(0)Y_n)$$

$$+ \quad Y_n^{1/2}\Big[\sum_{k=1}^{t} m(k)\Big]^{-1} \sum_{k=1}^{t} \sum_{j=1}^{k} a^{k-j}A_n(j) \tag{2.27}$$

where

$$\begin{aligned} a(t) &= \sum_{k=1}^{t} a^k \\ &= (1-a^{t+1})(1-a)^{-1} - 1 \ . \end{aligned} \tag{2.28}$$

It follows from equation (2.27) that, conditional on Y_n,

$$\hat{Y}_n(t) \overset{D}{\sim} N(Y_n, c(t)Y_n) \tag{2.29}$$

where c(t) is a function of the parameters a, m(t), and s, which, most notably, is decreasing in t. It follows from equation (2.29) that for each t, $\hat{Y}_n(t)$ is an unbiased state estimator of Y_n and that for t greater than s, $\hat{Y}_n(t)$ is a better estimator than $\hat{Y}_n(s)$.

Based on the preceding discussion we take our state estimator to be

$$M_n(t) = w(t)\hat{Y}_n(t) + (1-w(t))(\hat{Y}_{n-1} + \hat{b}_n) \tag{2.30}$$

where

$$w(t) = 1 - [(T-1-t)/T]^2 \tag{2.31}$$

The weight function is chosen to have the following properties

$$0 \backslash w(t) \backslash 1 \qquad t = 1,\ldots,T+1 \tag{2.32}$$

$$w(1) = 0 \tag{2.33}$$

$$w(T+1) = 1 \tag{2.34}$$

$$w(T/2 + 1) > .50 \tag{2.35}$$

On the first day of year n we have only observations from preceding years, so all of the weight must be on preceding years, that is, w(1) must equal 0. At the other extreme we have all of the data from year n available. The assumption that w(T+1) equals 1 implies that year-to-year dependence in the random mean process is weak. Specifically the assumption implies that for estimating Y_n from $H_n(T)$, previous years' data provide no further information once Y_n is available. Equation (2.35) implies that much of the information about Y_n is available at the mid-point of the year. Intuitively, relatively less "new" information about Y_n should be expected as the year progresses. The second half of the year, for example, contains less new information than the first (provided, of course, that the first half is observed).

2.4. APPLICATION OF THE WATER USE MODEL

The water use forecast system is used to schedule releases from two water supply reservoirs located in the Potomac River basin upstream of Washington D.C. (see Chapter 5 for a detailed discussion of water supply management in the Potomac River basin). A large reservoir, located far from the WMA, can provide water to the WMA with a travel time of approximately 5 days. Releases from a small local reservoir reach water utility intakes within the day of release. The water use model is developed for the "summer season" (May - September), during which forecast information is needed for scheduling water supply releases.

For the WMA water supply system the large upstream reservoir is operated to meet "average" water demand; the local reservoir meets shortfalls arising from "extreme" demands. Average demand is clearly a moving target. The estimated Kendall slope estimator $\hat{b}_n$ obtained from 1974-1986 WMA water use data is 8 mgd/year (mgd = million gallons per day; 1 mgd = 5680 cubic meters per day). The estimated weekly coefficients (see Figure 2.2) obtained from equation (2.21) range from a minimum of .92 at the beginning of May to a maximum of 1.06 in mid-July, from which they decrease below 1 by the end of September. For an annual mean water use of 500 mgd (the 1986 value for the WMA), the seasonal variation in mean water use is 70 mgd (from 460 mgd in early May to 530 mgd in mid-July). The estimated day-of-week coefficients $\hat{p}_1, \ldots, \hat{p}_7$ (equation (2.22)) range from a maximum of 1.02 on Wednesday to a minimum of .97 on Sunday. Differences in day-of-week water use result in part from the fact that there is a larger population in the service area during the week than on weekends (a significant number of people work in the region served by WMA water utilities but live outside of the area served by WMA water utilities). For a weekly mean water use of 500 mgd, the range in mean water use associated with day of week is 25 mgd. For scheduling an upstream release on day t, average water use (for day t+5) is estimated by

$$\hat{M}(t) = \hat{m}(t+5)[w(t)\hat{Y}_n(t) + (1-w(t))(\hat{Y}_{n-1} + \hat{b}_n)] \qquad (2.36)$$

To operate the small local reservoir, forecasts of 1-day-ahead water use are required. To apply the forecast equation (2.13) we need only specify the autoregressive parameter a. The estimate obtained from equation (2.25) is .76. The forecast obtained from equation (2.13) for one-day-ahead water use is

$$\hat{D}(t+1) = \hat{a}\cdot D_n(t) + [\hat{m}(t+1)-\hat{a}\hat{m}(t)][w(t)\hat{Y}_n(t)+(1-w(t))(\hat{Y}_{n-1}+\hat{b}_n)] \qquad (2.37)$$

Figure 2.3 shows errors of 1-day-ahead forecasts for WMA water use during the summer of 1986. Note that the errors are weakly correlated and that variability of the estimators decreases as the year proceeds. The per cent bias for 1986 1-day-ahead forecasts is -.1%; the standard error is 29 mgd.

2.5. SUMMARY AND CONCLUSIONS

A time series model of daily municipal water use is developed. Emphasis in model development is placed on long-term trend, seasonality, and day-of-week effects. The model, which is termed a conditional autoregressive process, can be interpreted as an autoregressive process with randomly varying mean. The random mean process, which represents long-term trend in mean water use, is treated in a nonparametric framework. Conditional on the random mean water use, the model reduces to a Gaussian autoregressive process with a modest number of parameters.

Seasonality and day-of-week effects are captured in the model through the unit demand function. The unit demand function at time t, m(t), is the ratio of mean daily water use on day t of the year to mean daily water use over the course of the year. An important model assumption is that the unit demand function does not vary from year to year, even if long-term trend in mean water use is present.

An attractive feature of the water use model is computational tractability. The forecast equation derive in Section 2.2 (equation (2.13)) can be easily implemented provided that two estimation problems are solved. To implement the forecast equation, parameter estimators for the model parameters m(t) and a are needed. Also, state estimators for the random (and unknown) mean Y_n are required. A notable feature of the parameter estimation procedure developed in Section 2.3 is inclusion of a state estimation step. Parameter estimates for m(t) and a are obtained after first applying a "detrending" procedure which is based on simple state estimators for the random mean process. The state estimator that is developed for use in the forecast equation is nonparametric and exploits special structure of the water use data set.

Parameter and state estimation procedures are applied to summer season water use in the Washington D.C. Metropolitain Area (WMA). The estimation results confirm that long-term trend, seasonality, and day-of-week effects are prominent features of WMA water use. The water use forecast system developed for the WMA is used to schedule releases from two water supply reservoirs. A large upstream reservoir is operated to meet "average" water demands. For operation of this reservoir, the state estimator of "current mean water use" (equation 2.36) provides the necessary forecast information. One-day-ahead forecasts, obtained from equation (2.13), are used to operate a small local reservoir, which covers shortfalls due to "extreme" water demands.

A potentially useful extension to the forecast system involves incorporation of precipitation data (see Chapter 1). This could be accomplished by expanding the model equation (1) to explicitly include precipitation variables. An alternative is to incorporate precipitation variables into the random mean process $\{Y_n\}$. With the second approach, precipitation forecasts are not required for implementing a forecast system. Instead, observed precipitation data are used, along with observed water use data, to update the estimate of current mean water use.

FIGURE 2.1
SUMMER WATER USE
(May - September)

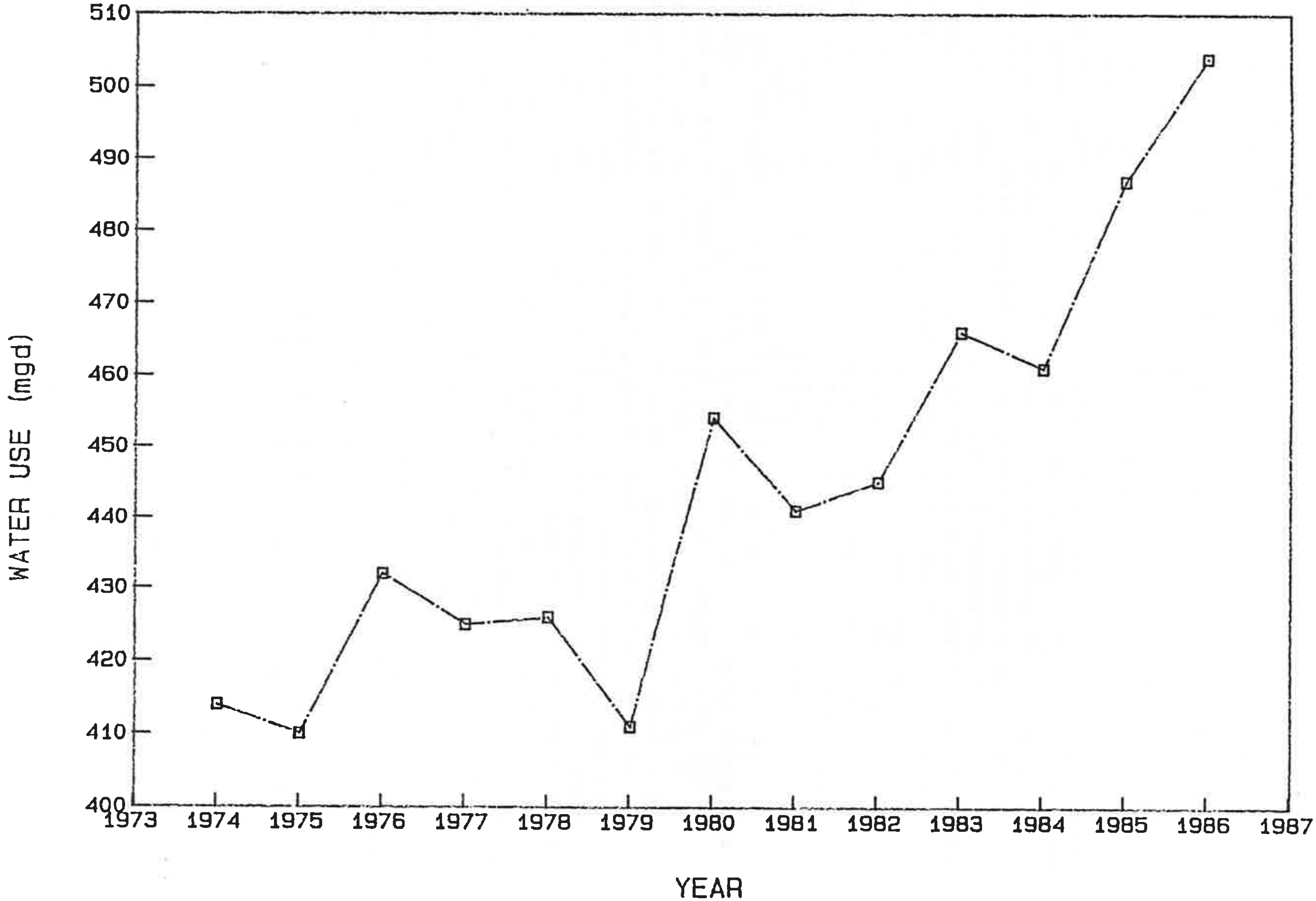


FIGURE 2.2
WEEKLY DEMAND FACTORS
(May - September)

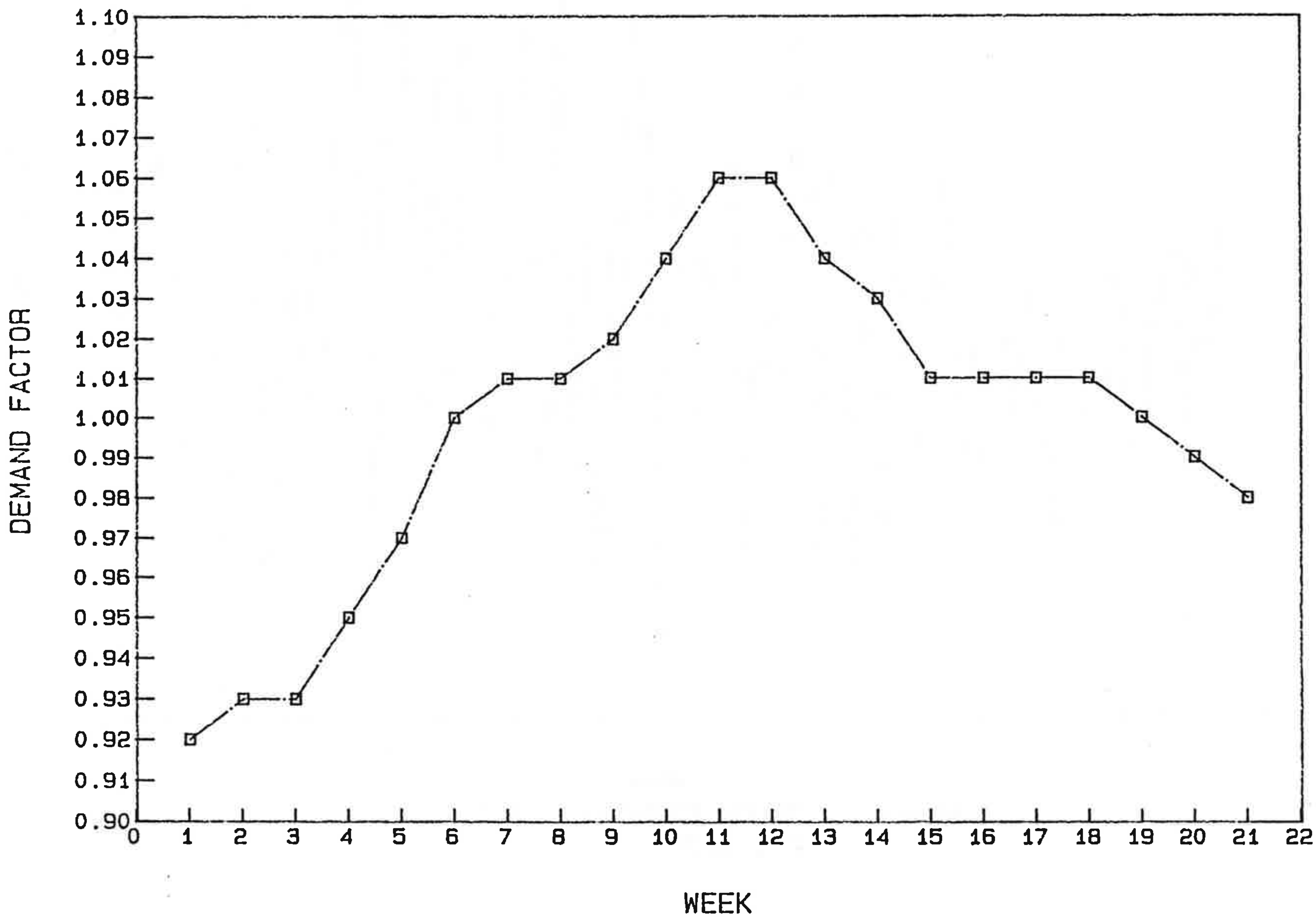
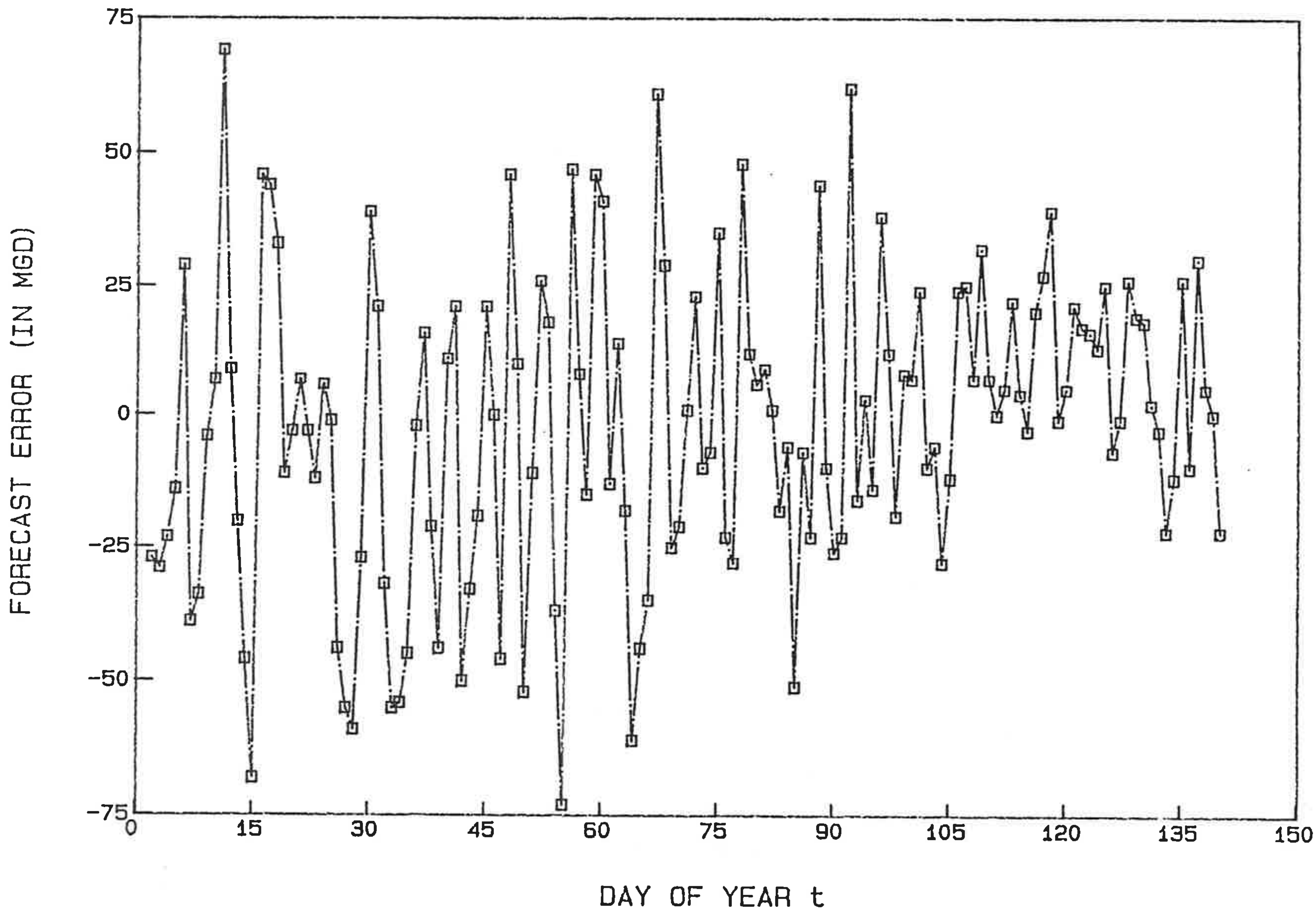


FIGURE 2.3
ONE-DAY-AHEAD FORECAST ERROR
1986



CHAPTER 3

NONPARAMETRIC TECHNIQUES FOR WATER SUPPLY FORECASTING

3.1. INTRODUCTION

The water supply forecasting problem considered in this chapter can be described as follows. On day t of the year we want to produce a forecast of a random variable Y(t), which is solely a function of future streamflow. It is assumed that the information available for constructing a forecast is historical daily streamflow data for the preceding n-1 years. From this data a sample $Y_1(t),\ldots,Y_{n-1}(t)$, with the same distribution as Y(t), can be obtained. The forecast we propose for Y(t) is simply a weighted average of $Y_1(t),\ldots,Y_{n-1}(t)$. The weight for year i depends on how similar hydrologic conditions on day t of year i are to hydrologic conditions on day t of the current year. Nonparametric regression (see Yakowitz [1985]) is the tool used to determine the weights.

The forecast model is applied to management of the Washington D. C. Metropolitan Area (WMA) water supply system. Mean daily water use for the WMA is approximately 450 mgd (maximum historical daily water use is 660 mgd). Figure 3.1 shows the sample distribution of minimum daily flow for the Potomac River at Washington D.C. "Normal" years clearly do not pose a threat of water supply shortage (the median value of minimum daily flow is 1000 mgd). As is often the case for water supply applications interest focuses on extreme drought conditions. The forecast system developed in this paper is designed to produce distributional forecasts for the "lower tail" of water supply variables (the distributional approach to long-range water supply forecasting is also adopted by Hirsch [1981] and Day [1985]).

During the period 1930-1980 annual minimum flow of the Potomac River was less than 700 mgd 10 times. The sample estimate of the probability of falling below 700 mgd in any given year is 10/51 (.20). Figure 3.2 shows a plot of minimum June flow versus minimum summer flow (August to October). If minimum flow during June is known, it should be possible to sharpen the estimate of minimum flow falling below 700 mgd. Nonparametric regression provides a flexible tool for incorporating information contained in the preceding record of streamflow into long-range water supply forecasts.

Contents of the sections are as follows. Section 3.2 contains a detailed problem formulation and description of the tools that are developed for implementing the forecast system. Application of the forecast system to WMA water supply management problems is the topic of Section 3.3. Section 3.4 contains a summary and conclusions.

3.2. DEVELOPMENT OF THE FORECAST SYSTEM

Streamflow on day t of year i is denoted by $X_i(t)$. A year consists of T=365 days and begins on January 1. The random process $\{X_i(t);t=1,\ldots T;i=1,2,\ldots\}$ can be partitioned on day t of year n into two sets, the "historical data set"

$$H_n(t) = \{X_i(s); i=1,\ldots,n-1; s=1,\ldots,T; X_n(1),\ldots,X_n(t-1)\} \quad (3.1)$$

and "future streamflow"

$$G_n(t) = \{X_i(s); i>n; s=1,\ldots,T; X_n(t),\ldots,X_n(T)\} \quad (3.2)$$

The sequence of forecast random variables will be denoted $\{Y_i(t); t=1,\ldots,T; i=1,2,\ldots\}$. For each n and t, $Y_n(t)$ is a function of future streamflow, that is, $Y_n(t)$ is a function of the random variables in $G_n(t)$. The forecast problem can be posed as follows. We wish to compute the conditional expectation of $Y_n(t)$ given the data available at the beginning of day t, $E[Y_n(t)|H_n(t)]$. The following example illustrates the form that the forecast problem may take.

EXAMPLE 1.

Let $\{X_i(t)\}$ denote daily streamflow of the Potomac River at Washington, D.C. We take

$$Y_n(t) = 1(\min\{X_n(s): t \le s \le T\} < y) \qquad t \le T \quad (3.3)$$

that is, $Y_n(t)$ equals 1 if Potomac streamflow drops below y before the end of the year. The forecast $E[Y_n(t)|H_n(t)]$ is the conditional probability that Potomac flow will drop below y, that is,

$$E[Y_n(t)|H_n(t)] = P\{\min(X_n(t),\ldots,X_n(T))<y|H_n(t)\} \quad (3.4)$$

The estimator that will be used for the forecast random variable $Y_n(t)$ is a weighted average of the preceding n-1 annual observations $Y_1(t),\ldots,Y_{n-1}(t)$. The weight for year i should reflect how similar basin conditions on day t of year i are to conditions on day t of year n. Heuristically, the weight should be large if streamflow values preceding day t of year i are very similar to the corresponding streamflow values for year n and small if streamflow values are very different.

To develop a useful model, the streamflow information in the data set $H_n(t)$ must be condensed. The approach we take is to condense streamflow information for each year i into a covariate random variable $Z_i(t)$. The covariate random variable may be yesterday's flow, minimum flow for the preceding d days, cumulative flow for the preceding d days, etc. We will attempt to select the covariate random variable so that little information is lost by condensing $H_n(t)$ to $Z_n(t)$, that is

$$E[Y_n(t)|H_n(t)] \approx E[Y_n(t)|Z_n(t)] \quad (3.5)$$

The weight for year i should be large if the covariate value for year i, $Z_i(t)$, is close to the covariate value for the forecast year, $Z_n(t)$. For real-valued z and positive b, let $K(z|b)$ be the Gaussian kernel

$$K(z|b) = (2\pi)^{-1/2}\exp\{-1/2(z/b)^2\} \quad (3.6)$$

The weight function is defined by

$$W_i(z) = K(Z_i(t)-z|b(t)) / \sum_{j=1}^{n-1} K(Z_j(t)-z|b(t)) \quad (3.7)$$

where, for each t, b(t) is a positive parameter termed the "bandwidth" (also "window width" or "smoothing factor"). We will interpret $W_i(z)$ as the weight for year i (and day t) given that the covariate value for the forecast year is z (that is, $Z_i(t)$ equals z). It is clear from equations (3.6) and (3.7) that if z is close to the covariate value for year i, the weight will be large; if z is very far from $Z_i(t)$ the weight for year i will be very small. Note also that the weights sum to 1.

The nonparametric kernel estimator of $Y_n(t)$, given that $Z_n(t)$ equals z is

$$\hat{Y}_n(t|z) = \sum_{i=1}^{n-1} W_i(z)Y_i(t) \quad (3.8)$$

The Gaussian kernel in equation (3.6) could be replaced in equation (3.7) by any symmetric density. As noted by Sheather [1986] performance of kernel estimators is not overly sensitive to the particular kernel that is chosen. Performance of kernel estimators is, however, sensitive to the bandwidth parameter.

The criterion we use for selecting the bandwidth parameter is integrated mean square error (IMSE), which is defined by

$$I_t = \int_R E[\hat{Y}_n(t|z) - E[Y_n(t)|Z_n(t)=z]]^2 dz \tag{3.9}$$

Unfortunately it is difficult to explicitly compute the IMSE. Following Haerdle [1986] we estimate the IMSE by

$$\hat{I}_t = (n-1)^{-1} \sum_{j=1}^{n-1} [\hat{Y}_j(t|Z_j(t)) - Y_j(t)]^2 \tag{3.10}$$

where $\hat{Y}_j(t|Z_j(t))$ is the "leave-one-out" estimator of $Y_j(t)$ given $Z_j(t)$ (to compute $\hat{Y}_j(t|Z_j(t))$ we "leave out" data from year j in computing (3.8)). Haerdle shows that minimization of (3.10) is asymptotically equivalent to minimization of (3.9) (as the number of years n goes to infinity). We will denote the bandwidth value that minimizes (3.10) by $\hat{b}(t)$.

Note that the weights in equation (3.7) can be made arbitrarily close to the inverse sample size, $(n-1)^{-1}$, by making the weight b(t) large. If the covariate random variable contains little information about the forecast random variable one would expect the weight $\hat{b}(t)$ to be "large". Conversely, the weights should be small if the covariate random variable can accurately specify the forecast random variable.

If $Y_n(t)$ is binary-valued, taking either the value 0 or 1, the following representation for forecast variance can be obtained:

$$S(t)^2 = E[(\hat{Y}_n(t|z) - Y_n(t))^2|H_n(t)]$$

$$= \hat{Y}_n(t|z)^2 - 2\hat{Y}_n(t|z)E[Y_n(t)|H_n(t)] + E[Y_n(t)|H_n(t)] \tag{3.11}$$

The proof is given in the Appendix.

From equation (3.11) we obtain an estimate of the forecast variance by replacing $E[Y_n(t)|H_n(t)]$ by $Y_n(t|z)$,

$$\hat{S}(t)^2 = \hat{Y}_n(t|z)(1 - \hat{Y}_n(t|z)) \tag{3.12}$$

The estimated forecast variance of equation (3.12) is solely a function of $\hat{Y}_n(t|z)$. It is symmetric about .5, increasing from

0 in the case that $\hat{Y}_n(t|z)$ equals 0 to a value of .25 in the case that $\hat{Y}_n(t|z)$ equals .5. Yakowitz and Szidarovszky [1985] develop data-based estimators of forecast variance that can be used if $Y_n(t)$ is not binary-valued.

The main restriction on the structure of the forecast random variables is that for each t in {1,...,T} the random variables $Y_1(t), Y_2(t), \ldots$ are identically distributed. In Example 1, the forecast random variable can be represented as

$$Y_n(t) = g_t(X_n(t), \ldots, X_n(T)) \qquad (3.13)$$

where g_t is a function of T-t+1 arguments. The identically distributed assumption requires both that 1) the function g_t of future streamflow does not depend on the year index n and 2) the random vectors $(X_1(1), \ldots, X_1(T))$, $X_2(1), \ldots, X_2(T)), \ldots$ are identically distributed. No further distributional assumptions are made on streamflow.

3.3- APPLICATION OF THE FORECAST MODEL

In this section application of the forecast model to WMA water supply management is illustrated. The Potomac River is the principal water supply source for the WMA. Historical minimum flow of the Potomac River at Washington D.C. (400 mgd) is less than current mean daily water use (450 mgd) and substantially less than maximum daily water use (660 mgd). To determine potential augmentation requirements for WMA water supply, long-term forecasts of minimum flow of the Potomac River are desired. A useful estimate of the likelihood of water supply releases is the probability that natural flow of the Potomac River falls below 700 mgd during the course of the summer.

To implement the forecast model of Section 3.2 the covariate random variables $\{Z_i(t)\}$ must be specified and bandwidth parameters b(t) must be estimated. The covariate random variable $Z_i(t)$ should contain information about baseflow potential during the remainder of the year. Current baseflow is taken to be the best indicator of future baseflow. As a surrogate for current baseflow we take $Z_i(t)$ to be minimum flow during the preceding 30 days,

$$Z_i(t) = \min \{X_i(t-j); j=1,\ldots,30\} \qquad (3.14)$$

We will use the notation $z_p(t)$ for the pth quantile of the covariate random variable $Z_i(t)$, that is

$$P(Z_i(t) \le z_p(t)\} = p \qquad (3.15)$$

Figure 3.3 shows the estimated bandwidth parameters for forecasting the conditional probability that Potomac streamflow drops below 700 mgd. Note that the parameters generally decrease over the course of the year. As noted in the preceding section, this behavior is consistent with a decrease in correlation between the forecast random variable and the covariate random variable with time separation.

The sample probability of minimum summer flow of the Potomac dropping below 700 mgd is .20. Figure 3.4 illustrates the range of information that can be provided by the forecast model to sharpen the estimate. The figure plots $Y_n(t|z_p(t))$ versus t for three quantile values; p=.1, p=.5, and p=.9. For p=.1, the figure shows the influence of low antecedent baseflow conditions at time t on future baseflow. For p=.9, the figure illustrates the influence of wet antecedent conditions. If antecedent conditions are normal, we obtain the plot with p=.5.

Selection of the covariate random variables is a critical part of developing and implementing the forecast system. We examine below sensitivity of model performance to selection of covariate random variables.

Table 3.1 shows IMSE values (from equation (3.10)) for two variations on the minimum flow definition of the covariate random variable in equation (3.14). The new covariate random variables are based on minimum flow for the preceding 60 and 90 days. Figure 3.5 shows plots of $Y_n(t|z_p(t))$ versus t for 1) two choices of covariate random variables (30 and 90 days preceding minimum flow) and two choices of p (.1 and .9). Note that forecast performance is not dramatically changed by increasing the window of the covariate random variable to 90 days.

To examine sensitivity of model performance to a different form of covariate random variable we consider cumulative flow over the preceding J days,

$$Z_i(t) = J^{-1} \sum_{j=1}^{J} X_i(t-j) \tag{3.16}$$

It could be argued that water balance variables like cumulative flow may perform better than the baseflow surrogate under certain conditions. Table 3.2 contains IMSE value for three values of J, corresponding to lags of 30, 60, and 90 days. Note that the IMSE values are larger than the IMSE values for minimum flow during late spring and summer.

To conclude this section we examine sensitivity of the forecast model to the discharge threshold y. It is hoped that the estimated bandwidth parameters do not vary much if y is slightly different from 700 mgd (in part because we would like to apply the model to discharge thresholds "near" 700 mgd). If the bandwidth parameters are stable in a region (y_0,y_1) about y, we can 1) assume the model to hold over the range of discharge thresholds (y_0,y_1) and 2) estimate the bandwidth parameters b(t) by averaging the IMSE estimate of equation (3.10) over the discharge range (y_0, y_1). Figure 3.6 shows the estimated bandwidth parameters for the discharge region (600 mgd, 800 mgd). Note that the parameters are close to the parameters for 700 mgd, especially beyond early spring.

Figure 3.7 shows the sample distribution of minimum summer flow, restricted to the range (600,800). Also shown are nonparametric regression estimates for July corresponding to covariate values $z_{.1}(t)$ and $z_{.9}(t)$ (the covariate random variable is minimum flow for the preceding 30 days).

3.4. SUMMARY AND CONCLUSIONS

In this paper a forecast system is developed for producing long-term distributional forecasts of water supply variables. The forecast procedure requires only daily discharge data for implementation. The forecast problem is formulated as follows. On day t of year n we want to produce a forecast of a random variable $Y_n(t)$, which is solely a function of future streamflow. The forecast we propose is a weighted average of the random sample $Y_1(t),...,Y_{n-1}(t)$. The weights are determined by nonparametric regression. The key step in implementing nonparametric regression is specification of "bandwidth parameters". Bandwidth parameters are chosen to minimize the integrated mean square error (IMSE) of forecasts.

Utility of the forecast system is illustrated for water supply management problems for the Washington Metropolitan Area. The central problem is to anticipate whether natural flow of the Potomac River will drop to levels requiring augmentation from water supply reservoirs. The procedures developed in Section 3.2 provide distributional forecasts for minimum flow of the Potomac River.

Several topics for future research can be noted. It is straightforward to allow multivariate covariate random variables. The Gaussian kernel of equation (3.10) is simply expanded to a multivariate Gaussian kernel. Is is also conceptually straightforward to include "exogenous" covariate information into our forecast model. For water supply forecasting in the western US it would be highly desirable to include any measurements of snowpack. Development of formalized procedures for selection of covariate random variables, analogous to those available in standard regression problems, would be useful.

APPENDIX

Proof of Equation (3.11).

$$E[(\hat{Y}_n(t|z) - Y_n(t))^2|H_n(t)] =$$

$$E[\hat{Y}_n(t|z)^2|H_n(t)] - 2E[\hat{Y}_n(t|z)Y_n(t)|H_n(t)]$$

$$+ E[Y_n(t)^2|H_n(t)]. \qquad (A.1)$$

Because $\hat{Y}_n(t|z)$ is a function of data in $H_n(t)$ it follows that

$$E[\hat{Y}_n(t|z)^2|H_n(t)] = \hat{Y}_n(t|z)^2. \qquad (A.2)$$

Similarly,

$$E[\hat{Y}_n(t|z)Y_n(t)|H_n(t)] = \hat{Y}_n(t|z)E[Y_n(t)|H_n(t)]. \qquad (A.3)$$

The result follows by inserting (A.2) and (A.3) in (A.1) and noting that

$$E[Y_n(t)^2|H_n(t)] = E[Y_n(t)|H_n(t)] \qquad (A.4).$$

FIGURE 3.1
MINIMUM DAILY FLOW
POTOMAC RIVER LITTLE FALLS

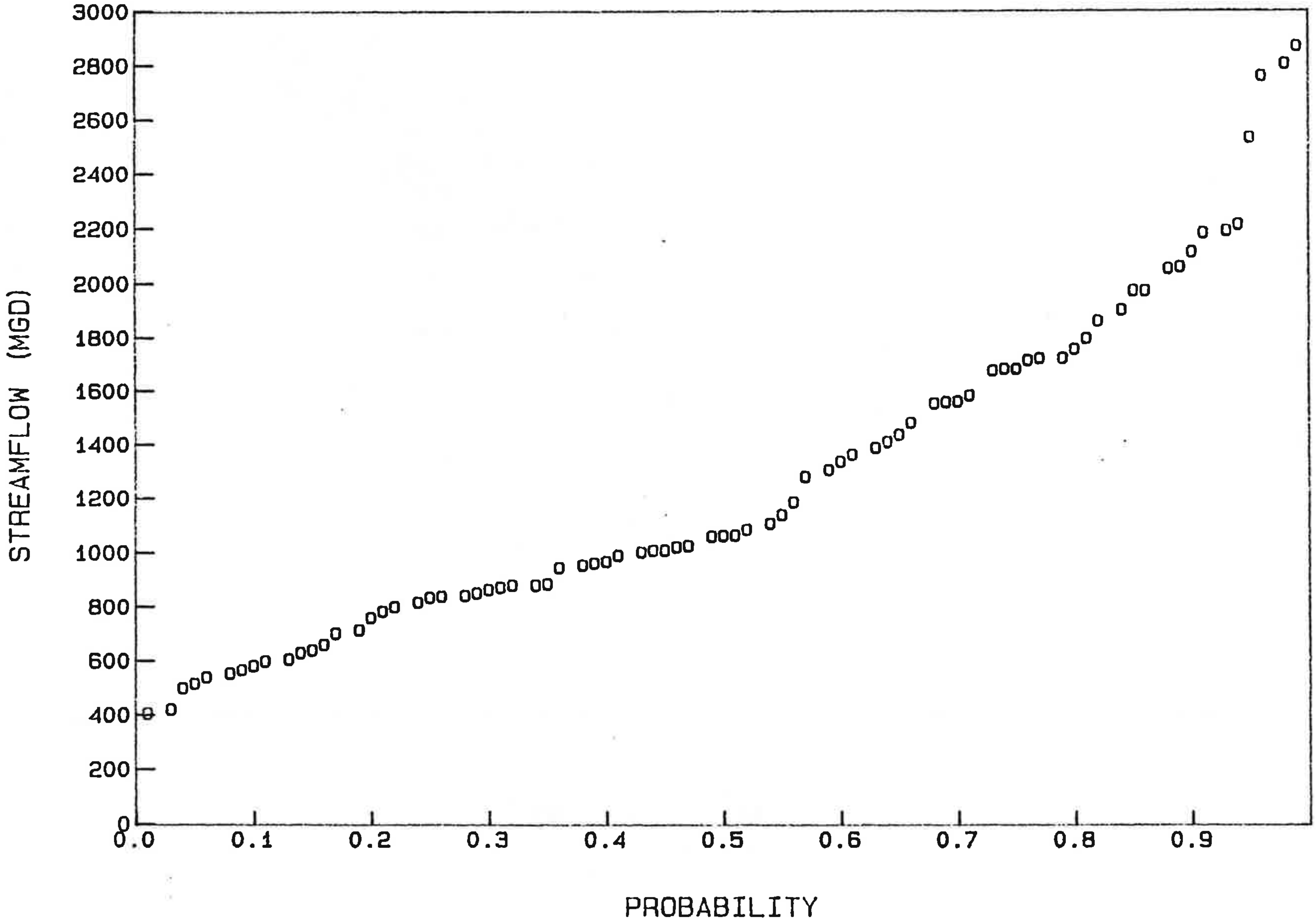


FIGURE 3.2
SUMMER IS AUGUST - OCTOBER

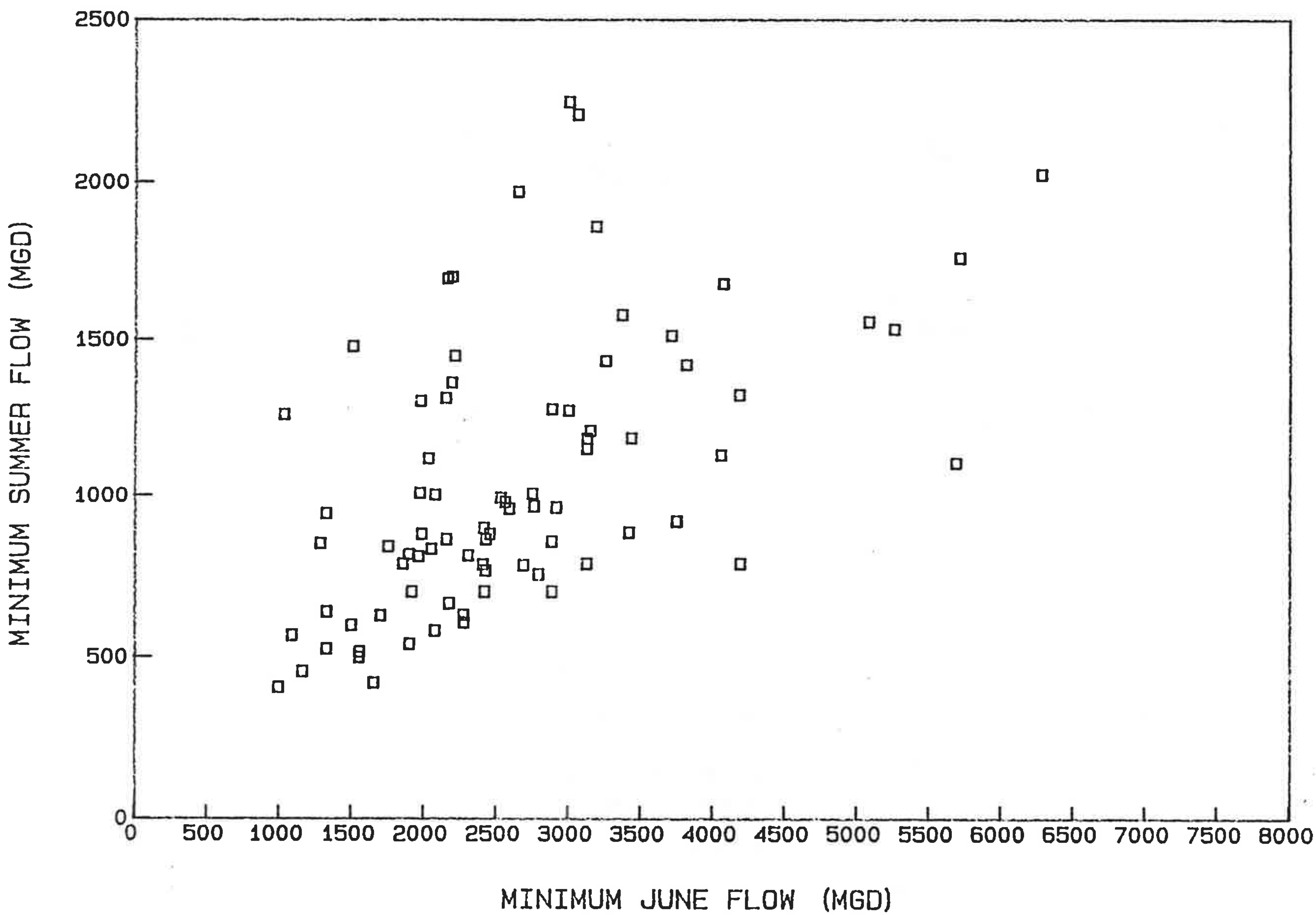


FIGURE 3.3
ESTIMATED BANDWIDTH PARAMETERS
BY MONTH

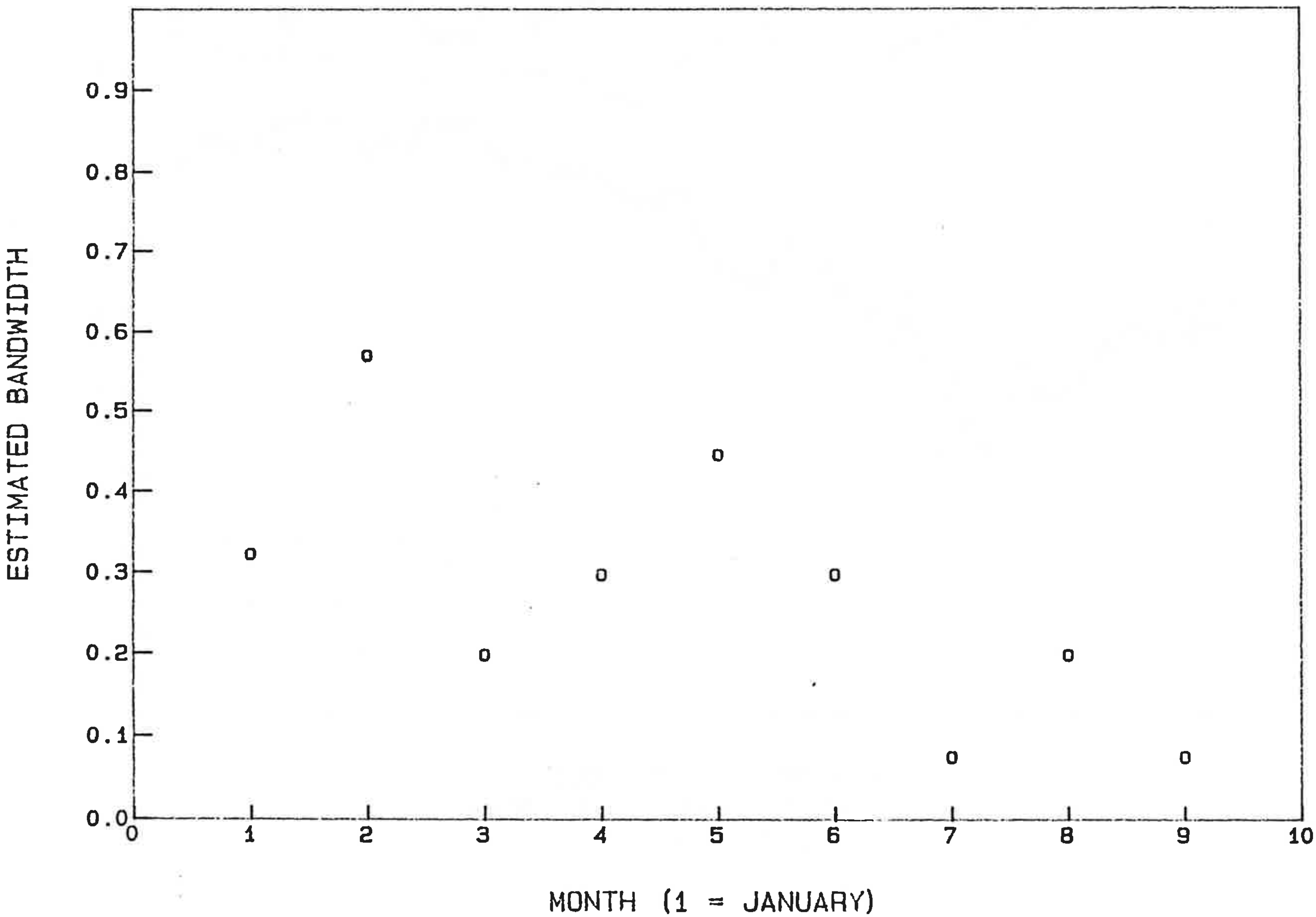


FIGURE 3.4
PROBABILITY THAT POTOMAC FLOW
DROPS BELOW 700 MGD.

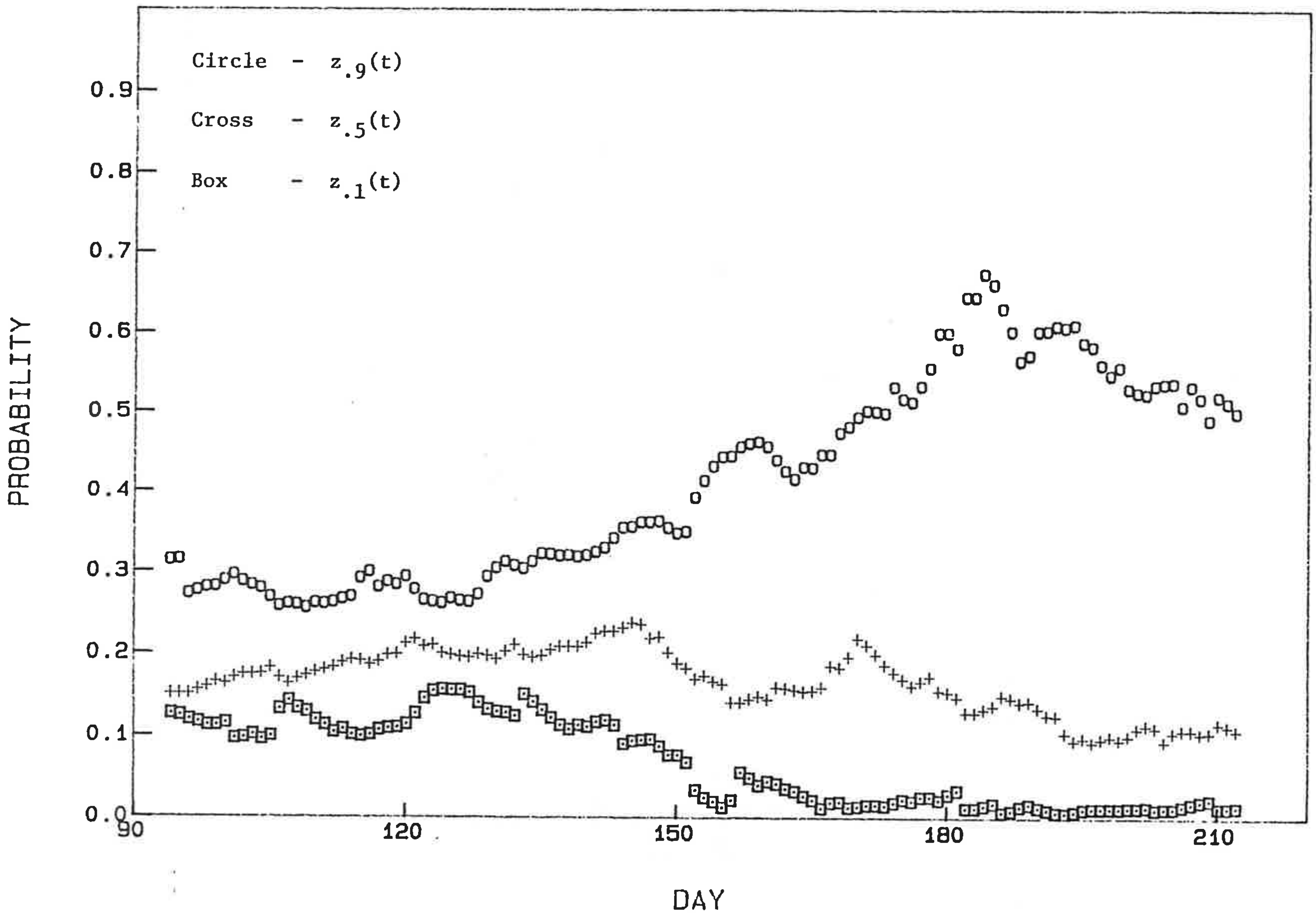


FIGURE 3.5
PROBABILITY THAT POTOMAC FLOW
DROPS BELOW 700 MGD

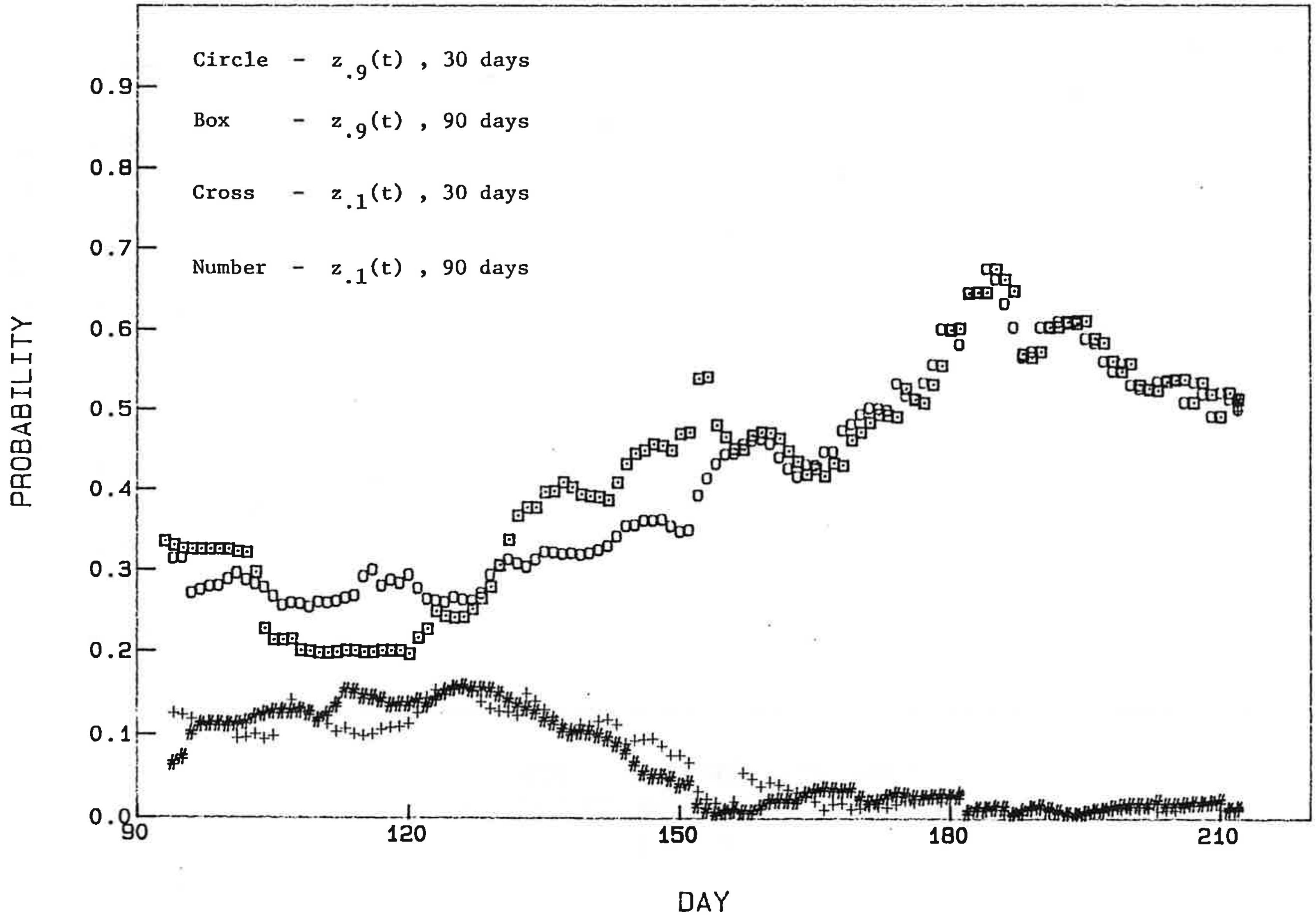


FIGURE 3.6
ESTIMATED BANDWIDTH PARAMETERS
600 - 800 MGD THRESHOLD

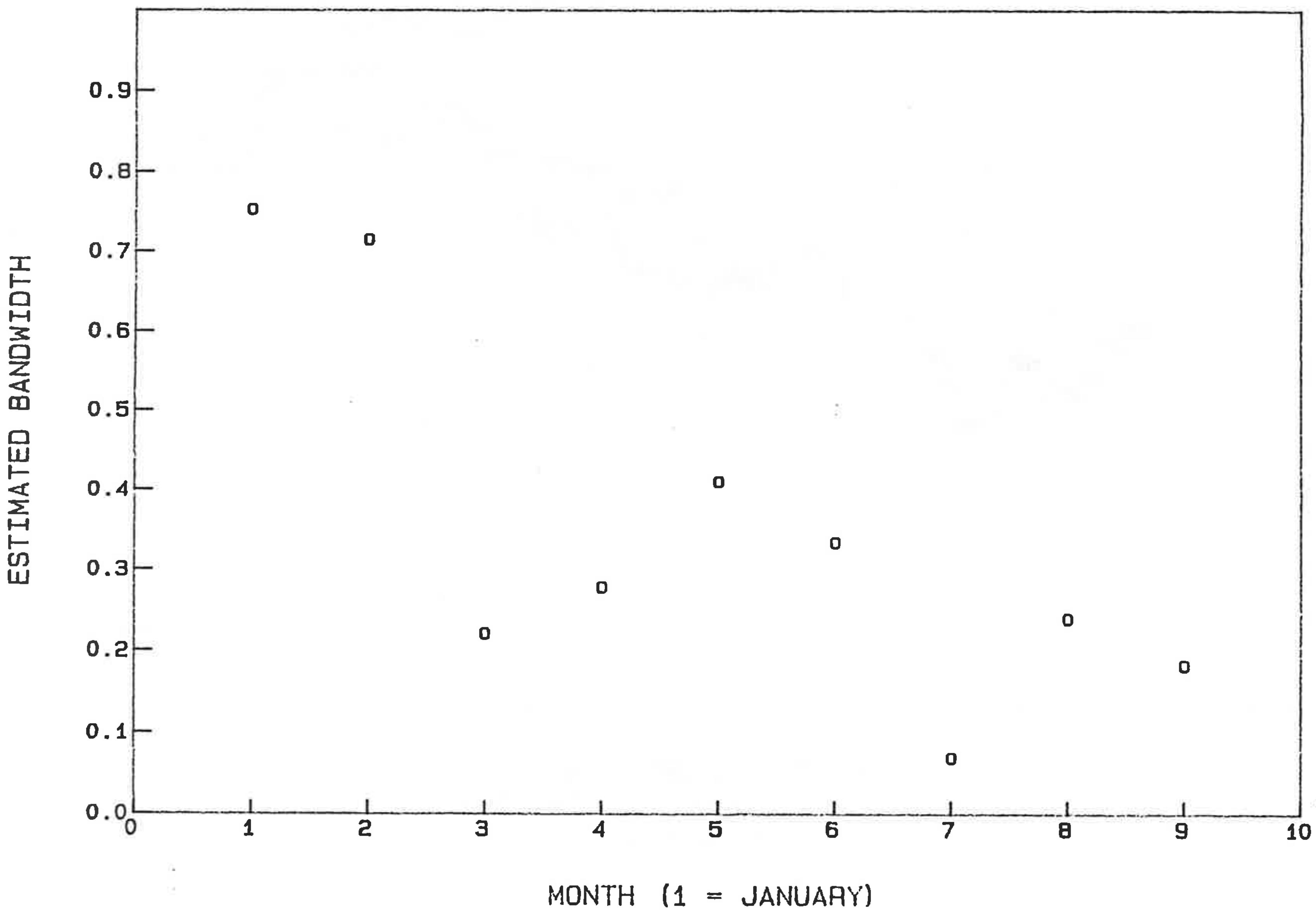
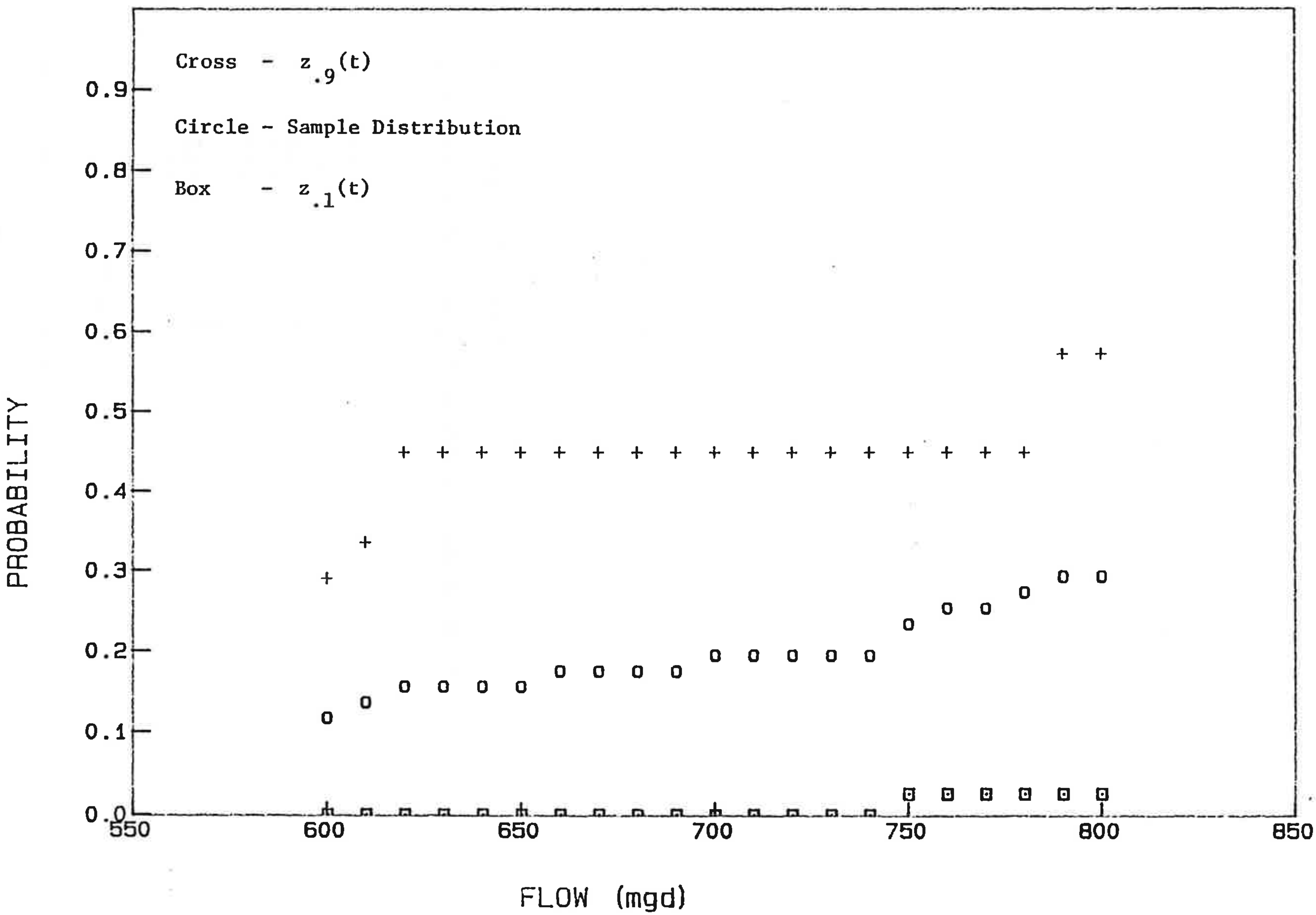


FIGURE 3.7
CONDITIONAL DISTRIBUTION OF
MINIMUM POTOMAC FLOW



IMSE Value

Month	Minimimum Flow for Preceding 30 days	60 days	90 days
1	.156	.137	.152
2	.150	.151	.135
3	.154	.155	.151
4	.144	.155	.155
5	.161	.147	.155
6	.142	.136	.114
7	.094	.093	.093
8	.115	.115	.115
9	.088	.101	.101

TABLE 3.1. IMSE values by month for minimum flow covariate random variables. Month 1 is January.

	IMSE Value		
	Accumulated Flow for Preceding		
Month	30 days	60 days	90 days
1	.149	.150	.151
2	.160	.153	.147
3	.163	.157	.152
4	.156	.161	.155
5	.162	.158	.157
6	.166	.149	.159
7	.125	.140	.148
8	.115	.120	.137
9	.085	.090	.101

TABLE 3.2. IMSE values by month for accumulated flow covariate random variables. Month 1 is January.

CHAPTER 4

PARAMETRIC OPERATING RULES FOR WATER QUALITY OPERATION

4.1- INTRODUCTION

Juhle [1986] notes that "it is surprising how little attention is given to low flow management. The Corps of Engineers has over 650 projects,... many of which are not being utilized to their maximum potential for low flow management." Juhle suggests that the traditional safe yield approach to low flow management should be replaced by seasonally varying strategies that push the reservoir beyond its safe yield. In this chapter a family of seasonally varying operating rules, termed "conditional yield operating rules", is introduced. Conditional yield operating rules extend safe yield rules in a straightforward fashion. The principal difference is that for conditional yield rules reliability remains constant over the course of a year, not release. An important feature of conditional yield operating rules is that they can be viewed as "parametric operating rules". Heuristically, a parametric operating rule specifies reservoir release in terms of previous inflow (and quantities that can be computed from inflow, such as reservoir storage) and a finite number of real-valued parameters.

Conditional yield operating rules are developed within the framework of the "maximum yield problem". The basic goal of the maximum yield problem is to balance the conflicting objectives of maximizing the quantity of releases and maximizing the reliability of releases. The maximum yield problem is formulated for operation of water quality storage in Jennings Randolph reservoir, located on the North Branch Potomac River in western Maryland.

Contents of the sections are as follows. Definitions and notation are given in Section 4.2. Also contained in Section 4.2 is an introduction to parametric operating rules. Quantile yield and conditional yield operating rules are the topics of Section 4.3. Safe yield operation is a special case of a quantile yield operating rule. The maximum yield problem is the principal topic of Section 4.4. Application of yield techniques to operation of Jennings Randolph is contained in Section 4.5. A summary and conclusions are given in Section 4.6.

4.2- DEFINITIONS AND NOTATION

We consider daily operation of a single reservoir with capacity C. A year consists of T days and begins on June 1 (the period of water quality operation for Jennings Randolph reservoir extends from June 1 - November 31; the number of days T during the year is 183). Reservoir storage on day t of the year is denoted $S(t)$. The reservoir release rule on day t is denoted $R(t)$ and reservoir inflow is denoted $X(t)$. It is assumed throughout that $S(0) = C$, that is, reservoir storage is at capacity on the first day of each year. When yearly dependence is important, we modify the notation to include a year index. Storage on day t of year i, for example, will be denoted $S_i(t)$.

Continuity is represented by the following equations:

$$S(t) = \min\{S(t-1) + X(t) - \tilde{R}(t) , C\} \; ; \quad t = 2,\ldots,T \qquad (4.1)$$

where

$$\tilde{R}(t) = \max\{S(t-1) - X(t) , R(t)\} \qquad (4.2)$$

Note that the actual amount of water released is given by the process $\{\tilde{R}(t)\}$. By (4.2) if our operating policy specifies a release larger than storage at time t-1 and reservoir inflow during day t, the actual release is reduced from the release rule value to the available storage plus inflow. In this case storage is completely depleted.

We say that $\{R(t),t=1,\ldots,T\}$ is a parametric operating rule with operating parameter, $a = (a_1,\ldots,a_k)$, and operating function $(f_t,t=1,..,T)$, provided that

$$R(t) = f_t(X(1),\ldots,X(t-1),a) \quad , \quad t = 1,\ldots,T \qquad (4.3)$$

where f_t is a non-negative function of (t-1)+k arguments. The operating function is treated as known, the operating parameters as unknown.

Parametric operating rules are encountered most often in implicit form rather than the explicit form of (4.3). Most notable is the following class of operating rules. If

$$R(t) = f_t(S(1),\ldots,S(t-1),R(1),\ldots,R(t-1),a) \qquad (4.4)$$

where f_t is a positive function of 2(t-1)+k arguments, then R(t) is a parametric operating rule with parameter a.

EXAMPLE 4.1. The linear decision rule (ReVelle et al. [1969]) is specified by

$$R(t) = a_1 \cdot S(t-1) + a_2 \qquad (4.5)$$

It is a parametric operating rule for which the operating function is a linear function of current storage; the operating parameter is (a_1,a_2).

It follows from our assumptions that reservoir inflow is the driving random process. Storage and release are random, but only through their dependence on inflow. It is assumed that the random vectors $\{X_i(t), t=1,\ldots,T\}$ $i=1,2,\ldots$ are independent and identically distributed (i.i.d). No further distributional assumptions will be made.

4.3- CONDITIONAL YIELD OPERATING RULES

The topic of this section is a particular class of parametric operating rules which will be termed conditional yield operating rules. In Section 4.4 we illustrate utility of these operating policies for the maximum yield problem. Conditional yield operating rules are straightforward extensions of safe yield operating rules. We begin this section with a statistical development of quantile yield and safe yield operating rules. Extension to conditional yield operating rules will follow.

The annual yield Y is the solution to the math programming problem

$$\begin{aligned}
&\text{maximize } y \\
&\text{such that } \quad S(0) = C \\
&\qquad\qquad\quad S(t) = \min\{C, S(t-1) + X(t) - y\} \\
&\qquad\qquad\quad S(t) > 0 \\
&\qquad\qquad\quad \text{for } t = 1,\ldots,T.
\end{aligned} \tag{4.6}$$

The annual yield is the maximum quantity of water that can be released on a constant daily basis without at any time during the year completely depleting storage. The random variable Y is a complicated function of $\{X(t), t=1,\ldots,T\}$. We denote its distribution function by

$$F(y) = P\{Y \leq y\} \tag{4.7}$$

The quantile function of F is defined by

$$Q(a) = \inf\ \{y\colon F(y) > a\} \qquad a \in [0,1) \tag{4.8}$$

The value of Q(a) is the reservoir yield that can be maintained in a given year with probability 1-a. A "quantile yield" operating rule is of the form

$$R(t) = Q(a) \tag{4.9}$$

Note that a quantile yield rule is a parametric operating rule for which the operating function is a constant, determined by the operating parameter a. The operating parameter a is termed the "operational reliability". If a is very low, reservoir yield will be low but reliability of delivering that yield will be high.

With the preceding framework safe yield y_F can be defined as the lower bound of the distribution F. It follows from (4.8) that

$$y_F = Q(0) \tag{4.10}$$

so that a safe yield policy is a special case of a quantile yield operating rule.

Conditional yield operating policies are based on the notion that yield of a reservoir may vary substantially over the course of a year. If, for example, a large storm fills the reservoir in mid-summer, yield of the reservoir has increased from what it was just before the storm. Similarly, yield reliability for a given storage s, should be strongly dependent on whether we are early or late in the annual operating cycle.

The "conditional yield" on day t of the year, Y(t), is the maximum quantity of water that can be released from day t onward on a constant daily basis until the end of the year without at any time depleting storage. Given that S(t) equals s, the conditional yield Y(t) is the solution to the math programming problem

$$\begin{aligned} &\text{maximize } y \\ &\text{such that } S(t) = s \\ &\qquad S(u) = \min\{C, S(u-1) + X(u) - y\} \\ &\qquad S(u) > 0 \\ &\qquad \text{for } u = t+1, \ldots, T. \end{aligned} \tag{4.11}$$

Clearly the conditional yield on day t is strongly dependent on the storage S(t).

The conditional distribution of yield at t is given by

$$F_t(y|s) = P\{Y(t) \le y \mid S(t) = s\} \tag{4.12}$$

Similarly the conditional quantile function at t is

$$Q_t(a|s) = F_t^{-1}(a|s) \qquad a \in (0,1) \tag{4.13}$$

A conditional yield operating rule is specified by

$$R(t) = Q_t(a|S(t)) \tag{4.14}$$

where a, the conditional system reliability, is the one unknown parameter.

A problem that arises in implementing either a quantile yield or conditional yield operating policy is that the distributions F and F_t are not known a priori and must be estimated from streamflow data. Given n years of historical streamflow data $\{X_i(t), t=1,...,T; i=1,...,n\}$ we obtain a random sample $Y_1,...,Y_n$ from F (or $Y_1(t),...,Y_n(t)$ if we are estimating F_t). The order statistics of $Y_1,...,Y_n$ are denoted $Y_{(1)}<...<Y_{(n)}$. The quantile estimator we use is obtained by inverting the linearized sample distribution function, F. The estimated quantile function is

$$\hat{Q}(a) = \hat{F}^{-1}(a) \qquad 1/(n+1) < a < n/(n+1) \tag{4.15}$$

The procedure described above does not provide an estimator of the safe yield y_F. Smith and Weissman [1985] and Loaiciga and Marino [1987] describe procedures for estimating the "lower tail" of a distribution that can be used to estimate the the safe yield.

4.4- MAXIMUM YIELD RESERVOIR MANAGEMENT

Maximum yield reservoir management has two conflicting objectives: 1) maximizing the quantity of releases and 2) maximizing the reliability of releases. A natural measure of performance for the first objective is the total release during the course of the year

$$U = T^{-1} \sum_{t=1}^{T} \tilde{R}(t) \tag{4.16}$$

given here as average daily release. In practice, the most common way of measuring reliability is in terms of the minimum release

$$\tilde{V} = \min\{R(t), t=1,\ldots,T\} \qquad (4.17)$$

If a minimum release r_m and reliability p can be specified, the two objectives can be combined to form the maximum yield problem

$$\begin{aligned} &\underset{R \in R^*}{\text{maximize}}\ E[U] \\ &\text{such that} \quad P\{V < r_m\} < p \end{aligned} \qquad (4.18)$$

where R^* is the class of operating rules we optimize over. Note that we do not guarantee a minimum release of r_m. We only guarantee that a minimum release of r_m can be maintained in a given year with probability 1-p.

In specifying R^*, r_m, and p, we determine the flexibility that is allowed in reservoir management. At one extreme we obtain the following maximum yield problem

$$\begin{aligned} &\underset{a \in [0,1)}{\text{maximize}}\ E[U(a)] \\ &\text{such that } R(t) = Q(a) \\ &\qquad\qquad P(V(a) < r_m\} \le p. \end{aligned} \qquad (4.19)$$

In this case R^* is the family of quantile yield operating rules. The minimum flow V(a) will differ from the average daily flow U(a) only if storage is depleted at some time during the year.

The optimal operating rule for (4.19) satisfies

$$R(t) \ge Q(p) \qquad (4.20)$$

if Q(p) is greater than or equal to r_m. If Q(p) is less than r_m, (4.19) has no feasible solution. To obtain (4.20) note that if $a_1 > a_2$,

$$E[U(a_1)] > E[U(a_2)] \qquad (4.21)$$

that is, the larger the constant release rule, the larger the expected cumulative release (strict inequality in (4.21) requires a minimal assumption on the streamflow process, such as existence of continuous density for $X_i(t)$). If $Q(p)$ is greater than or equal to r_m

$$P\{V(p)<r_m\} < P\{V(p)<Q(p)\} = p \tag{4.22}$$

It follows from (4.21) that $R(t)$ is at least as large as $Q(p)$. In practice $R(t)$ will likely be very close to $Q(p)$, because in years for which storage is depleted, actual release is reduced to inflow. The minimum release r_m will typically be much larger than reservoir inflow during extreme reservoir drawdown periods.

If we take the class of operating rules R^* to be the conditional yield operating rules we obtain the following formulation of the maximum yield problem.

$$\begin{aligned} &\underset{a \in (0,1)}{\text{maximize}} \quad E[U(a)] \\ &\text{such that } R(t) = Q_t(a|S(t)) \\ &\qquad P\{V(a) < r_m\} \le p. \end{aligned} \tag{4.23}$$

In this formulation R^* is the family of conditional yield operating rules.

Because we treat inflow in a nonparametric framework we have little hope of analytically evaluating $E[U(a)]$ and $P\{V(a)<r_m\}$. We can, however, estimate them. If we have n years of historical inflow data $\{X_i(t);t=1,\ldots T;i=1,\ldots,n\}$ our estimators are

$$\hat{G}_n(a) = n^{-1} \sum_{i=1}^{n} U_i(a) \tag{4.24}$$

and

$$\hat{H}_n(a) = n^{-1} \sum_{i=1}^{n} 1(V_i(a) < r_m) \tag{4.25}$$

Note that the inflow data set is serving two functions. To compute U_i and V_i we use streamflow data from year i as the inflow sequence needed for evaluating (4.16) and (4.17). We also use streamflow data from the n-1 remaining years to estimate the conditional quantile functions (as described at the end of Section 4.3) necessary for implementing the conditional yield rules.

We now choose $\hat{a}$ as any solution to the optimization problem

$$\text{maximize } \hat{G}_n(a)$$

$$a \in [0,1) \tag{4.26}$$

$$\text{such that } \hat{H}_n(a) = p \quad .$$

We will use $\hat{a}$ as our operating parameter. To solve the optimization problem (4.26) a numerical search procedure is used.

4.5- EXAMPLE

Jennings Randolph is a multipurpose reservoir located on the North Branch Potomac River. Drainage area of the North Branch above the dam is 240 square miles. The reservoir has a total capacity of 30 billion gallons of which 16.6 billion gallons are authorized for water quality. The principal goals of water quality operation are maintenance of aquatic habitat in the river below the dam and dilution of effluents from downstream industrial dischargers. For both purposes it is desirable to release as much water as possible.

The Corps of Engineers has determined that a successful flow augmentation season is one in which minimum release is maintained above 100 mgd. Juhle [1986] notes that 8 or 9 good years (out of 10) are much preferable to 10 mediocre years.

Maximum yield problems are formulated below for operation of water quality storge in Jennings Randolph. The minimum release r_m is taken to be 100 mgd and the probability level p is taken to be .1.

Figure 4.1 shows the annual yield distribution for Jennings Randolph water quality storage. It follows from (4.20) that the optimal policy for the maximum yield problem of (4.19) is approximately, R(t) = 160 mgd. Using the estimator (4.24) the optimal mean yield is 145 mgd.

The optimal conditional yield rule was also determined using r_m equal to 100 mgd and p equal to .10. The optimal parameter obtained from (4.26) is .40. Figure 4.2 shows the sequence of relases using a conditional yield rule during a dry year (1966).

The optimal mean yield obtained from (4.24) is 210 mgd. It follows that the quantity of water released from the reservoir is 60% larger using conditional yield rules rather than a constant quantile yield rule. The reliability is the same for both rules.

4.6- SUMMARY AND CONCLUSIONS

The statistical approach we propose for developing reservoir operating rules contrasts sharply with stochastic dynamic programming. In stochastic dynamic programming structure of operating rules is general and strict parametric assumptions are made on the random component of the process, reservoir inflow (see Yakowitz [1982]). We take the opposite approach. We severely restrict the structure of operating rules and treat the random component, reservoir inflow, in a nonparametric framework.

The procedure we recommend for developing reservoir operating rules conforms closely to traditional engineering practice. Specification of the form of operating rules is carried out prior to the optimization step and is generally based on the experience of system operators. Operating rules have several unknown parameters, which are determined by simulation (using historical data) to optimize a specified measure of system performance. The optimization step is present in our approach but in a diminished role, as compared with stochastic dynamic programming. Our approach is motivated in part by Rogers and Fiering's [1986] suggestion that "the use of optimizing models be softened in favor of systematic analysis".

Uitlity of parametric operating rules extends beyond the water quality problem discussed in this section. Parametric operating rules are used for operation of water supply reservoirs in the Potomac River basin (see Chapter 5). Parametric operating rules have also been developed for flood control problems (Smith and Karr [1986] and Chapter 6).

FIGURE 4.1
JENNINGS RANDOLPH YIELD
WATER QUALITY STORAGE

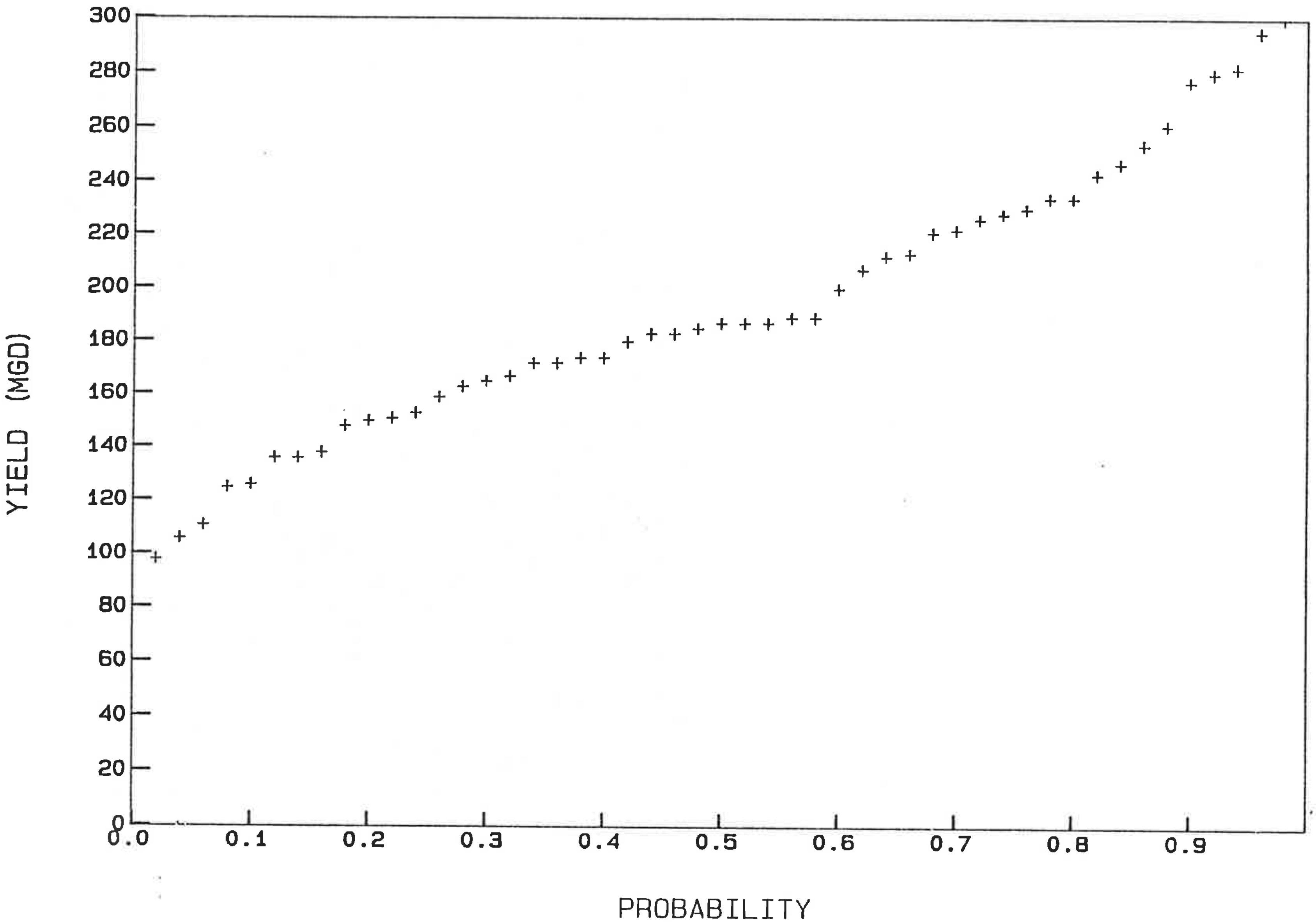
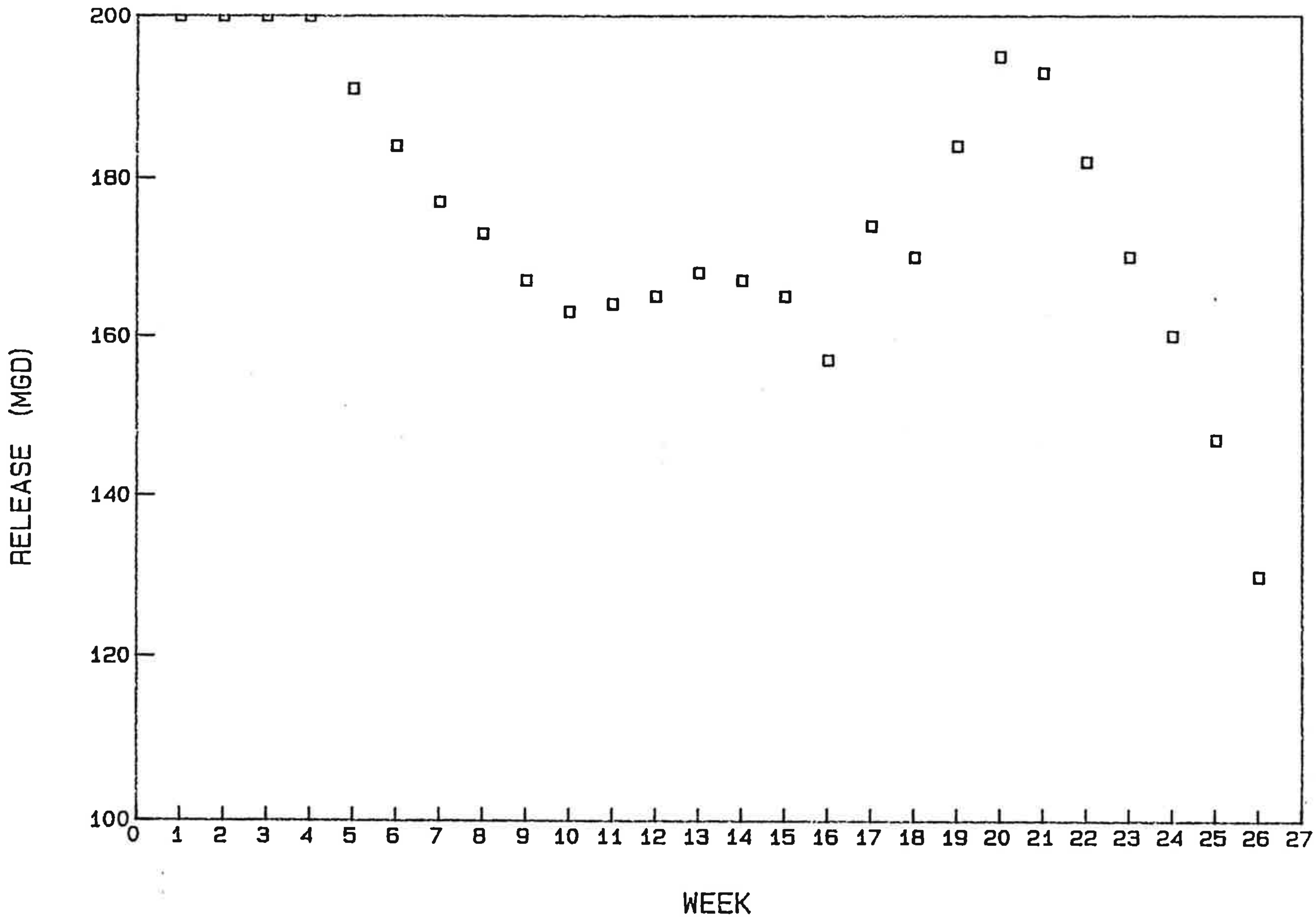


FIGURE 4.2
JENNINGS RANDOLPH RELEASE



CHAPTER 5

PARAMETRIC OPERATING RULES FOR WATER SUPPLY OPERATION

5.1. INTRODUCTION

The Washington Metropolitan Area (WMA), consisting of the District of Columbia, the Maryland suburbs of D.C., and the Virgina suburbs of D.C., has experienced rapid growth in the 1980's. Associated with population growth is a sharp increase in water use (see Figure 2.1). In this chapter yield analysis techniques are developed to assess adequacy of the current WMA water supply system to meet escelating future demands.

Three major water utilities (one for each of the principal geographic subregions) provide drinking water for the WMA. Water supplies for the three utilities are operated jointly to minimize the risk of water supply shortage (see Sheer and Flynn [1985]). The direct link among the three water utilities is reliance on the Potomac River. Natural flow of the Potomac River can be augmented by water supply releases from two upstream reservoirs, Jennings Randolph reservoir, located on the North Branch Potomac River in western Maryland, and Little Seneca reservoir, located on Little Seneca Creek in the northwest corner of the WMA.

The yield analysis techniqes developed in this chapter are used to examine the "operational" yield of an existing water supply system rather than the potential yield of a water supply system under design. Three features of the yield analysis techniques are key to our use of the term operational: 1) the modeling time step is daily, 2) actual reservoir operating rules are incorporated, and 3) variability in daily water use is accomodated.

Techniques developed in this chapter borrow from several sources. The work of Palmer et al. [1982], in which a statistical treatment of water supply yield is developed, is a direct antecedent of this paper (see also Vogel and Stedinger [1987]). The search for increased water supply yield from joint reservoir operation is motivated by synergystic gain concepts introduced by Hirsch et al. [1977]. The emphasis on water use variability in the yield model of Section 5.4 builds upon developments in short-term water use modeling (see, for example, Maidment and Parzen [1985a] and [1985b] and Smith [1987]).

Contents of the sections are as follows. Section 5.2 contains definitions, notation, and development of the statistical model used for yield analysis. Yield analysis models for the WMA are developed in Section 5.3. In Section 5.4 the yield analysis techniques are extended to account for variability in water use. A summary and conclusions are presented in Section 5.5.

5.2. DEFINITIONS AND NOTATION

Storage in Jennings Randolph reservoir on day t of year i is denoted by $S1_i(t)$; the capacity of water supply storage for Jennings Randolph is denoted C_1 and equals 13.4 billion gallons. Storage in Little Seneca reservoir is denoted $S2_i(t)$; the capacity of Little Seneca is denoted C_2 and equals 4.0 billion gallons. The release rules are denoted $R1_i(t)$ and $R2_i(t)$, reservoir inflow $Z1_i(t)$ and $Z2_i(t)$. Natural flow of the unregulated portion of the Potomac River is $X_i(t)$. A "water supply year" begins on June 1 and consists of $T = 365$ days. It is assumed that reservoir storage is at capacity at the beginning of each year, that is, $S1_i(0)$ equals C_1 and $S2_i(0)$ equals C_2.

Catchments of the two reservoirs differ significantly in their hydrologic characteristics. The drainage area of the North Branch Potomac River above Jennings Randolph is 240 square miles. Mean annual runoff for the mountainous North Branch is 20 inches. The drainage area of Little Seneca Creek above the reservoir is 21 square miles. Mean annual runoff of Little Seneca Creek is 12 inches. Jennings Randolph not only has a much larger ratio of drainage area to capacity than Little Seneca, it also has a much larger ratio of mean inflow to drainage area. Hydrologic characteristics and reservoir location play a major role in determining the form of reservoir operating rules (see Section 5.3).

We introduce in Sections 5.3 and 5.4 two definitions of "annual yield." The following examples illustrate the form annual yield random variables may take in special cases.

EXAMPLE 1-

The yield of a single reservoir for year i, Y_i, is the solution to the math programming problem,

$$\begin{aligned}
&\text{maximize } y \\
&\text{such that } R_i(t) = y \\
&\qquad S_i(t) = \min\{S_i(t-1) + Z_i(t) - R_i(t), C\} \\
&\qquad S_i(t) > 0 \\
&\qquad \text{for } t = 1,\ldots,T .
\end{aligned} \tag{5.1}$$

Because we consider only a single reservoir, dependence on the reservoir index is suppressed in the release, storage, and inflow notation of (5.1). The reservoir yield Y_i is the largest constant release that can be maintained in year i without at any time completely depleting storage, given that the reservoir begins the year with storage at capacity.

EXAMPLE 2-

The yield for an unreglated River for year i is simply the minimum daily flow, that is,

$$Y_i = \min \{X_i(t),\ t=1,\ldots,T\}. \qquad (5.2)$$

EXAMPLE 3-

The combined yield of a river and upstream reservoir can be defined, in certain cases, as the solution to

$$\begin{aligned} &\text{maximize } y \\ &\text{such that } R_i(t) = \max\{y - X_i(t), 0\} \\ &\qquad S_i(t) = \min\{S_i(t-1) - R_i(t)_1 + Z(t), C_1\} \\ &\qquad S_i(t) > 0 \\ &\qquad \text{for } t = 1,\ldots,T. \end{aligned} \qquad (5.3)$$

Releases from the reservoir are made only if natural flow of the river falls below the yield value y we wish to maintain. This model is not appropriate for assessing yield of the WMA water supply system. Implicit in equation (5.3) is the assumption that reservoir releases are immediately available at the demand point. Releases from Jennings Randolph reservoir require approximately 5 days to reach Washington, D.C. (Trombley [1982]). Furthermore, the WMA system consists of two reservoirs. Extension of equation (5.3) to multiple reservoir systems requires explicit specification of operating rules to allocate the deficit, $\max\{y-X_i(t),0\}$, among the reservoirs.

Yield random variables developed in Sections 5.3 and 5.4 are defined implicitly by representations like equations (5.1) - (5.3). We assume that yield random variables $Y_1,\ldots,Y_n$ are independent and identically distributed (i.i.d.). Their common distribution is denoted by

$$F(y) = P\{Y_i \le y\} \qquad (5.4)$$

Parametric assumptions on yield distributions are not made. The independence assumption rests on critical periods (or minimum annual flow) being separated from year to year by sufficiently long non-drawdown periods (see Loaiciga and Marino [1987] for detailed discussion; see also Matalas [1963]). For reservoir yield samples, the identically distributed assumption is valid and useful only if reservoirs refill annually.

For water supply yield analysis interest focuses on the lower tail of the distribution F. A useful tool for describing the lower tail of F is the quantile function

$$Q(p) = \inf \{y : F(y) > p\} \qquad p \in [0,1). \tag{5.5}$$

The quantile function evaluated at p gives the yield that can be maintained in a given year with probability 1-p. Of particular importance is the "safe yield" y_F, which is defined to be the lower bound of F, that is

$$y_F = Q(0) . \tag{5.6}$$

In analyzing yield data it is useful to deal with the order statistics $Y_{(1)}<...<Y_{(n)}$. We term the minimum value $Y_{(1)}$ the "historic yield" (following Palmer et al. [1982]). The "interior" portion of the distribution F (see Section 5.3) can be estimated from order statistics using standard procedures (see, for example, David [1970]). Smith and Weissman [1985] and Loaiciga and Marino [1987] develop techniques that can be used to estimate "exterior" characteristics of the yield distribution, including the safe yield y_F.

5.3. YIELD ANALYSIS - CONSTANT DEMAND

The traditional method for assessing yield of a water supply system is to combine the historic yield values for each of the system components. The historic yield values for Jennings Randolph and Little Seneca reservoir, obtained from equation (5.1), are 63 mgd and 13 mgd, respectively. The historic yield for the Potomac River, obtained from equation (5.2) is 406 mgd. Combining the historic yield values, we obtain a system yield of 482 mgd. Hirsch et al. [1977] show that if components of a water supply system are operated jointly the system yield may be substantially larger than the sum of the individual historic yields. They term the increased yield due to joint operation the synergistic gain. In this section a yield model is formulated for assessing the joint yield of the Potomac River, Jennings Randolph reservoir, and Little Seneca reservoir.

To obtain an analysis of system yield relevant to operational conditions we must explicitly incorporate reservoir operating rules. The basic guidelines for operation of Jennings Randolph and Little Seneca are straightforward. Releases are not made when demand can be met from natural flow of the Potomac River (which is likely to be the case for the better part of any year). When reservoir releases are needed, Jennings Randolph provides the "average" shortfall between demand and natural flow of the Potomac. Little Seneca fills in holes created by "extreme" shortfalls between demand and Potomac flow (releases from Little Seneca reservoir reach the WMA within the day of release). These operating guidelines are based on the different travel times of reservoir releases to the WMA and the contrasting hydrologic characteristics of the two catchments (see Section 5.2).

The operating rules that are used for Jennings Randolph and Little Seneca are of the following form. To meet a constant demand of y for the WMA, the Jennings Randolph release rule is

$$R1_i(t) = \max\{a_1(y - X_i(t)) + a_2,\ 0\} \tag{5.7}$$

where a_1 is a nonnegative constant and a_2 is a real constant. The release rule for Little Seneca is

$$R2_i(t) = \max\{y - \tilde{X}_i(t),\ 0\} \tag{5.8}$$

where

$$\tilde{X}_i(t) = X_i(t) + R1_i(t-5) \tag{5.9}$$

is natural flow of the Potomac plus the routed releases from Jennings Randolph (recall from Section 5.2 that releases from Jennings Randolph reservoir require 5 days to reach the WMA). By increasing either a_1 or a_2, Jennings Randolph releases are increased and reliance on the smaller local reservoir is reduced.

With the preceding operating rules, the system yield for year i is defined by

$$\begin{aligned}
&\text{maximize} \quad y \\
&\text{such that } R1_i(t) = \max\{a_1(y - X_i(t)) + a_2, 0\} \\
&\qquad R2_i(t) = \max\{y - \tilde{X}_i(t), 0\} \\
&\qquad S1_i(t) = \min\{S1_i(t-1) + Z1_i(t) - R1_i(t), C_1\} \\
&\qquad S2_i(t) = \min\{S2_i(t-1) + Z2_i(t) - R2_i(t), C_2\} \\
&\qquad S2_i(t) > 0 \\
&\qquad \text{for } t = 1, \ldots, T\ .
\end{aligned} \tag{5.10}$$

Note that the system fails when Little Seneca runs out of water.

The yield distribution of random variables Y_1, Y_2,... obtained from equation (5.10) is dependent on reservoir operating rules. The dependence is simple because the operating rules are completely specified by streamflow observations and the two parameters a_1 and a_2. We will denote by $Q_a(p)$ the quantile function of the yield distribution indexed by the parameters $a = (a_1, a_2)$. In other words, $Q_a(p)$ is the quantile function of yield random variables $Y_1, Y_2, \ldots$ obtained from equation (5.10), given that the operating parameters are (a_1, a_2).

The parameters a_1 and a_2 are chosen to optimize specific characteristics of the yield distribution. The optimization problem can be formulated as follows: choose the parameters (a_1, a_2) to maximize the "weighted yield"

$$I(a) = \int_0^1 w(p)Q_a(p)dp \qquad (5.11)$$

where

$$\int_0^1 w(p)dp = 1 .$$

The weight function w(p) determines attributes of the yield distribution that are to be optimized. The following examples illustrate flexibility of (5.11).

EXAMPLE 4. If the weight function is the Dirac delta function

$$w(p) = \delta_{p_0} \qquad (5.12)$$

then

$$I(a) = Q_a(p_0) \qquad (5.13)$$

In particular if p_0 equals 0, the objective is to maximize the safe yield $Q_a(0)$.

EXAMPLE 5. If the weight function is constant and equal to 1,

$$I(a) = E_a[Y_i] \tag{5.14}$$

The objective in this case is to maximize the expected yield.

EXAMPLE 6. If

$$\begin{aligned} w(p) &= 10 \qquad p \in [0,.1] \\ w(p) &= \;\,0 \qquad \text{otherwise,} \end{aligned} \tag{5.15}$$

then the objective is to maximize the "average yield" over the lower 10% of the yield distribution, that is,

$$I(a) = 10 \cdot \int_0^{.1} Q_a(p)dp \tag{5.16}$$

Because the yield distribution is treated in a nonparametric framework, there is little hope of evaluating the weighted yield I(a) analytically. The weighted yield can, however, be estimated by

$$\hat{I}(a) = \sum_{i=1}^{n} w_i Y_{(i)} \tag{5.17}$$

where

$$w_i = (n+1)^{-1} w(i/(n+1)) \tag{5.18}$$

and $Y_{(i)}$ is the ith order statistic obtained from equation (5.10).

EXAMPLE 4a. if we take

$$w_i = 1 \qquad i = j \tag{5.19}$$
$$w_i = 0 \qquad i \neq j$$

equation (5.17) provides an estimator of

$$I(a) = Q_a(p_0) \qquad j/n \in [p_0-(2n)^{-1}, p_0+(2n)^{-1}]. \tag{5.20}$$

EXAMPLE 5a. If we take

$$w_i = 1/n \qquad \forall \; i \tag{5.21}$$

equation (5.17) provides an estimator for

$$I(a) = E_a[Y_i] \tag{5.22}$$

EXAMPLE 6a. If we take

$$w_i = 10/n \qquad i < 10/n \tag{5.23}$$
$$w_i = 0 \qquad i > 10/n$$

equation (5.17) provides an estimator for

$$I(a) = 10 \cdot \int_0^{.1} Q_a(p)dp \tag{5.24}$$

Table 5.1 contains parameter estimates and historic yield values for several choices of weight function. To maximize the historic yield we can take w_1 equal to 1 and all other weights equal to 0. The optimal historic yield in this case is 718 mgd, an increase of 236 mgd above the sum of the component historic yields!

By placing all weight on the historic yield year, operations are closely tied to a particular sequence of hydrologic events. In practice, it may be desirable to take a smoother weighting of the lower tail of the yield distribution. Note that if we take w_1 equal to .5, $w_2 = w_3 = .2$, and $w_4 = .1$, we lower the historic yield to 712 mgd, but significantly increase the remainder of the lower tail of $Q_a(p)$.

5.4. YIELD ANALYSIS - VARIABLE DEMAND

The yield results of the preceding section are not directly comparable to the information available concerning long-term trend in water use (see Figure 2.1). In this section annual yield random variables will be represented in terms of mean daily water use so that yield and demand are directly comparable. A simple model of daily water use (see Chapter 2) is used to assess the effect of water use variability on water supply yield.

Water use for day t of year i is denoted $D_i(t)$. If mean daily water use for year i is y, the water use model is specified by

$$D_i(t) = m(t) \cdot y + a \cdot [D_i(t-1) - m(t-1) \cdot y] + y^{1/2} A_i(t) \tag{5.25}$$

where a is a real-valued parameter and m(t) is the "unit demand function"

$$m(t) = \frac{E[D_i(t)]}{\sum_{j=1}^{T} E[D_i(j)]} \tag{5.26}$$

The unit demand function at time t, m(t), is the ratio of average water use on day t of the year to average daily water use over the course of the year. The unit demand does not vary from year to year even if long-term trend in mean water use is present. This assumption implies that, although mean water use may exhibit trends over time, seasonal and day-of-week structure of water use do not. The error process $A_i(t)$ is assumed to be an i.i.d. sequence of Gaussian random variables with mean 0 and variance s^2.

The reservoir operating rules of equations (5.7) and (5.8) are easily modified to account for daily variability in water use. If mean daily water use for the year is y, the Jennings Randolph release is

$$R1_i(t) = \max \{a_1[m(t+5)\cdot y - X_i(t)] + a_2,\ 0\} \quad (5.27)$$

The Little Seneca release is

$$R2_i(t) = \max \{D_i(t) - \tilde{X}_i(t),\ 0\} \quad (5.28)$$

The system yield for year i, Y_i, is the solution to,

$$\text{maximize } y \quad (5.29)$$

$$\text{such that } D_i(t) = y\cdot m(t) + a[D_i(t-1)-y\cdot m(t-1)] + y^{1/2}A_i(t)$$

$$R1_i(t) = \max \{a_1[m(t)\cdot y-X_i(t)] + a_2,\ 0\}$$

$$R2_i(t) = \max \{D_i(t) - \tilde{X}_i(t),\ 0\}$$

$$S1_i(t) = \min \{S1_i(t-1) + Z1_i(t) - R1_i(t), C_1\}$$

$$S2_i(t) = \min \{S2_i(t-1) + Z2_i(t) - R2_i(t), C_2\}$$

$$S2_i(t) > 0$$

$$\text{for } t = 1,\ldots,T .$$

In previous formulations, randomness in annual yield is attributed solely to randomness in streamflow. In the current formulation randomness is additionally attributed to random fluctuations in daily water use. Because water use exhibits marked trend over time, it is not possible to use water use observations directly, as we do with streamflow. Instead we use historical water use data to estimate parameters of the water use model (see Section 2.3 of Chapter 2). A "data set" of error variables $\{A_i(t);t=1,\ldots,T;i=1,\ldots,n\}$ for use in equation (5.29) is then obtained by simulation.

For the case in which all weight is place on the first order statistic, the historic yield obtained from (5.29) is 724 mgd. Surprisingly, the historic yield increases from the previous formulation despite the additional variability introduced in daily water use. The explanation is to be found in Figure 5.1, which shows the estimated unit demand function m(t). Note that mean water use peaks in July; mean daily water use in July is 18% larger than mean daily water use for the entire year. The trough in mean water use occurs in winter. Mean daily water use in late winter is 10% less than mean daily water use for the entire year. Water use peaks several months before natural flow of the Potomac reaches its trough. During the extended reservoir drawdown period in fall and early winter, mean water use is at its minimum.

5.5. CONCLUSIONS

In this chapter a sequence of water supply yield models have been developed. The models are used to assess adequacy of the current WMA water supply system to meet escalating water demands. The following features of the yield models are noteworthy.

1) In each of the models the fundamental concept is "annual yield". Annual yield for a given year is a nonnegative random variable which represents the maximum yield that the water supply system can provide in that year. Randomness in annual yield may be attributed soley to randomness in supply (as in the yield model of Section 5.3) or to randomness in both supply and demand (as in the yield model of Section 5.4).
2) The statistical model that is used to analyze yield random variables is nonparametric. It is assumed that annual yield random variables are i.i.d. It is noted in Section 5.3 that justification for the yield model rests in part on the fact that reservoirs refill annually. For application to water supply systems with multi-year reservoir drawdown (as in much of the western U.S.), the yield models would require modification.
3) Annual yield random variables are dependent on reservoir operating rules. For the WMA water supply system, dependence is simple because operating rules are completely specified by streamflow observations and two real-valued parameters (a_1,a_2). The form of reservoir operating rules for the WMA water supply system is dictated by reservoir size and travel time. The large reservoir, which is located far from the WMA, provides "average" shortfalls between demand and natural flow of the Potomac River. The small reservoir, located close to the WMA, covers shortfalls arising from extreme demands. The operating parameters (a_1,a_2) determine the extent to which the large reservoir is utilized (in effect, determining the boundary between "average" and "extreme").
4) Operating parameters (a_1,a_2) are chosen to optimize a specified attribute of the yield distribution. The "weighted yield" I(a) is introduced as the criterion for selecting the

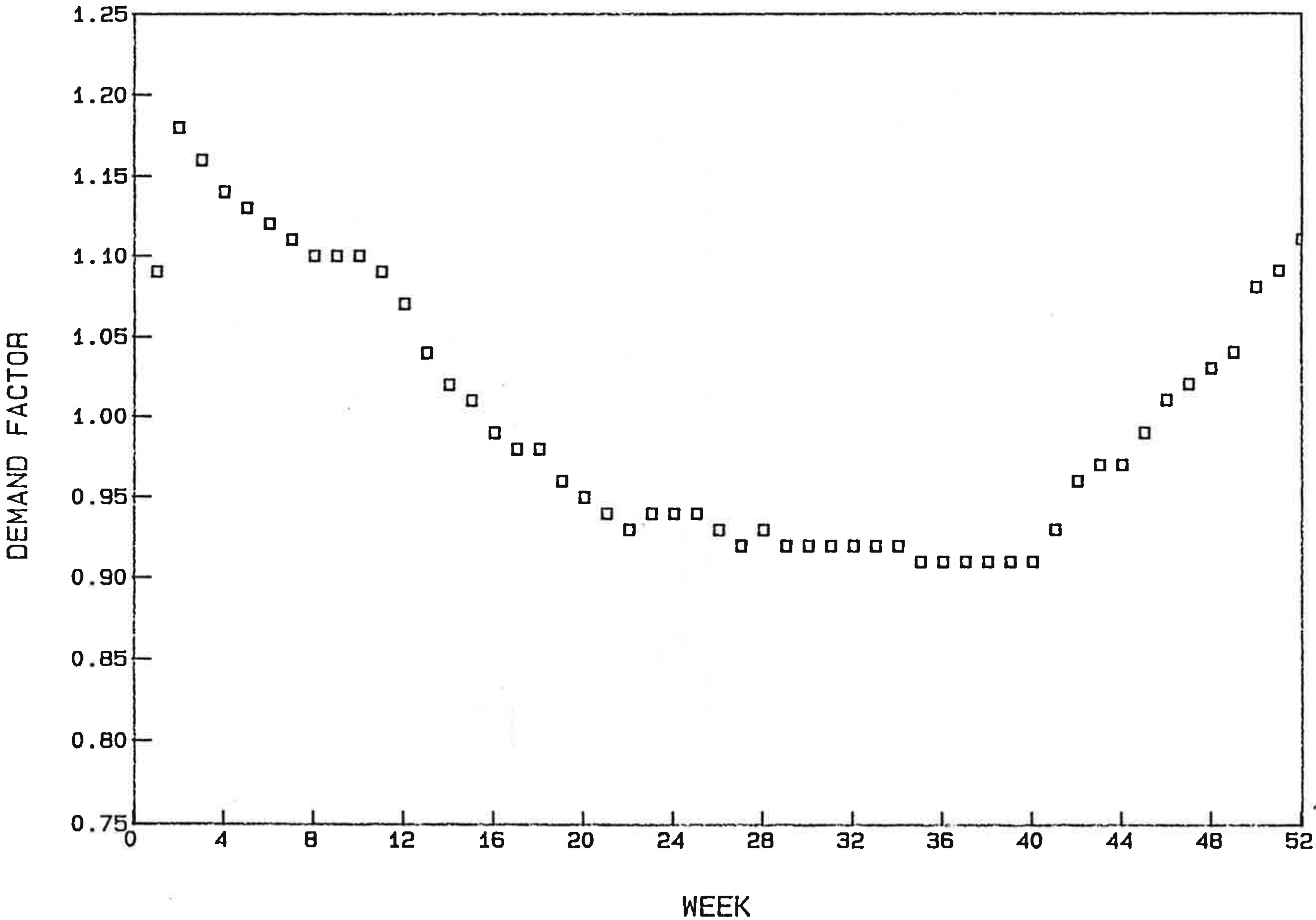
operating parameters. It is shown that the sample estimator I(a) can be used to obtain solutions to the optimization problem.

5) The yield model is extended in Section 5.4 to accommodate variability in water use. In the yield model of Section 5.4, annual yield is interpreted as the largest mean daily water use that the system can provide without experiencing supply shortfalls on any day. This formulation allows direct comparison of water supply yield with trends in mean water use.

6) The historic yield values of Jennings Randolph reservoir, Little Seneca reservoir, and the Potomac River, are, respectively, 63 mgd, 13 mgd, and 406 mgd. Combining the yield values of the three components of the WMA water supply system produces a yield of 482 mgd. The historic yield values for joint system yield models of both Sections 5.4 and 5.5 exceed 700 mgd. The synergistic gain attributed to the system operating rules is in excess of 200 mgd. Most importantly, synergistic gain places water supply yield well above the current mean water use.

The yield analysis results are encouraging. For the present, the WMA water supply system is clearly quite reliable. If water use continues to grow at the rate of the past 5 years, mean daily water use will equal the historic yield of the water supply system in approximately 30 years. Additional water supply will ultimately be needed for the WMA. The techniques used in this chapter have relevance to future design problems. If water supply storage is to be added to the WMA system, it should be added to functionally augment the current water supply system.

FIGURE 5.1
WMA WATER USE DEMAND FACTORS
(June - May)



WEIGHTED YIELD

a_1	a_2	(w_1, w_2, w_3, w_4) (1.0,0,0,0)	(.7,.3,0,0)	(.6,.3,.1,0)	(.5,.2.,2,.1)
.38	65.	719*	745	772	794
1.1	0.	709	754*	784	811
.91	43.	707	753	786*	817
.79	73.	699	747	783	820*

Table 5.1 Weighted yield values. Stars indicate optimal parameters. The parameter set (.38,65), for example, is optimal for the weights (1,0,0,0). The optimal weighted yield in this case is 719. All weights for indices larger than 4 equal 0.

CHAPTER 6

PARAMETRIC OPERATING RULES FOR FLOOD CONTROL OPERATION

6.1 INTRODUCTION

In this chapter a family of reservoir operating rules is developed that can be used to make hourly flood control releases in both a design context, and in real-time operation. The regulation of a flood through reservoir operation is considered and general characteristics of a damage reducing operating plan are described. While the regulation of damage producing runoff is viewed generally, each flood event varies in its runoff volume, peak discharge, and timing. From the general family of operating rules considered, parameters can be chosen to best achieve flood management goals for a particular high flow event. The parametric operating rule specifies reservoir release in terms of previous inflow (and quantities that can be computed from inflow, such as reservoir storage) and a finite number of real-valued parameters.

Two observations motivate the development of a simple parametric operating rule for hourly flood control operation. Part of the operating procedures at major flood control reservoirs include "standing instructions to damtenders". The detailed operating procedures at a flood control reservoir are designed to utilize a wide range of hydrometeorologic data to guide operation (see section 7). The information available to the reservoir operator will include Quantitative Precipitation Forecasts issued every 12 hours by the National Weather Service, telemetered observations of precipitation and streamflow both upstream of the reservoir as well as for critical locations downstream. From the synthesis of this data careful control of reservoir outflow will be maintained to utilize the limited volume of flood control most effectively. Nevertheless, the failure of communication channels providing this information must be anticipated, and contingency operating procedures using only the observations available at the reservoir must be a part of each reservoir operating manual.

The U.S. Army Corps of Engineers (1959) describes this need:

> "It is essential that damtenders at the various flood control and multiple-purpose reservoirs be supplied with regulation schedules to be followed in case of failure of communications with the headquarters from which instructions are normally issued during flood periods. The emergency regulations are usually expressed in terms of pool elevation, rate of rise, and release so as to obtain a reasonable amount of flood regulation over a wide range of storm conditions and still function in a manner similar to that assumed for design in the event of a flood of project magnitude."

The parametric operating rules developed in this section can be used by a "blind" reservoir operator with a minimal description of operating goals for the flood.

In addition, parametric rules can provide a quick screening of operating benefits in storage allocation studies. Reallocation studies typically consider the expected value of flood damages resulting from alternate allocations of flood control storage. The operating rules used in these studies are rarely optimized to reflect the change in available storage. The parametric operating rules we develop are easily modified to find a good regulation for changing storage volumes, and could be a useful part of screening level design studies of flood control alternatives.

6.2 CHARACTERISTICS OF FLOOD CONTROL OPERATION

Flood control storage mitigates flood damages by storing streamflow that cannot be released without causing damages. Beard (1963) describes good flood control practice as "releasing water whenever necessary at the highest practical rates so that minimum space need be reserved for flood control". As conditions over the basin change, so too will reservoir operations. Kelman (1980) modeled the rising and falling limbs of the hydrograph separately, reflecting conditions that were dominated by fundamentally different hydrologic processes. While the rising limb reflects conditions originating external to the basin, the falling limb reflects the spatial and geomorphic properties of a basin that determine its drainage characteristics.

Flood risks will also vary over the rising and falling limbs of the hydrograph, and flood control operations should reflect this difference. On the rising limb of a flood hydrograph inflow will be stored to reduce the peak discharge downstream from the reservoir. This detention of inflow prevents runoff from contributing to the flood peak at critical downstream locations. On the falling limb of the flood hydrograph a rapid controlled evacuation of the flood pool is desired.

Operating rules for the rising limb will be cautious, storing inflow to meet operating targets determined from analyzing the flood forecast. On the falling limb, operating rules will set an acceptable release rate to prudently empty flood storage. The operating rules for the rising and falling limb are structured to reflect damage prevention downstream from the reservoir. We include a third flow state, in which operating rules tend to pass the inflow, without modification. When flow is rising and reservoir storage is nearly full, there is a danger of overtopping the reservoir. Under these risky conditions damage prevention must become secondary to protecting the structure. The parametric operating rules include simple criteria for identifying this risky situation, and adjusting releases to avoid overtopping the reservoir.

6.3 FLOOD REGULATION - ATTENUATION AND DELAY

The regulation of a flood hydrograph can be characterized as a transformation of the magnitude and timing of runoff. When flood protection is being provided to an area that is mainly influenced by the controlled drainage area, the most efficient use of flood storage is to minimize the maximum discharge of the regulated flood hydrograph. When flood prone areas are influenced by considerable uncontrolled drainage from watersheds downstream from the reservoir, the timing of flood releases may be more critical in reducing flood damages than the magnitude of releases.

The two parameters for the rising limb operating rule are the delay, in hours, and the maximum permissible release, in cfs, during rising inflow. In general flood control regulation will both delay and attenuate the inflow hydrograph. Good regulation of the flood hydrograph will balance the use of limited storage to control both the magnitude and timing of releases. In practice, high flow events of moderate magnitude can fill reservoir storage. In the limit, attenuation and delay converge to identical operating decisions as the entire flood hydrograph is stored.

When river stages are clearly falling over the basin falling limb operation begins to release the stored flood waters. Some common operating procedures for falling reservoir levels and decreasing inflow are 1) drawdown to static full pool level within a specified number of hours; 2) maintain maximum gate openings; 3) release a fixed percentage (greater then 100%) of the mean inflow for the last three hours. The falling limb operating rule uses a prespecified drawdown discharge to accelerate the releases of stored inflow. The next section describes the operating rules in detail.

6.4.1 INITIAL OPERATING DECISIONS

Prior to beginning operation, the parameters Rmax, n, and Rfall are determined, setting the maximum target release, the delay in the flood hydrograph and the maximum release rate to drawdown the reservoir, respectively. Operation begins when the observed inflow during time t, Q_t exceeds a predetermined threshold approaching flood stage. For inflows below this level, with an empty reservoir, all inflow is released. If reservoir storage in time t, S_t is close to the capacity, CAP, of the flood pool, the release rate for period t, R_t is set as:

$$R_t = \mathrm{Max}(\,Q_t\,,\,\mathrm{Rmax}) \qquad (6.1)$$

If neither of these extreme conditions exist, the parametric operating rules determine release rates to delay and attenuate the inflow hydrograph.

6.4.2 RISING LIMB RELEASES

When flows are rising release decisions must be cautious, storing inflow that may contribute to damages. To reflect this caution we are extremely conservative in defining the state of the basin as rising. First the current storage is compared with storage in the previous period. If storage is not falling we use cautious rising limb operating rules. We similarly compare the change in inflow with the observed inflow n hours ago (where n is the 'delay' operating parameter. If current inflow, Q_t is not less then Q_{t-n} we consider the state of the system to be rising. Finally we check the hydrograph from 'n' hours ago, if $Q_{t-n+1} \geq Q_{t-n}$ we use the rising limb operating rule.

On the rising limb we try to delay runoff by 'n' hours, while limiting the maximum release to Rmax. The proposed release is therefore:

$$R_{proposed} = Min(Q_{t-n}, Rmax) \qquad (6.2)$$

To keep rising limb releases less then natural inflow we modify (6.2) as :

$$R_{proposed} = Min\ (Q_t, Q_{t-n}, Rmax) \qquad (6.3)$$

Releases greater than $R_{proposed}$ will only be made under two conditions. If prior rising limb releases have already exceeded Rmax, and Rmax is greater than $R_{proposed}$, the higher previous release will be maintained. This prevents secondary peaks downstream, after floodwaters have started to recede. To accomplish this we add the memory of a prior release at Rmax as R_p.

$$R_p = Max\ (R_p, Min(R_{t-1},Rmax)) \qquad (6.4)$$

As reservoir storage becomes full, rising limb operations become more cautious. Release rates will increase in order to pass inflows that cannot be stored. The transition to a cautious operating state is imperfectly perceived. When the reservoir is full, all operating goals become secondary to protecting the structure. When storage is empty, release decisions are guided by prescribed damage preventing operating goals. A cautious release decision is proposed in every time period using the fraction of available storage as an indication of the degree of caution to be employed in the current state. The cautious release is calculated as:

$$R_c = \frac{S_t}{CAP} Q_t \qquad (6.5)$$

The fraction of inflow to be released in order to protect the reservoir is here defined as a linear function of the available storage. This implicitly represents a risk preference. The cautious release function could be an arbitrary concave or convex function of the relative storage availability, reflecting more or less cautious operation, respectively. The significant feature of the cautious release is that it increases to 100% of inflow as reservoir storage fills.

The rising limb release is now determined as:

$$R_t = Min\ (S_t+Q_t, Min(R_{proposed}, R_c, R_p) \qquad (6.6)$$

6.4.3 FALLING LIMB RELEASES

The conservative definition of a rising state, combined with the cautious release R_c ensures that falling limb releases will only be called for when flood conditions are diminishing. The goal of falling limb operation is to drawdown reservoir storage at the target discharge rate Rfall. For large values of delay, Q_{t-n} may be greater than Rfall. In this case release follows the falling limb of the inflow hydrograph until lagged inflow is less then Rfall. To ensure operating targets will not be violated by a large delay, the falling limb release can never exceed Rmax. Finally the release cannot exceed total storage plus inflow. The falling limb release is determined as:

$$R_t = \text{Min}\ (\text{Rmax},\ \text{Max}\ (\text{Rfall},\ Q_{t-n}),\ S_t + Q_t) \qquad (6.7)$$

The parameter Rfall is optional. If Rfall is set to zero, the falling limb operating rule releases will follow the falling limb of the inflow hydrograph, delayed 'n' hours.

6.5 PARAMETRIC OPERATION

In this section examples of flood control releases generated with the parametric operating rules are presented. The utility of the parametric rules is suggested in both real-time operation, as part of the standing instructions to damtenders, as well as a planning context.

6.5.1 STANDING INSTRUCTIONS TO DAMTENDERS

To suggest the utility of the parametric rules as part of standing instructions to damtenders, a loss of release guidance is simulated. We suppose normal operation for a 65000 cfs-hr flood control reservoir relies on a sophisticated operating procedure, utilizing radar imagery and telemetered precipitation and streamflow to make release decisions that hedge against hydrologic uncertainty. After an initial evaluation of expected flood control operation based on the current forecast, all communications with the control center is lost. The only information the reservoir operator has is a tentative release trajectory leading to a maximum release of 20,000 cfs, and a delay of the peak discharge of six hours. These parameters are used to provide a regulation of the flood using only the observed inflow at the reservoir to determine the release in each hour.

Figure 6.1 shows the inflow hydrograph and the releases resulting from a parametric operating rule calling for a 6 hour delay and a maximum release of 20,000 cfs. The threshold flow that initiates operation is arbitrarily chosen as 3000 cfs for this example. A number of features of the parametric rules are illustrated in this example. Even though the maximum release target was 20,000 cfs, the maximum release was actually 21,571 cfs at hour 32. At this time the state of the system was still rising (since inflow was greater than release) and storage was 92% full. The cautious release R_c dominated the parametric operating targets, and a larger release was made to reduce the risk to the dam.

Although the drawdown release is arbitrarily set at 10,000 cfs for this example, falling releases are never made at that release rate. To preserve the 6 hour delay target, releases at hour 38 attempt to release the inflow from hour 32. Since this exceeds the release target, Rfall, the falling limb release is set at Rmax, 20,000 cfs. At this high release rate reservoir storage is emptied before the 6 hour delayed inflows fall below 10,000 cfs (which would have caused releases at the rate Rfall)

The parametric regulation of the hydrograph in Figure 6.1 uses a maximum of 92% of the available 65,000 cfs-hr flood storage.

This particular example was chosen to enable a comparison with the operating rule developed in chapter 7. The stochastic multiple objective operating rule developed in chapter 7 uses forecast information each hour to identify operating decisions that hedge against hydrologic uncertainty. The operation of that rule on the same flood hydrograph as Figure 6.1, is shown in Figure 7.14. The operating parameters of 6 hours and 20,000 cfs come from the operating rule in Section 7.

6.5.2 SCREENING OPERATING ALTERNATIVES

The parametric rule can also serve as a planning tool in evaluating feasible operating plans during a flood, as well as providing operating alternatives for design events in design studies for flood control storage. The flood considered in Section 6.5.1 is reexamined using other combinations of delay and attenuation to evaluate the most efficient use of storage. For runoff volumes that are significant compared to the volume of flood control storage, the storage cost of even a few hours of delay is high. The parametric rule suggests considerable reductions in peak discharge can be achieved if delay of the flood hydrograph is reduced. The value of this tradeoff will depend on the configuration of the basin, the location of the key damage centers, and the timing of uncontrolled downstream runoff.

Considering the uncertain environment in which real-time decisions must be made, an error of only a few hours in the choice of operating goals can have severe consequences in terms of flood crests produced. Parametric operation suggests risk averse reservoir operators may prefer to attenuate rather then delay large floods when forecast uncertainty is high.

Figure 6.2 shows three alternate regulations for the flood hydrograph considered in Section 6.5.1. These parametric rules were chosen to emphasize the relative value of peak attenuation over delay for large runoff volumes. The three parametric rules considered are summarized in Table 6.1.

Accepting no delay in the inflow hydrograph makes a substantial volume of storage available for peak attenuation. Attempting to reduce the peak to 10,000 cfs is infeasible. Storage is inadequate, and the cautious release leads to a 19,623 release to protect the structure. A maximum release of 15,000 cfs is feasible and only uses 51,300 cfs-hrs of storage. The value of the six hour delay specified in the parametric rule in Section 6.5.1 must be weighed against a reduction of the maximum controlled flow of over 25%. The relatively small volume of storage required for a maximum discharge of 20,000 cfs suggests a number of compromise operating rules are feasible that will significantly reduce the flood peak while allowing a delay in the timing of runoff. The storage cost of delay can be large when the runoff volume is significant compared to the volume of flood storage available. For real-time reservoir operation in an uncertain environment an error of only a few hours in delaying the flood hydrograph can deplete the available volume of flood storage. Cautious operating strategies may favor parametric operating rules emphasizing attenuation rather than delay for large runoff events in an uncertain environment.

6.5.3 PARAMETRIC OPERATION OF JENNINGS RANDOLPH RESERVOIR

In this section the Standard Project Flood (SPF) for Jennings Randolph reservoir is examined using parametric operating rules. Figure 6.3 shows the Jennings Randolph SPF hydrograph. In studying the reallocation of flood control storage in Jennings Randolph reservoir, the Baltimore District of the U.S. Army Corps considered operating consequences of reducing the size of the flood control pool. The Corps' regulation of the SPF effected a delay of the time to peak of 5 hours, with a maximum discharge of 40,000 cfs. The parametric operating rule with these operating targets was used to produce the regulation of the SPF shown in Figure 6.4. Figure 6.5 shows the result of two operating rules with parametric targets of 5 hours and 40,000 cfs, and 10 hours and 20,000 cfs. The 10 hour 20,000 cfs rule, while more desirable, is not feasible. The storage fills so rapidly that large releases must be made to protect the structure. Although the maximum release target was only 20,000 cfs, a caution dominated release of 46,000 cfs is made at hour 36.

Finally Figure 6.6 shows the result of two feasible parametric rules. The 5 hour - 40,000 cfs rule is compared to a 0 -hour 35000 cfs rule. The relative value of 5 hours of delay versus 5000 cfs of peak reduction must be evaluated in terms of downstream flow conditions, as well as the uncertain environment in which these decisions must be made. The parametric rule can be a useful tool for quickly evaluating a wide range of operating alternatives in flood regulation studies.

6.6 SUMMARY

A family of reservoir operating rules has been identified that captures many of the characteristics of flood control operation. The operating rules can be parameterized from operating plans based on forecast information and full communication of radar, precipitation, and streamflow data in real-time. Once the parameters are estimated, the parametric rules appear to offer acceptable releases based only on flow observed at the reservoir. The parameterized operating rules, in conjunction with standard, sophisticated real-time operating procedures may be suitable for providing contingency reservoir releases as part of the standing instructions to damtenders.

In a planning context, simulation over a range of parameters can quickly screen feasible operating alternatives for more rigorous analysis. The rapid evaluation of closely related operating rules would be useful in both planning studies, evaluating alternate storage allocations, as well as in real time operation to estimate operating targets and risks based on forecast information.

FIGURE 6.1

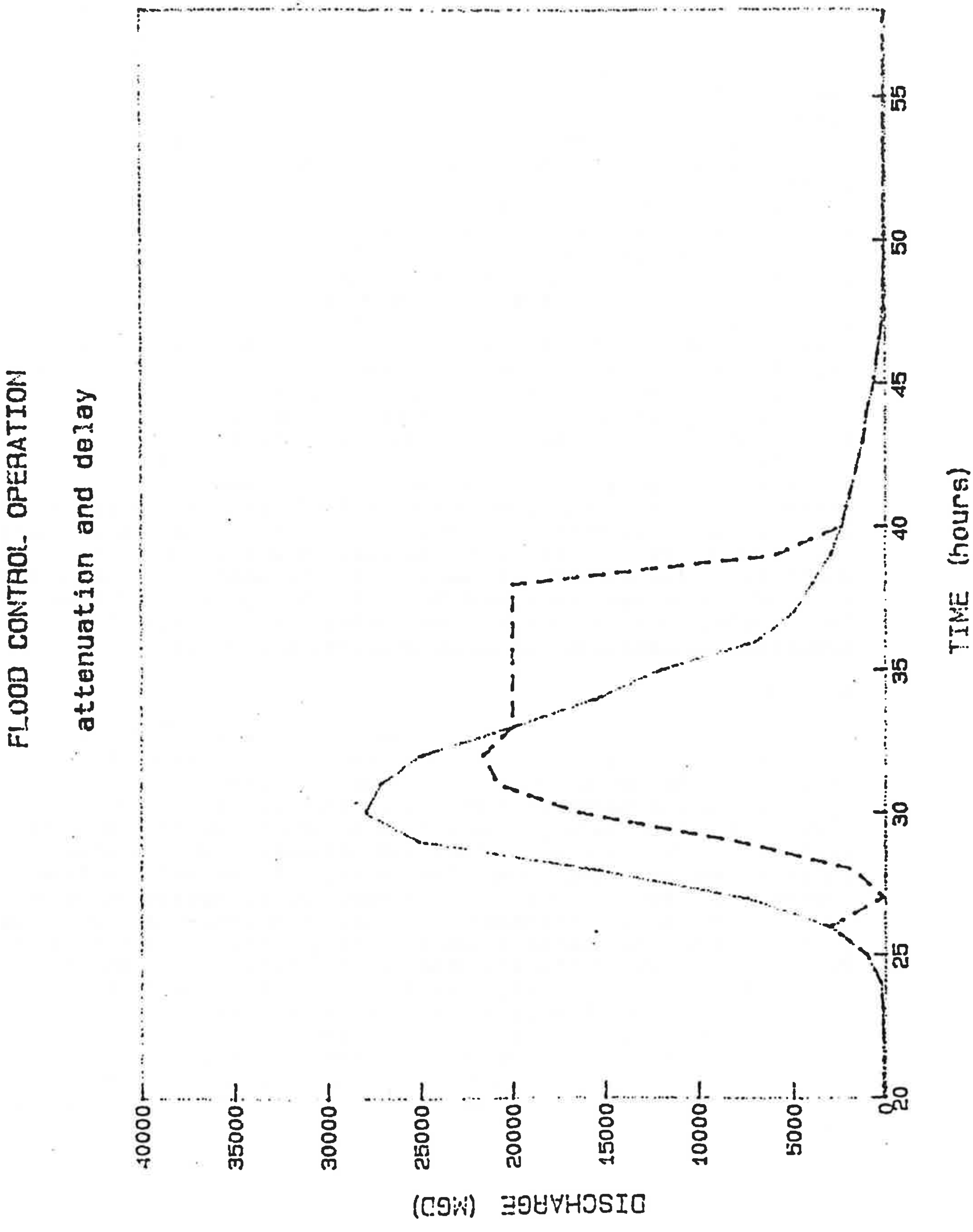

FLOOD CONTROL OPERATION
attenuation and delay
TIME (hours)
DISCHARGE (MGD)
20
25
30
35
40
45
50
55
0
5000
10000
15000
20000
25000
30000
35000
40000

FIGURE 6.2

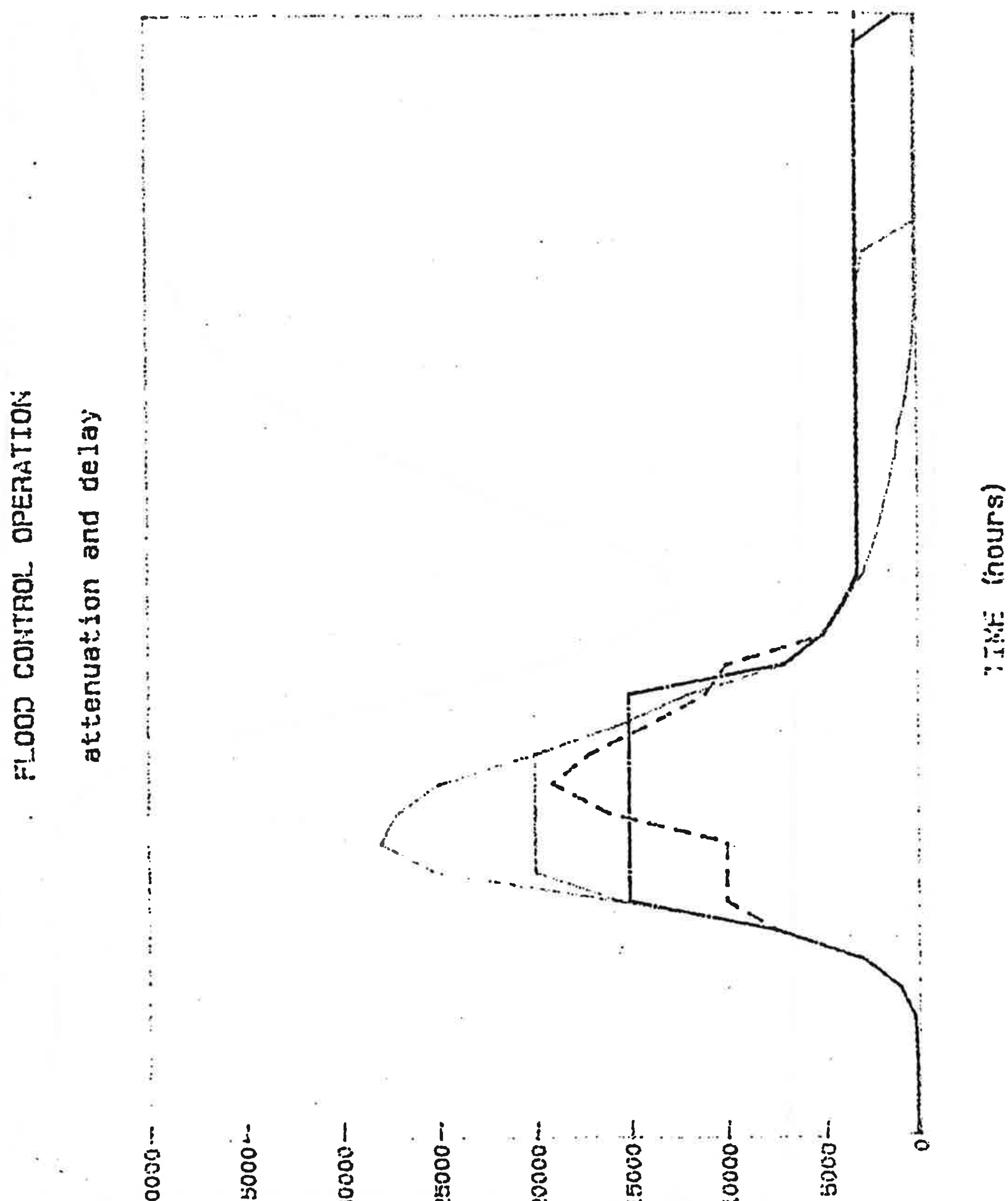
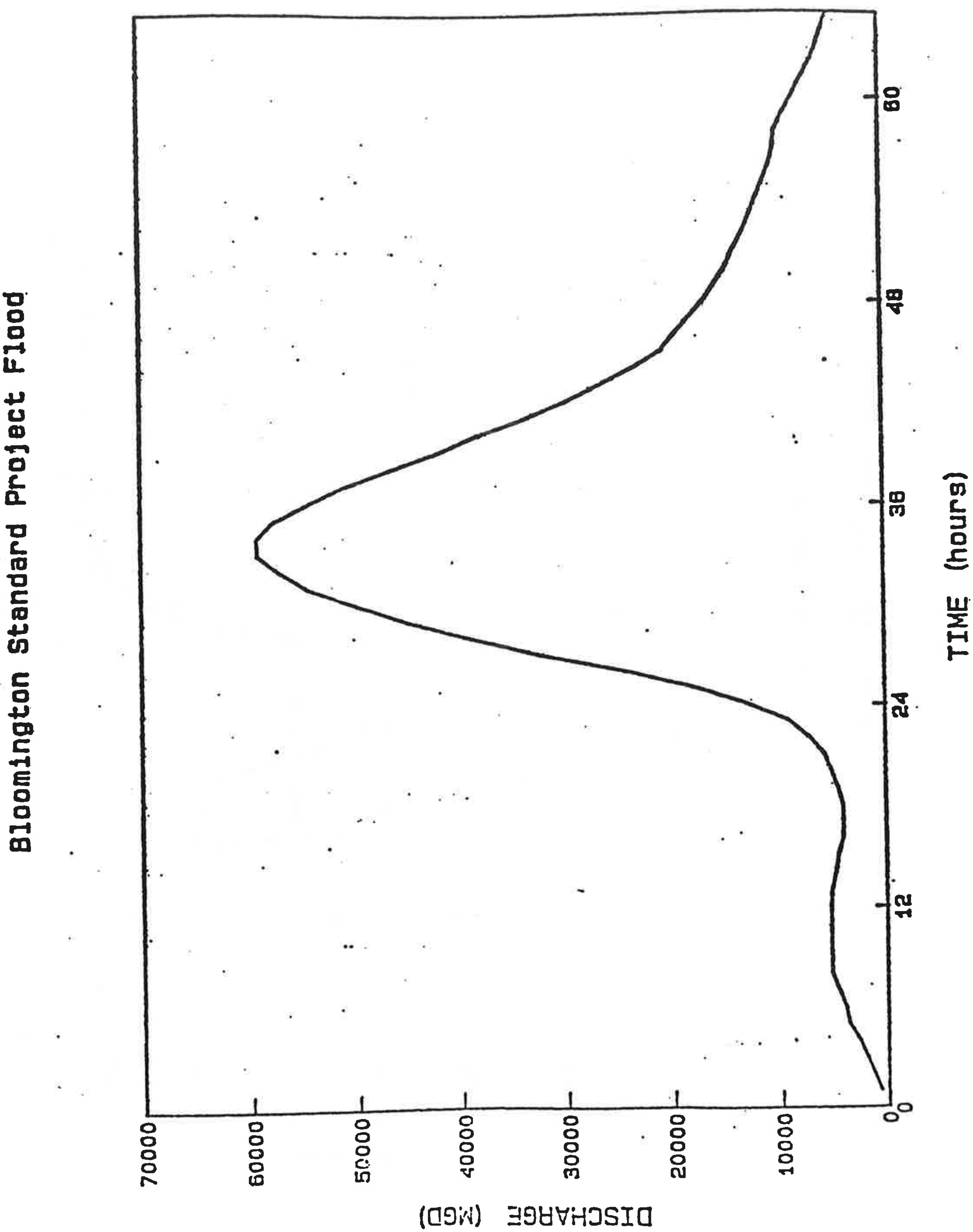

FLOOD CONTROL OPERATION
attenuation and delay
TIME (hours)
DISCHARGE (MGD)
40000
35000
30000
25000
20000
15000
10000
5000
0

FIGURE 6.3



Bloomington Standard Project Flood
DISCHARGE (MGD)
70000
60000
50000
40000
30000
20000
10000
0
0
12
24
36
48
60
TIME (hours)

FIGURE 6.4

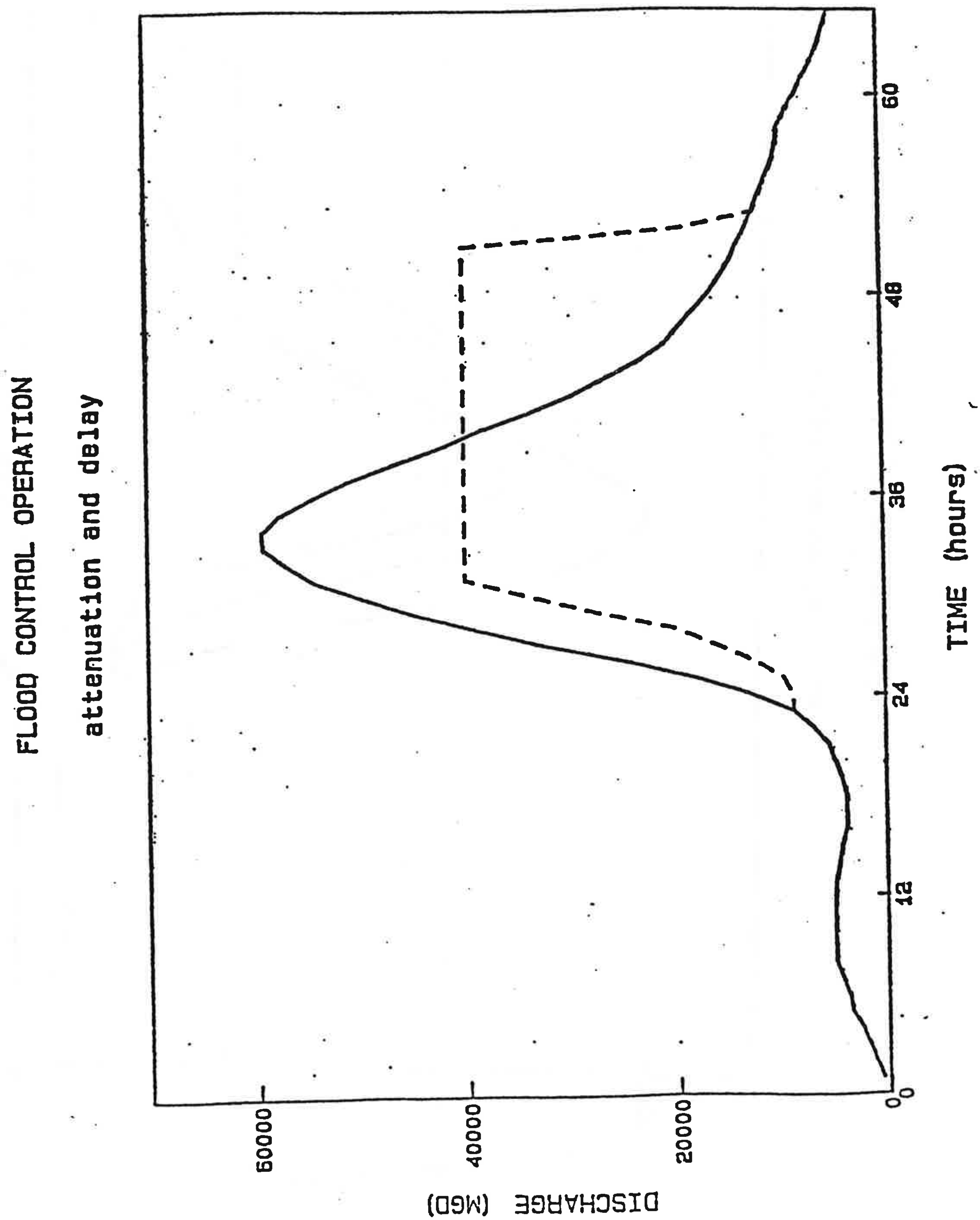

FLOOD CONTROL OPERATION
attenuation and delay
DISCHARGE (MGD)
60000
40000
20000
0
0
12
24
36
48
60
TIME (hours)

FIGURE 6.5

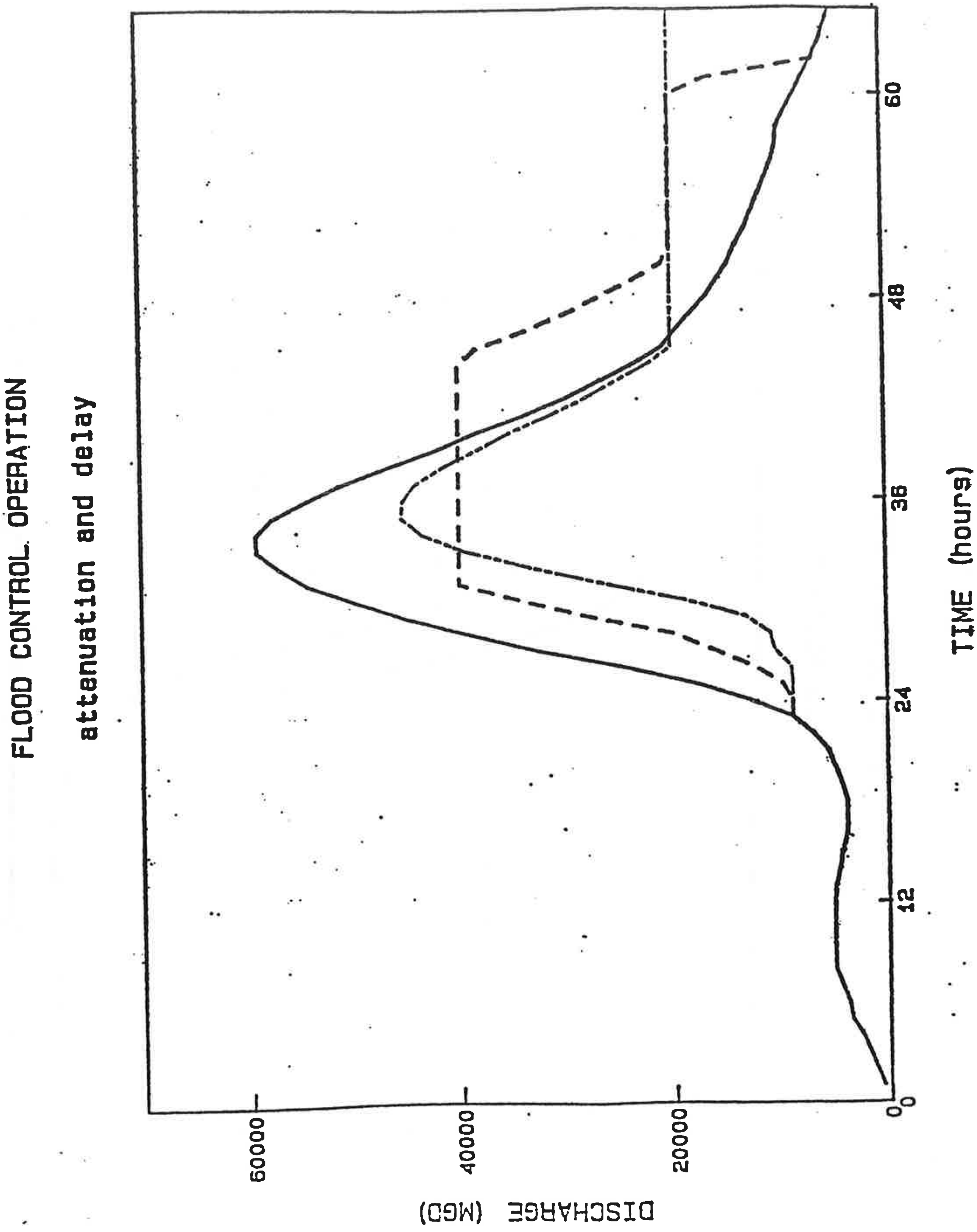

FLOOD CONTROL OPERATION
attenuation and delay
TIME (hours)
DISCHARGE (MGD)
0
12
24
36
48
60
20000
40000
60000

FIGURE 6.6

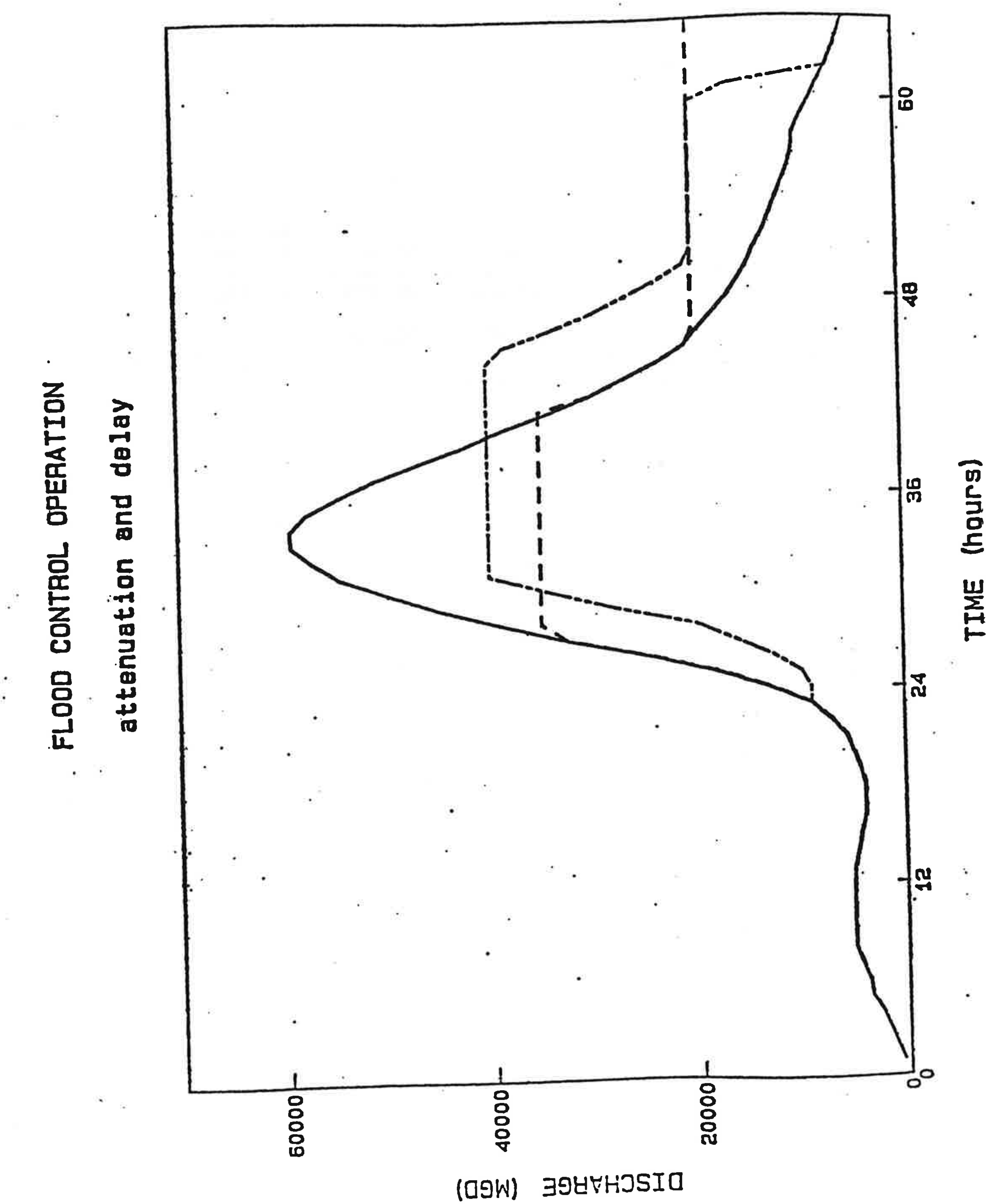

FLOOD CONTROL OPERATION
attenuation and delay
TIME (hours)
0
12
24
36
48
60
DISCHARGE (MGD)
0
20000
40000
60000

Table 6.1

delay	Rmax	Smax	maximum release
0	10,000	60,938	19,623
0	15,000	51,300	15000
0	20,000	25,300	20,000

CHAPTER 7

STOCHASTIC MULTIOBJECTIVE REAL-TIME FLOOD CONTROL OPERATION

7.1 INTRODUCTION

In this chapter the use of forecast information in multiobjective reservoir operation is developed for the problem of real-time operation of a flood control reservoir. The probabilistic nature of streamflow and precipitation combined with finite storage capacity requires operating decisions that trade-off conflicting risks. The reservoir operator's problem is presented as a reliability programming problem, noting the symmetry of the operation and design problem. Reliable damage prevention fully utilizes reservoir storage, increasing other operating risks. The goal of the operating problem is to determine the optimal non-damaging temporal allocation of inflow to available storage, utilizing both reservoir and channel capacity.

Decisions to commit reservoir storage to inflow in real-time flood control operation are viewed as a stochastic sequential allocation problem. Some general optimality criteria for sequential allocation problems are considered and related to the real-time operation of flood control reservoirs. Extending the analogy with sequential allocation problems, a stochastic multiobjective algorithm is developed for hourly flood control operation.

The real-time operating problem for a flood control reservoir is formulated as a two-stage recourse problem with feedback. A simple parameterization of storm structure is used to describe the distribution of runoff during the storm event. Two-stage programming yields the current period release which minimizes the expected value of downstream damage under the current estimate of storm parameters.

Using a multiple objective formulation, decisions are generated that trade-off the expected value of downstream damage and the risk of overtopping. In addition to choices that are non-inferior based on first moment information, the joint distribution of stage and storage is also presented to the decision maker, clarifying the risky alternatives that are available.

7.2.1 FLOOD CONTROL OPERATION - HEDGING AGAINST UNCERTAINTY

Given the uncertainty of reservoir inflow during a storm event reservoir operation must hedge flood control release decisions against estimation errors in both the timing and the magnitude of flood runoff. Each operating decision is a commitment of the limited storage that is available. This commitment of storage must balance the damages prevented by storing inflow against the risks of using limited storage inefficiently.

Consider the reservoir operator's problem when faced with a forecast simply represented in Figure 7.1. For illustration, we suppose the operator has been observing a flood rise, and has been provided a forecast in the form of three possible inflow hydrographs, for which probabilities of occurrence have been determined. Without loss of generality we assume the only source of uncertainty is in the magnitude (not the timing) of runoff. To simplify the operating problem further, we assume flood damages are determined only by peak discharge (not duration of damaging flow), and that the reservoir completely controls the drainage area upstream from the damage site of interest. With no uncontrolled intervening drainage between the reservoir and the damage site, the optimal regulation of a flood hydrograph is to store the peak discharge using all available storage. This regulation will achieve the greatest reduction in peak discharge and, under the damage assumption, greatest reduction in damage. For a fixed capacity reservoir the optimal regulation of each of the three likely flood futures is represented in Figure 7.2.

The operator's problem is now more clear. Given the probability of each likely hydrograph, and the optimal regulation for each, the operator must determine how much runoff should be stored in the current period. If the operator was certain that the low runoff hydrograph would be realized, regulation would clearly begin in the current time period. The operation for the high runoff hydrograph would delay any regulation until later in the event. The uncertainty in the forecast requires the operator to hedge his operating decisions.

There are two ways in which the operator can err in the face of uncertain inflow. If the magnitude of runoff is underestimated, the operator will commit too much storage too soon in the flood event. When the true flood peak is realized insufficient storage will be available. In this case the operator will be forced to release inflow, causing damages that could have been prevented. If the magnitude of runoff is overestimated, the operator will release the rising inflow, reserving storage for a flood peak which will not appear. In this case the operator will fail to store the true peak flow, again causing damages that could have been prevented.

The operator needs to make prudent operating decisions while hedging against hydrologic uncertainty. In this allocation decision hedging implies balancing the expected loss of benefits in the short-term against much greater probabilistic losses in the future.

Traditionally, hedging against hydrologic uncertainty in flood control operation is achieved through structural means by constructing sufficient capacity to store damaging flows with long recurrence intervals. Design based on the Standard Project Flood (SPF) has furnished a structural measure of reliability in most large flood control reservoirs built in the U.S. Regulation of SPF designed flood control reservoirs, often follows operating rules that completely store inflow when levels at a downstream control point approach flood stage, retaining this runoff in storage until either

a) stages are clearly falling or
b) storage is nearly full requiring regulating releases to prevent over topping.

This regulation implicitly stores inflows which might cause damages if released. In this sense, we can formalize a parametric regulation for which our objective is to minimize the probability of contributing to damages downstream. If we denote by $\underline{Q}$ the complete, current, probabilistic forecast of flood runoff, and let R_t be the release in time period t we can define the operator

$M_a(\underline{Q},S_t)$ as the release, for which the conditional probability of causing any damage is = a i.e.,

$$Pr\ (R_t>R^*, R_{t+1}, >R^* \ldots R_T>R^* \mid \underline{Q}, R_{t-1}, R_{t-2} \ldots R_0) = a$$

where R^* corresponds to the threshold of damage and S_t is the volume of runoff held in flood storage during time period t.

In this sense the reservoir operator's problem can be stated as:

Min a

s.t.

$$R_t = M_a(\underline{Q},S_t) \qquad t=1,2,\ldots T$$
$$S_t - S_{t-1} - I_t + R_t = 0 \qquad t=1,2,\ldots T$$
$$S_t \leq S_{cap} \qquad t=1,2,\ldots T$$

Implicit in the operator $M_a(\underline{Q},S_t)$ are conditional release decisions through the end of the current operating horizon. In practice $M_a(\underline{Q},S_t)$ will embody heuristic procedures or optimization algorithms for determining reservoir releases conditioned on changing hydrologic conditions.

The importance of incorporating optimal operating rules in reservoir design is emphasized by the symmetrical design problem:

Min S_{cap}

s.t.

$$R_t = M_a(\underline{Q},S_t) \qquad t=1,2,\ldots,T$$
$$S_t - S_{t-1} - I_t + R_t = 0 \qquad t=1,2,\ldots,T$$
$$S_t \leq S_{cap} \qquad t=1,2,\ldots T$$

where S_{cap} is the capacity of the flood control reservoir.

In general, the reliability of damage reduction, and the volume of flood control storage are linked in a multiobjective optimization problem:

Min(a, S_{cap})

s.t.

$$R_t = M_a(\underline{Q},S_t) \qquad t=1,2,\ldots,T$$
$$S_t - S_{t-1} - I_t + R_t = 0 \qquad t=1,2,\ldots,T$$
$$S_t \leq S_{cap} \qquad t=1,2,\ldots,T$$

Consider the reservoir operating problem in which S_{cap} is fixed. As a is reduced more storage will be utilized throughout the storm event. While increased reliability in damage reduction is desirable, increased storage utilization represents an additional operating risk.

In an informal way a full reservoir represents an increased risk of being "caught" with inadequate storage if a second, unexpected storm system effected the basin before draw down is complete. The release of stored floodwaters is a critical part of flood control operation. The U.S. Army Corps of Engineers' Reservoir Regulation Manual (1959) notes:

> "...the nondamaging release of such stored water frequently presents greater difficulties than the storage phase. The primary objective is to empty the storage space allocated to flood control as rapidly as possible without increasing or unduly prolonging the duration of damaging stages at downstream locations. ... In the event of a succession of severe storms, the storage operations during the storm periods may be entirely dependent upon the degree to which the storage space can be emptied between the storms."

In a more general context this is just another case of forecast uncertainty, or underestimation of runoff. As the reservoir fills, the risk of overtopping increases as well, raising the possibility of a catastrophic structural failure. A more general statement of the problem recognizes hedging of current releases against future damages, as well as hedging against inefficient drawdown and structural failure that can result from overfilling reservoir storage.

7.2.2 HEDGING AND RESOURCE ALLOCATION

Real-time operation takes place in a changing, uncertain environment. The sensitivity of operations to forecast uncertainty will determine the need and value of a complex stochastic operating policy. Stedinger (1985) and Mays et al. (1987) have considered large reservoir systems in which the main source of hydrologic uncertainty is river routing. For the High Aswan Dam, the runoff volume is observable upstream, weeks before reaching the reservoir. In the Texas high lakes, major flood waters are observed on tributaries several days before reaching the controllable flood reservoirs. For these applications hydrologic uncertainty is relatively small. The operating problem is largely one of optimal storage allocation rather than optimal hedging.

In contrast, the real-time flood control problem generally has a significant hedging component. The greater the volume of flood runoff, the greater is the need to hedge. For floods in which the runoff volume is significantly less than the flood storage volume, reservoir operation is largely an allocation problem. As the probability of filling the flood pool increases, so too does the need to hedge against hydrologic uncertainty. SPF design provides incremental flood storage which is infrequently used. The most frequent flood operating problems become dominantly allocation rather than hedging decisions. The transition from allocation to hedging dominated operation is related to the risk of filling the flood pool.

The need to hedge was considered by Bellman in a general sequential allocation problem in which a policy choice y is to be made to optimize the allocation of a fixed resource x. Defining f(x) as the total return from the optimal allocation of an initial resource x,

$$f(x) = \max_{y} [R(x,y) + f(x-D(x,y))]$$

where: R(x,y) - denotes the immediate return from choosing policy y when an amount x of the resource to be allocated is available.

D(x,y) - denotes the immediate cost of choosing policy y when an amount x of the resource is available.

Then

$$f(x) \sim \max_{y} [R(x,y) + f(x) - D(x,y)f'(x)]$$

for $0 < D(x,y) \ll x$

or

$$0 \sim \max_{y} [R(x,y) - D(x,y)f'(x)]$$

At optimality,

$$f'(x) = \max_{y} \frac{R(x,y)}{D(x,y)} \quad .$$

The maximum net short-term value of the current policy decision will equal the long-term marginal value of the available resource. If the resource cost of a policy is small, the optimal policy simply maximizes the immediate, short-term return.

An analogous conclusion can be reached for an explicitly stochastic problem. Consider now a distribution of a set of returns (z) ~ R(x,y,z) and of costs w ~ D(x,y,w). Maximizing the total expected returns, implies:

$$f(x) = \max_{y} [\int_0^\infty z \, d\, R(x,y,z) + f(x - \int_0^\infty w \, d\, D(x,y,w))]$$

now, as before, for small expected costs, i.e.

$$\int_0^\infty w \, d\, D(x,y,w) \ll x$$

it is approximately true that

$$0 = \underset{y}{\mathrm{Max}}\left[\int_0^\infty z \, d\, R(x,y,z) - f'(x) \int_0^\infty w \, d\, D(x,y,w) \right]$$

and

$$f'(x) = \underset{y}{\mathrm{Max}} \frac{\int z \, dR}{\int w \, dD}$$

$$= \underset{y}{\mathrm{Max}} \frac{E(\text{Immediate Return})}{E(\text{Immediate Cost})}$$

where E(.) is the expectation operator.

The conclusion is that myopic, short-term benefit maximizing decisions will be nearly optimal for resource allocation decisions in which the immediate resource cost of policy decisions is small compared to the availability of the resource.

This intuitively reasonable result is applicable to the flood control problem. We can view real-time flood control operation as a problem in allocating the limited volume of the flood control reservoir to storing reservoir inflow. When runoff is small compared to flood storage capacity, the resource cost is small for any policy. (We could store the entire flood event, preventing all damages without exhausting reservoir storage). Allocating storage using a simple myopic policy will be optimal; hedging against hydrologic uncertainty is unnecessary. As successively larger floods are considered the resource cost of many policies is no longer insignificant; the small cost assumption:

$$0 < D\,(x,y) << x$$

will no longer hold, and the myopic policy will become increasingly suboptimal.

The general stochastic allocation problem suggests that decisions which maximize the relative expected short-term gain will be nearly optimal much of the time. A number of workers have reached similar conclusions for specific applications. Based on Monte-Carlo simulation Young (1967) concluded in a water supply context that reservoir draft equal to mean inflow was an optimal or near optimal control law. Klemes (1977) reached a similar conclusion. Simon (1956) examined a set of stochastic sequential decision problems and showed:

> "When the criterion function is quadratic, the planning problem for the case of uncertainty can be reduced to the problem for the case of certainty simply by replacing, in the computation of the optimal first-period action, the "certain" future values of variables by their unconditional expectations."

For a water resource system Cornish (1979) used simulation to calculate the sensitivity of the present value of benefits to various moments of stochastic flows. Cornish concluded benefits are most sensitive to changes in the mean of inflow, but changes in the second and third moments of inflow have a negligible effect. These results suggest near optimal decisions can result from the reduction of a stochastic sequential decision problem to a deterministic problem by substituting the stochastic inputs with their expected values. Dantzig (1963) analyzed a two stage recourse problem with uniformly distributed random inputs. He showed that the optimal solution (first stage cost plus the expected value of the second stage cost) for the stochastic two-stage problem, was identical to the solution for the deterministic problem formed by substituting the stochastic input with its expectation.

Nevertheless there are times when hedging is required.

7.2.3 STOCHASTIC SEQUENTIAL ALLOCATION

Manne (1967) analyzed the two stage recourse problem, reconciling the apparent optimality of deterministic solutions with the need to hedge against uncertainty. The two-stage recourse problem is formulated as:

$$\operatorname{Min} \underset{b}{E} (c'x + f'y)$$

$$\text{s.t.} \quad Ax + By = b$$

$$x,\ y,\ >0$$

In the recourse problem we observe the realization of the random vector b after choosing the first stage decision variable x, and then choose y in order to minimize $c'x + f'y$. (noting $y=B^{-1}[b-Ax]$)

We want to find

$$\underset{x}{\operatorname{Min}}\ z = \underset{b}{E} (c'x + f'y^{*})$$

$$= \underset{b}{E} \underset{y}{\operatorname{Min}} (c'x + f'y)$$

Let

$$C(b,x) = \underset{y}{\operatorname{Min}} (c'x + f'y) \quad \text{for a given } x$$

Then

$$\underset{x}{\operatorname{Min}} \underset{b}{E}\ C(b,x)$$

is referred to as the "here and now problem".

In other words we choose x "here and now", before b is observed in order to minimize the expected cost of x and the optimal recourse decision y. Following Manne,

$$\underset{b}{E}\ \underset{x}{\text{Min}}\ C(b,x)$$

is called the "wait and see problem". This is the expected cost of choosing both x and y after waiting to observe the realization of the random variable b.

One operating heuristic frequently employed is to solve the two-stage problem by forming a deterministic optimization problem in which the random vector is replaced with its expectation, $E_b b$. This problem, denoted

$$\underset{b}{E}\ C(b, x(\underset{b}{E}b))$$

is referred to as the "Deterministic Approximation".

Define

$$x(b^*) = \{x \mid \underset{x}{\text{Min}}\ C(b^*,x) = C(b^*,x(b^*))\}$$

for any realization b^* of b.

Since the "here and now" problem admits x(Eb) as a feasible solution it is clear that

$$\underset{b}{E}\ C(b, x(\underset{b}{E}b)) \geq \underset{x}{\text{Min}}\ \underset{b}{E}\ C(b,x)$$

Since

$$\underset{x}{\text{Min}}\ C(b,x) \leq C(b,x)$$

and

$$\underset{b}{E}\ \underset{x}{\text{Min}}\ C(b,x) \leq \underset{b}{E}\ C(b,x)$$

it follows that

$$\underset{b}{E}\ \underset{x}{\text{Min}}\ C(b,x) \leq \underset{x}{\text{Min}}\ \underset{b}{E}\ C(b,x).$$

Therefore,

$$\underbrace{\underset{b}{E}\ C(b, x(Eb))}_{\text{deterministic approximation}} \geq \underbrace{\underset{x}{\text{Min}}\ \underset{b}{E}\ C(b,x)}_{\text{Here \& Now problem}} \geq \underbrace{\underset{b}{E}\ \underset{x}{\text{Min}}\ C(b,x)}_{\text{Wait \& See problem}}$$

In general the deterministic approximation will be bounded from below by the here-and-now problem. However, if

C(b,x) is linear in b then $\underset{b}{E}\ C(b,x) = C(Eb,x)$

and, in particular the deterministic approximation will yield an optimal solution for the two-stage recourse problem. As myopic decisions were optimal for some allocation problems, myopic decisions are also optimal for some, but not all stochastic sequential decision problems.

More generally the expected returns from allocation decisions based on expected value information only, will be bounded from below by stochastic optimization problems which explicitly take into account future uncertainty. Similarly, real-time sequential decisions will be bounded from below by the "true" optimal solution calculated with perfect forecast information. While solutions based on expected value information may prove optimal under some circumstances, this will not in general be the case. The optimal real-time decision problem must hedge against future uncertainty.

7.3 STOCHASTIC MULTIOBJECTIVE FLOOD CONTROL OPERATION

In this section algorithms are considered for real-time flood control operation. We consider a single flood control reservoir, located above a critical reach influenced by significant uncontrolled drainage. The operating goal is to minimize potentially damaging releases by temporarily storing runoff from the controlled watershed. The use of a stochastic multiobjective algorithm in flood control operation is a unique approach to the real-time operating problem.

Real-time decisions must utilize the description of hydrologic uncertainty from the current forecast, to balance the conflicting risks of operating a flood control reservoir. The stochastic sequential decision problem is approximated with a two-stage recourse formulation. The current forecast provides an estimate of the joint density function of hydrologic inputs. The current period release, determined in the two-stage recourse problem reflects current operating risks.

Conflicting operating risks are managed with a multiple objective formulation. The regulation decision given a storm forecast can be represented as a tradeoff of peak discharge (damage) reduction and peak storage utilization. This tradeoff captures the balance which must be struck between the risk of flood damages and the risk of structural failure. These conflicting risks are probabilistic products of release decisions which transform current storage and inflow along with current forecast conditions, into a net, effective reservoir regulation for a flood event.

A stochastic multiobjective two-stage recourse algorithm with feedback is proposed for the real-time operation of a flood control reservoir. The algorithm is developed for a single flood control reservoir protecting a downstream damage site. The effects of significant uncontrolled drainage and channel routing are explicitly included in the operating formulation.

Using a parameterized estimate of current hydrometeorologic conditions, the expected value of both peak discharge at the downstream damage site, as well as peak storage utilization during the flood are considered. From this expected value information, first moment non-inferior operating alternatives are generated in each hour. Peak discharge is taken as a measure of the damage potential downstream. Peak reservoir storage represents the risk of exhausting reservoir storage before flood risks have subsided, as well as overtopping the structure.

The proposed multiobjective algorithm allows the joint distribution of peak storage and peak discharge to be estimated for each non-inferior solution, giving the reservoir operator a complete description of the associated risks. Both chance constraints and expected value optimization are shown to be inadequate in representing the full range of operating risks associated with each first moment non-inferior solution.

Throughout this analysis we assume maximum discharge corresponds to maximum damage. The duration of flows above flood stage is not considered. A stage-damage function could be incorporated in the operating algorithms considered in this section. Such a function would allow recognition of damage zones which will significantly guide operations in specific applications. Using a stage-damage function, for example, an operator could recognize the level at which a flood plain is likely to be inundated. For areas with limited flood plain development, but significant structural development above the flood plain, operations may well allow inundation of the flood plain, reserving flood storage for flows which would cause damage to the development at higher elevations. Without loss of generality we consider only peak discharge as the measure of damage, recognizing the value of incorporating the site specific non-decreasing stage damage function for all applications.

7.3.1 CHANNEL PACKING

Real-time flood control operation occurs in an uncertain, evolving environment in which current forecasts as well as observed precipitation and runoff are combined to make sequential management decisions. If the inflow and uncontrolled hydrographs were known with certainty, simple algorithms could provide an optimal regulation of the flood hydrograph. Figure 7.3 shows a representative hydrograph for a basin downstream from our flood control reservoir. Assuming peak discharge alone determines the level of damage, releases could be made contributing to this hydrograph, without increasing the maximum discharge. The shaded area in Figure 7.3 represents the equivalent volume of channel storage which is available to pass controlled runoff. Exploiting this transient channel storage allows the flood wave to be passed in less time, using less reservoir storage, without increasing the damage producing stage.

Uncertainty in the timing of uncontrolled runoff reduces the channel storage available for flood routing, as Figure 7.4 indicates. As this uncertainty increases, the risk that a reservoir release contributes to downstream damages also increases. Greater uncertainty requires increased use of reservoir storage. Uncertainty in the magnitude of uncontrolled runoff similarly reduces the volume of channel storage available to safely pass the flood wave, increasing the use of reservoir storage.

For real-time flood control operation in which hydrologic forecasts guide operation, the "optimality" of a flood regulation can only be judged in terms of optimal hedging against current hydrologic uncertainty. In contrast to the optimal deterministic regulation which can only be evaluated after flooding runoff has been observed, (the "wait and see problem" of section 7.2), the optimal real-time regulation must find releases which will allow near optimal operation over the current estimate of the probability density function of flood runoff (the "here and now problem"). Uncertainty in the flood hydrograph will cause the real-time regulation to be bounded from below by the deterministic a posteriori regulation of the flood hydrograph. The challenge of real-time operation is to achieve operating benefits that approach this lower bound while prudently hedging against hydrologic uncertainty.

7.3.2 PARAMETERIZING THE FLOOD FORECAST

A simple parameterization of the current hydrologic forecast for the multiple basin system is presented which can be derived from a variety of flood forecasting methodologies. This reflects the way in which the operating algorithm utilizes feedback of hydrologic state observations at each time step to correct for the separation of hydrologic state estimation from the solution for an optimal release.

The current forecast provides an estimate of the magnitude and timing of runoff on each watershed in the system. The simple system considered here consists of two watersheds. One is controlled by the flood control reservoir, the other contributes unregulated runoff effecting the downstream damage site. For this system the hydrologic forecast can be represented as:

$$Q_t = \{\underline{q}_c, \underline{q}_u, dt\} \qquad (7.1)$$

where:

$\underline{q}_c$ and $\underline{q}_u$	represents the current estimate of the runoff hydrograph from the controlled and uncontrolled basins respectively
dt	is the difference in hours between the peak discharge for each of the runoff hydrographs

Since the hydrologic state Q_t represents the current forecast, Q_t is a random vector whose components have an estimated mean vector $\mathbf{M}_t$ and estimated covariance matrix $\mathbf{V}_t$. At each time step we assume a current flood forecast has been prepared for each basin in the system. A great diversity of flood forecasting techniques are available. The general approach taken in this operating model is to separate the operating algorithm from any one forecasting technique. We assume the general description of magnitude and timing of runoff in eq. 7.1 can be derived from the forecast technique or techniques being used. The distinguishing feature of this representation is that any forecast can be incorporated in the operating algorithm when represented as a joint probability density function.

The probabilistic forecast representation of eq. 7.1 could be derived from statistical streamflow forecasting models (Moore 1982, Kitanidis and Bras 1978, Ozaki 1980). Conceptual models used in adaptive forecasting procedures (Kitanidis and Bras 1980a, Kitanidis and Bras 1980b, Pegram and Stretch 1982, Logan et al. 1982) could similarly provide a probabilistic forecast.

The major source of uncertainty for short-term flood control operation is the depth and distribution of precipitation. Short-term precipitation forecasting procedures using raingage networks (Johnson and Bras 1979, 1980) or a conceptual meteorological model (Georgakakos and Bras 1984a, 1984b) can drive a conceptual hydrologic model to provide a probabilistic forecast of runoff (Troutman 1982). The use of rain-gage data combined with radar images can provide maximum likelihood estimates of storm rainfall (Eddy 1979). The use of radar imagery in short term precipitation forecasting provides resolution of the velocity and spatial extent of storm systems (Austin and Bellon 1974, Klatt and Schultz 1983). Automated algorithms using pattern recognition have been used to project storm trajectories, providing temporal resolution for flood forecasting (Unny 1982).

Without employing a particular algorithm for pattern recognition and projection, we note that many such procedures have been developed to provide real-time storm predictions. We assume the availability of such a forecast system for precipitation, as well as the availability of calibrated hydrologic models for the generation of likely storm hydrographs. The simple forecast structure envisioned transforms the current set of meteorologic observations into a distribution of precipitation and runoff over each basin.

We assume the flood forecast at each time period can be represented as a conditional joint distribution of precipitation and peak runoff:

$$Q'_t = \{p_c \ , \ p_u \ , \ dt\}$$

where p_c and p_u represent the storm hyetographs for the controlled and uncontrolled watersheds, respectively, and dt represents the distribution of the time between peak discharges for the two watersheds. The components of the state vector again have an estimated mean and estimated covariance structure arising from the forecast procedure.

This conceptual forecast structure combined with a hydrologic model allows the generation of likely flood futures and their associated probabilities for the system. We wish only to suggest the plausibility of forecast information in real-time which provides a joint probability distribution for the hydrologic inputs of the system. With this representation of hydrologic uncertainty we turn to the stochastic multiobjective operating algorithm.

7.3.3 RECURSIVE UPDATING - THE NEED FOR FEEDBACK

The multiobjective flood control operating algorithm is formulated as a tradeoff between peak discharge reduction and peak storage utilization in a two-stage programming problem. This tradeoff captures the balance which must be struck between the risk of flood damages and the risk of structural failure. These conflicting risks result from the transformation of stochastic precipitation and runoff forecasts into reservoir releases. We use an open loop feedback algorithm, incorporating real-time updating of the system throughout a storm event. Ideally the entire reservoir system could be represented in state space form, incorporating the dynamic equations governing the stochastic state transitions to identify closed loop operating decisions. Such solutions of the optimal control problem would give the optimal release decision conditioned on the current state of the system, and the current time step within the flood event.

Two factors have made this desired result elusive. First, for real-time operation of water resource systems the relevant components of the state vector may be too complex for the true state dynamics to be known. For systems in which atmospheric circulation and cloud physics determine the state dynamics, it is doubtful that the physical laws which generate probabilistic state transitions from stochastic inputs will ever be known exactly. This fundamental uncertainty in the mechanics of state evolution necessitates the recursive use of current hydrometeorologic observations to update the current estimate of the state vector, prior to solving for the current period's control decision.

Second, as is frequently noted in real-time operating problems, most of the true closed loop algorithms that have been applied to water resource problems suffer from the exponential growth in computational requirements, i.e. the "curse of dimensionality". A number of approximately optimal algorithms have been derived from the elegant theory of linear systems with quadratic criteria functions (LQ). LQ systems have the appealing property that for gaussian or exponential inputs, the optimal control decision (optimal in an expected value sense) in each discrete time period can be determined analytically, using first moment information. This analytic solution can be derived because the state and control variables are assumed unbounded, and continuously differentiable. For a reservoir in which storage and release cannot normally be negative, and for which reservoir capacity imposes a hard upper bound, this assumption is not valid. These bounds can be incorporated if we are willing to expand the computational complexity of the algorithm to a search over Kuhn-Tucker points. Such a search will grow exponentially with the number of bounds in each stage.

One computationally feasible approach employs an approximation of the LQ system to get an approximately optimal control decision when storage is not too close to its upper or lower bound. These approximately optimal algorithms generate the current period's release or control decision using a perturbation algorithm around a feasible nominal trajectory.

While the optimal trajectory so derived will provide a nearly optimal decision for the current period, as uncertainty in state transitions accumulate the state of the system must be re-estimated, and the operating algorithm re-solved. These algorithms have extremely attractive computational characteristics and have been demonstrated on sizable water resource systems. Nevertheless, adaptive state estimation, followed by a new solution iteration is common to all these procedures.

7.3.4 GROWTH OF THE STATE VECTOR

A true real-time closed loop reservoir operating algorithm would provide a unique optimal control decision conditioned on the system state and stage. In general no real-time non-adaptive closed loop reservoir operating algorithm has been proposed which adequately captures the state dynamics. One major reason for this is the unavoidable complexity of the hydrologic state vector. For example, a single watershed represented with the Sacramento soil moisture accounting model would have a bounded component of the state vector for each of the six conceptual soil moisture zones. On the Potomac River Basin the calibration of the National Weather Service River Forecast System partitioned the basin into 23 separate watersheds with a total of 138 conceptual storage zones. For flood routing, the channel segments connecting the separate watersheds further increase the dimensionality of the hydrologic state vector. As the need for improved hydrologic resolution increases in real-time operating problems, it is difficult to avoid an exponential increase in the computational burden. The dimensionality curse is very real indeed.

We note, however, that the growth in the state vector with increasing complexity is largely restricted to that part of the state vector representing the uncontrollable hydrologic system. If we accept the need to recursively solve an approximately optimal problem with feedback, we may consider algorithms that, while clearly not closed loop, reduce the dimensionality of the inherently imperfect representation of the hydrologic system. Stedinger et al. (1984) showed that the choice of an informative representation of the hydrologic state, was as important in identifying efficient operating decisions as the choice of algorithm used to compute the optimal control decision for the system. The growth of the hydrologic state vector with hydrologic complexity continues to make closed loop solutions computationally unattractive. We approach this problem using an approximation of the hydrologic system in an open loop algorithm. Feedback is employed at each time step to limit the error introduced in the simplified hydrologic state representation.

We develop an open loop feedback algorithm in which hydrologic observations are used to approximate future transitions in the hydrologic state vector at each time step. The two-stage recourse algorithm explicitly separates the representation of the uncontrolled hydrologic state vector from the optimal release for the controllable reservoir component of the reservoir system. Feedback from updated hydrometeorologic observations is utilized (although the availability of these future updates is not explicitly considered in the current period's operating problem) to keep this state approximation accurate and specify current boundary conditions.

7.3.5 MULTIPLE OBJECTIVE TWO-STAGE RECOURSE WITH FEEDBACK

The two-stage recourse problem is a stochastic optimization problem in which a decision must be made determining the allocation of a finite resource of unknown amount. The available resource is known only to a probability density function and is represented by the random vector Q. An allocation decision, x, must be made in the current period, prior to observing the realization q, of the random process, Q. After observing the realization q, a second stage decision, y, is made to allocate the remaining resource. The optimization problem is to find the optimal first stage decision variable, x^* that minimizes the expected value of the combined costs of the first and second stage decisions. The two-stage recourse problem is formulated as:

(2SR-1)

$$\min_{x} \left[E_{Q} \min_{y} \left(c'x + d'y \right) \right]$$

s.t.

$$Ax + By = Q$$

$$x, y, \geq 0$$

where the coefficient matrices A and B embody the operative system constraints, and Q is a random vector of known probability density. In this problem feasibility requires :

$$y = B^{-1}(q - Ax)$$

The reservoir operating problem is a stochastic sequential decision problem. We need to identify an "optimal" decision for the current period that accounts for hydrologic uncertainty in future periods. To consider this multi-stage problem as a two stage decision problem requires the introduction of an approximation of the probabilistic hydrologic state transitions. This approximation maintains a forward looking property of closed loop algorithms in the open loop algorithm. We consider the first stage decision as the release for the current time period. The set of future releases in all subsequent time periods is treated as the second stage decision vector. This approximation is equivalent to assuming 1) the probability density function of all future flow states is known, and 2) after the current period's decision has been made, the particular realization of Q, is unambiguously observed.

The two-stage representation of the multistage operating problem produces the current period release that minimizes the expected value of the objective function, as well as the set of conditional release trajectories corresponding to the set of optimal releases that would be made for the remainder of the operating horizon after the true flood hydrographs were revealed. Clearly all hydrologic uncertainty will not be resolved in the next time period, after making the current release. To correct for this approximation, a feedback step is incorporated in the operating algorithm. After making the first period decision the next realization of the hydrometeorologic process (represented by q in problem 2SR-1) is observed. The state of the reservoir is updated based on the newly observed inflow and the release prescribed as the first stage decision variable. Using the next observation of the hydrologic process, a new two-stage problem is formulated using the updated level of reservoir storage as a new boundary condition.

We formulate the two-stage operating problem by assuming a forecast has been prepared from which the joint probability density function of reservoir inflow I and uncontrolled downstream runoff Q can be estimated. From this joint density we can derive a number J, of discrete realizations of streamflow as well as the probability of that joint realization. The joint density is sampled to generate hydrograph realizations for the basin $\{Q_j, I_j\}$ where

$$Q_j = \{Q_{j,t}, Q_{j,t+1}, \ldots Q_{j,T}\} \quad \text{and}$$

$$I_j = \{I_{j,t}, I_{j,t+1}, \ldots I,_{j,T}\}$$

are the hydrographs for the uncontrolled and controlled watersheds under discrete realization j. The probability of realization j is calculated from the estimated joint density function as p_j where

$$p_j = Pr\{Q_j, I_j \mid H(t,t-1,t-2,\ldots t_0)\}$$

and $H(t,t-1,t-2,\ldots t_0)$ denotes the current estimate of the hydrologic state, using the observed history of the system to date.

The two-stage operating problem is formulated as:

(2SR-2)

$$\text{Min } w[\sum_{j=1}^{J} \sum_{t=1}^{T} p_j u_{j,t}) + (1-w) \sum_{j=1}^{J} \sum_{t=1}^{T} (p_j v_{j,t}) + e \sum_{j=1}^{J} \sum_{t=1}^{T} S_{j,t}$$

s.t.

$$S_{j,t+1} - S_{j,t} - R_{j,t} = I_{j,t} \qquad 7.10$$

$$Q_{j,t} + M_{j,t} = F_{j,t} \qquad 7.11$$

$$M_{j,t} - KR_{j,t} - K'M_{j,t-1} = 0 \qquad 7.12$$

$$S_{j,t+1} - S_{,j,t} + u_{j,t} - u_{j,t} = 0 \qquad 7.13$$

$$F_{j,t+1} - F_{j,t} + v_{j,t} - v_{j,t} = 0 \qquad 7.14$$

$$S_{j,T} = 0 \qquad 7.15$$

$$R_{j,1} - R_1 = 0 \qquad 7.16$$

Constraints 7.10 - 7.14 hold for j = 1,2,...J, t = 1,2,...T where J is the number of hydrologic realizations considered and T is the length of the operating horizon, and:

- $S_{j,t}$ - volume in flood storage at the end of time t under hydrologic future j
- $R_{j,t}$ - release from flood storage during time t under hydrologic future j
- $I_{j,t}$ - inflow to flood storage during time t under hydrologic future j
- $Q_{j,t}$ - uncontrolled runoff at the downstream damage center at time t under hydrologic future j
- $M_{j,t}$ - Muskingum routing of reservoir releases contributing to downstream damage at time t under hydrologic future j
- $F_{j,t}$ - discharge at the downstream damage site at time t under hydrologic future j
- $u_{j,t}$ - changes in flood storage during time t under hydrologic future j
- $v_{j,t}$ - changes in stage at the downstream damage site during time t under hydrologic future j
- R_1 - flood control release for the current period
- p_j - the probability of hydrologic future j under the current forecast
- e - an arbitrarily small constant
- K,K' - Muskingum channel routing constants

The objective function minimizes the expected value of the cumulative rise in reservoir storage and the cumulative rise in discharge at the downstream damage center. The third term in the objective function is the cumulative volume of flood control storage for all hydrologic futures. This term is included in the objective function with an arbitrarily small weight. The small weight on total storage is chosen to ensure this term will not effect the cumulative rises in either storage or discharge. Inclusion of the total storage term drives the solution toward alternate optima with respect to storage and discharge in which flood control storage is reduced from peak levels most rapidly.

Constraint 7.10 is the continuity equation which applies to all hydrologic realizations, in all time periods. The discharge at the downstream damage site is defined in 7.11 as the sum of the current uncontrolled runoff plus the routed component of reservoir releases. The routing in this formulation is defined using a linear Muskingum channel in 7.12. It is clear from constraints 7.10 - 7.12 that each hydrologic realization results in the solution of a conditionally deterministic optimization problem as part of the second stage solution. Each conditional solution provides a conditional peak discharge and peak storage resulting from a conditionally optimal reservoir regulation. Constraints 7.13 and 7.14 have been added to ensure the operating plans for each hydrologic realization are plausible from the reservoir operator's perspective.

The two measures of operating performance in this formulation are peak discharge and peak storage. A formulation simpler than (2SR-2) could be developed in which constraints 7.13 and 7.14 are replaced by :

$$S_{j,t} - SMAX_j \leq 0 \qquad 7.17$$

$$F_{j,t} - FMAX_j \leq 0 \qquad 7.18$$

and the first two terms in the objective function were simply

$$\sum_{j=1}^{J} p_j SMAX_j \quad \text{and} \quad \sum_{j=1}^{J} p_j FMAX_j$$

The specification of maximum storage and discharge in terms of the cumulative rises defined in 7.13 and 7.14 rather than 7.17 and 7.18 ensures the optimal operating increase in storage and discharge will be monotone, and will not admit spurious secondary peaks. Without constraints 7.13 and 7.14 and their contributions to the objective function, optimal release trajectories would allow erratic fluctuations in storage in which inflow was alternately stored and released every few hours.

Such an operating policy, though unacceptable to the reservoir operator would be a feasible solution to problem 2SR-2. Indeed such a solution could be an alternate optima in a formulation that minimized only $SMAX_j$ and $FMAX_j$. The more complex formulation of 2SR-2 avoids alternate optima with unrealistic release trajectories.

Constraint 7.16 imposes the two-stage structure on the operating problem. This constraint requires each operating trajectory for each hydrologic realization to utilize the identical release for the current period. Regardless of the hydrologic regime that is realized in later time periods, the conditionally optimal operating trajectory for each of the likely hydrologic realizations must begin with the identical first period release - the current period's release decision. In this way we generate the current period's release along with the set of conditionally optimal future releases to be followed for each possible hydrologic realization being considered.

Constraint 7.15 simply imposes an end of horizon boundary condition on drawdown. In practice the total storage term in the objective function will accelerate drawdown by driving solutions toward alternate optima that satisfy constraint 7.15.

The two-stage formulation allows future optimal releases to be part of the current release decision. In this way the two-stage algorithm provides a current period release decision (conditioned on the current level of flood storage and recent releases currently in the channel) that approximates the forward looking property of closed loop solutions.

If the probabilistic evolution of the hydrologic system could be known with certainty, the future joint inflows (I_j, Q_j) could be directly incorporated in the operating model. Our imperfect understanding of the evolution of flood producing storm systems limits our ability to completely describe future state transitions. This limitation necessitates recursive updating of the current estimate of the hydrologic state and the operating forecast of flood conditions over the basin.

We emphasize that this limitation in prescribing the hydrologic state dynamics is an inherent part of the real-time flood operating problem and is in no way limited to the two-stage algorithm presented here. Recursive estimation of the hydrologic state using updated observations must arise in real-time operating algorithms, where state dynamics are imperfectly known.

The second significant difference between the two-stage algorithm and a true closed loop algorithm is the implicit inclusion of the reservoir-channel system in the state variable. Current reservoir storage and channel contents are prescribed as initial conditions for the two-stage formulation. In a true closed loop solution the optimal policy would be conditioned on the observed state variable including storage and channel conditions explicitly. The implicit use of storage and channel conditions requires the two-stage algorithm to be re-solved at each time step with new boundary conditions. This requirement is not restrictive. Given the need for recursive estimation of the hydrologic state variable. A real-time operating algorithm will have to recalculate the optimal solution for the current estimate of the system state. The two-stage algorithm developed here is no exception.

7.3.6 FIRST MOMENT NON-INFERIOR SOLUTIONS

In contrast to a single objective optimization problem that provides the optimal release for the current period, the multiobjective algorithm admits a set of release decisions, each of which is non-inferior with respect to the conflicting objectives considered. Reflecting the conflicting risks of reservoir operation, the stochastic optimization problem is formulated as a tradeoff of peak discharge and peak storage. In formulation 2SR-2 these conflicting objectives appear in the objective function with weights w and (1-w) respectively. Any choice of weight w will allow the problem to be solved for a non-inferior solution. The choice of weights can be used to define the operator's preferences if they can be unambiguously elicited, Haimes(1974). The variation of weight w can also be used to generate the non-inferior set Cohon (1978). Generating the non-inferior set allows the decision maker to assess the choice of operating decisions in both an absolute sense (e.g. whether or not any alternative can avoid damaging stages) as well as in comparison to other non-inferior choices.

Although each solution of problem 2SR-2 provides an estimate of the joint distribution of storage and discharge, the multiobjective solutions are only non-inferior in terms of their expected values. For this reason we refer to the solutions of the stochastic multiobjective two-stage problem as **first moment non-inferior solutions**. Using generating methods (Cohon 1978) such as the NISE (non-inferior set estimation) technique (Cohon, Church and Sheer 1978) the first moment non-inferior set may be accurately estimated and presented to the reservoir operator in the form of a trade off curve as in Figure 7.5. The first moment non-inferior set gives the operator a graphical summary of the feasible performance alternatives for the flood control system under current forecast conditions. Each non-inferior solution of the multiobjective two-stage recourse problem can be represented by the expected values of peak storage and peak discharge resulting from the proposed operating strategy.

Each first moment non-inferior solution also has a current period reservoir release associated with it, as well as an estimate of the joint density of storage and discharge that will result from that particular strategic choice.

7.4 REAL-TIME FLOOD CONTROL OPERATION

In this section we apply the two-stage recourse formulation to a representative single reservoir flood control problem. The system as shown in Figure 7.6 consists of a 125 square mile drainage basin upstream from an 8,000 acre-foot (AF) flood control reservoir that can store 1.25 inches of runoff. A damage center is located downstream from the reservoir. The damage center is also influenced by a significant uncontrolled drainage. Channel effects are significant between the reservoir and the damage site and are represented using Muskingum routing. Hydrographs for the two drainage areas are available with concentration times, for this example of 6 hours.

7.4.1 THE FORECAST ENVIRONMENT

We assume forecast information is available for each time period and without loss of generality we suppose meteorologic radar images and precipitation gages are providing data both within and beyond the boundaries of the watershed. Flood control operation is most critical for storms producing runoff volumes exceeding the annual flood. The large frontal systems with which these storms are associated are generally observed before they deliver precipitation to the basin. We assume estimates of storm depth & storm structure are available prior to the onset of runoff producing precipitation.

We simulate operation for the storm system by assuming a forecast has been issued calling for 1 to 3 inches of rain sometime during the next 12 hours. This "forecast" is a great simplification of the prudent steps that the operator of a flood control reservoir would take in monitoring the progress of a storm system. This simple forecast was chosen to emphasize two points. First, the operating algorithm uses a probabilistic description of the current forecast that is separable from the use of any particular forecasting technique. The current estimate of the storm attributes can be developed from a large number of the existing, or developing forecast technologies. Second, the assumption of a uniform distribution for both precipitation depth and storm timing represents a case of near maximum uncertainty. As observations on the storm system accumulate, the estimates of depth and timing become more accurate, converging to their true values. This convergence will be expressed in a reduction in the variance of the distributions and the emergence of a unimodal rather than a uniform distribution of the random components of the forecast. While the uniform distribution assumed here may represent too poor a forecast, the operating algorithm nevertheless provides operating alternatives for this highly uncertain environment. In the face of significant uncertainty early in the storm the two-stage recourse formulation provides non-inferior alternatives that hedge against hydrologic uncertainty.

Forecasting tools are currently available that would provide better forecast information to a reservoir operator than that assumed here. As forecasting technology continues to improve so to will the accuracy and timeliness of forecast products. Improved forecasts (manifested in smaller variances about unbiased predictions) will lead to more efficient utilization of reservoir storage. Improved operation will allow incremental reductions in flood crests, as well as alternate uses of flood storage without an increase in flood risk. The stochastic multiobjective operating algorithm is readily adapted to use a wide variety of forecast products producing hedging releases in real-time.

In each time period we generate J hydrologic realizations $\{I,Q\}$ and their associated probabilities of occurrence $P=\{p_1,p_2,... p_J\}$. The hydrograph pair varies in magnitude and timing according to a current joint density resulting from some forecast process.

Conceptually we imagine the availability of a sequence of radar images, from which the velocity of the storm system is estimated. Radar reflectivity and precipitation gages are used to estimate precipitation intensity, and distribution within the storm. Projecting likely storm trajectories provides a distribution of coverage for each basin. Routing precipitation forecasts through a calibrated hydrologic model provides likely hydrographs for the basin, varying in both runoff volume timing over the watersheds.

To demonstrate the algorithm we have assumed the depth of precipitation is a uniformly distributed random variable. Considering a significant frontal passage over our relatively small basin we further assume that the depth of effective precipitation over the two watersheds will be identical. We allow random variation of the actual storm track by considering the difference in the timing of precipitation over the two basins as a random variable. This admits an uncertain situation in which not only the depth of precipitation (and hence volume of runoff), but also the relative timing of runoff is uncertain. The operating decisions will account for both sources of uncertainty

Each realization of precipitation depth is distributed temporally with the dimensionless hyetograph depicted in Figure 7.7. While estimates of the structure and development of the storm can be made (Johnson and Bras 1979) the distribution of precipitation will in general be known only to an estimated probability density. The estimated stochastic structure of the hyetograph would also be a random variable for which discrete realizations would be required to further specify the set of probabilistic flood hydrographs for the basin.

In this example the hyetograph is assumed known and the only sources of uncertainty are in the depth of precipitation and the relative timing of runoff over the two basins. For illustration we represent the current forecast as a precipitation depth that is uniformly distributed between 1 and 3 inches and a difference in the time of peak runoff between the two basins uniformly distributed between -6 and 6 hours. In other words the peak runoff into the reservoir may arrive as early as 6 hours before the downstream peak, or as late as 6 hours after that peak (i.e. a range of twice the concentration time for the two basins).

Using this probabilistic forecast, we simulate flood control operation by using the probabilistic forecast to generate likely runoff events. The uniform distributions assumed for both precipitation depth and the time lag between precipitation over the two basins, represents a forecast with relatively low information content. The initial uncertainty is increased by assuming the depth and timing distributions for the storm are uncorrelated. Knowledge of depth of precipitation gives no skill in estimating the relative precipitation timing over the two basins.

For these very general assumptions, the distributions for depth and timing of precipitation are independently sampled and combined to produce 25 equally likely synthetic storms. The storm totals are distributed with the dimensionless hyetograph of Figure 7.7. Using the time lags for each depth and the dimensionless hyetograph, a set of 25 equally likely flood hydrographs are generated using the calibrated unit hydrographs for the two watersheds. In this way a set of flood hydrographs are generated that are consistent with the current forecast.

7.4.2 THE CURRENT RELEASE

Evaluating the operating risks represented in this set of likely hydrologic conditions, the operator must decide the degree and type of risk he is willing to accept, and translate this risk preference into an operating decision for the current period.

To assist in this decision, we solve the stochastic multiobjective operating problem to present the operator with a description of the efficient probabilistic choices available to him at the current time. One solution that is always available is to open the flood gates and release all inflow. While no storage would be used, the expected value of the downstream flood peak based on current forecast information would be 55,000 cfs. Solution of the multiobjective operating problem generates the tradeoff curve in Figure 7.8. The most aggressive regulation of the flood could reduce the expected value of the downstream peak discharge to 26,600 cfs with an expected peak utilization of 55,752 cfs-hrs of storage. If reservoir storage is used less aggressively (e.g. to hedge against the possibility of a second storm) the downstream flood peak is likely to be increased. The extreme point shown in Figure 7.8 reduces the expected flood peak to 36,947 cfs, but the expected storage required for this operation is only 7,869 cfs-hrs.

Associated with each first-moment non-inferior solution is a reservoir release for the current period. Figure 7.9. shows the current period release to be made in order to achieve the expected value result of each non-inferior point on the trade off curve. The curious result shown in Figure 7.9. indicates that the optimal release in the current period is 100 cfs regardless of the strategic operating targets for discharge and storage utilization favored by the operator. Even though a wide range of operating alternatives are feasible based on the current flood forecast, in the current time period no action is required. Although the operator will have to choose operating targets from the set of risky alternatives depicted in Figure 7.9, there is no need to hedge against forecast uncertainty in making the current period's operating decision. The current operating problem is not dominated by the need to hedge against hydrologic uncertainty. The opportunity to evaluate the risky alternatives that are feasible before a hedging decision must be made is viewed as a particularly useful feature of the proposed algorithm.

In addition to the expected value of storage and discharge, each non-inferior solution provides information about the probability distribution of storage and discharge. For example, in Figure 7.10 the probability of completely filling reservoir storage is shown for each non-inferior solution. If the reservoir operator was willing to use 10,000 cfs-hrs less storage, resulting in an increase in the expected flood peak from 26,000 to 28,000 cfs, the probability of filling the reservoir could be reduced from .64 to .28. The operator may be willing to accept an 8% increase in the expected flood peak in order to provide a dependable reserve of storage during the flood event.

7.4.3 THE OPERATING RISKS

Filling flood storage represents a very real risk the operator may wish to avoid. The probability of completely filling storage is, however, an insufficient measure of this risk. An operating plan that expects to use 99.9% of available flood storage will appear just as risky to a reservoir operator as a plan using 98% or 100%. The probability of crossing any one storage level may be a poor description of risk. To illustrate this point Figures 7.11 and 7.12 present the estimated joint probability density of storage and discharge for the two non-inferior solutions for which the probabilities of filling storage are .64 and .28 respectively. It is clear from the probability contours how much more likely the flood pool is to fill if operations follow the first non-inferior solution.

Comparing the two density surfaces, it is also clear that reservoir operation exerts a great deal of control over the mapping of hydrologic inputs into storage-discharge space. In expressing a preference for an arbitrary probability of using a prespecified volume of storage, the operator implicitly assumes the location of a particular quantile of the storage distribution contains information about the shape of the tail of that distribution. For a water resource system in which operating decisions can dramatically shape the distribution of system outputs, the location of a single quantile of the distribution is an insufficient descriptor of the tail of the distribution.

For the multiobjective flood control algorithm presented here, the complete joint probability density function of system outputs contain important information describing the feasible operating alternatives. Attempts to reduce this distribution to a single metric such as the mean, or an arbitrary quantile will not accurately convey the risks associated with a strategic choice. This observation has significant implications for stochastic operations of water resource systems using chance constraints or expected value formulations.

The "solution" of a stochastic optimization problem is really a choice of the optimal transformation of stochastic inputs to a probability density of returns. When the inputs can be described with a known parametric probability density function, and the resulting distribution of outputs is a smooth, well defined transformation of system inputs, a simple location parameter such as a mean or an arbitrary quantile is sufficient to completely specify the distribution of returns. In this situation, an expected value solution or a chance constrained solution are equivalent in choosing the optimal distribution of returns.

As the mapping of stochastic inputs to expected returns becomes more complex, the unique specification of the distribution of returns becomes increasingly difficult. Neither an expected value optimization, nor the specification of a single quantile through, for example, a chance constraint, are sufficient. Figures 7.11 and 7.12 show, the asymmetrical, multimodal mapping of the uniformly distributed precipitation forecast into the joint distribution of storage and discharge.

The flood release decisions clearly have a profound and dramatic effect on the shape of the distribution function of outputs. If a constraint were added restricting the probability of filling flood control storage, many solutions would be generated in which the difference between peak storage and reservoir capacity could be made arbitrarily small. The constraint would be satisfied but the reliability of the system would not be significantly improved.

Chance constraints or expected value objective functions alone, implicitly attempt to choose the optimal distribution of outputs by locating a single quantile of the pdf of returns. For systems which exert great control over the distribution of outputs, the location of a single quantile of the output distribution contains little information. We suggest that the nature of many structural solutions to water resource problems is to provide the means to fundamentally alter the distribution of system outputs. Operating decisions that effectively choose the pdf of system benefits should be based on the most complete representation of the distribution of system returns possible. For water resource problems utilizing significant structural means of flow modification, decisions based only on the location of a single quantile may provide an inaccurate description of the risks an operating decision creates.

The complete description of the available operating alternatives includes not only the current trade off curve with release decisions for each first moment non-inferior solution, but also the joint pdf of storage and discharge for each alternative. This description of the risky alternatives available at each decision point is superior to any single metric or univariate reduction of the mapping into objective space.

7.4.4 OPERATING THROUGH A STORM

With this background we simulate reservoir operation through the entire flood event. Operation consists of the generation of the non-inferior set at each time period, along with the optimal release and joint distribution of storage and discharge for each non-inferior solution. The reservoir operator uses this information to choose a strategy for managing the current flood, where the strategy refers to the preferences of risks implicit in identifying one non-inferior solution as most desirable. From this strategic choice, the current period release represents the tactical means to achieve this objective.

We simulate the routing of the flood in Figure 7.13 by assuming the reservoir operator always chooses the non-inferior solution that minimizes the downstream discharge. The trade-off curve in Figure 7.11 shows the first moment non-inferior set generated at time t=21. Initially the full 65,000 cfs-hrs of flood storage is available, and the steady inflow of 100 cfs is being completely released.

The uncontrolled inflow hydrograph along with the stochastic multiobjective release pattern for the reservoir is shown in Figure 7.14. The maximum release shown was 20775 cfs compared to the maximum inflow of 28532 cfs. The main feature of the regulation in Figure 7.14 is a delay in the peak controlled runoff of approximately 6 hours. Despite the uncertainty in each operating decision the recourse formulation with feedback offers a plausible means to use forecast information to identify real-time operating decisions for flood control operation.

The maximum volume of water in storage was 50,921 cfs-hrs, although inflow is stored for a total of 10 hours. Storage from hour 25 through 28 reflects hedging against the possibility of a low flood peak or early downstream crest. Storage from hour 30-32 is mainly hedging against continuing flood inflow and a late downstream flood crest. Although the non-inferior solution always sought to minimize the expected flood peak, the full 65,000 cfs-hrs of storage was not used. This is attributable to the uncertainty of the forecasts used.

Even after the runoff peak at time 29, the forecasts still allowed for the possibility of a higher peak due to added precipitation (with low probability). The unused storage was reserved for this unlikely event. The algorithm again hedged against this uncertainty.

Hedging, both early and late in the storm event, has a significant storage cost which must be reflected in downstream discharge. Although hedging is suboptimal a posteriori, the need to hedge was imposed by the uncertainty of the runoff forecast. We assume flood forecasters will become increasingly confident in their runoff estimates through a storm. As this reduced uncertainty is used in the operating algorithm, release decisions will use storage more aggressively. In the face of considerable uncertainty, releases must be hedged. The Multiobjective two-stage algorithm provides hedging releases along with a full description of the resulting operating risks in each time period.

7.5 SUMMARY

Flood control operation of reservoirs provides benefits by storing damaging flows until conditions allow a safe controlled release of storm runoff. Release decisions that store runoff represent risky commitments of limited reservoir storage. The use of real-time forecast information can provide improved damage protection and risk reduction in real-time flood control operation.

Real-time flood control operation can be viewed as a stochastic sequential decision problem in which the finite volume of reservoir storage is sequentially allocated over time to maximize flood protection benefits. The risks associated with flood damage reduction require the reservoir operator to hedge release decisions against hydrologic uncertainty. Damage minimization leads to the full utilization of reservoir storage. Filling the flood pool imposes additional risks from unanticipated inflow as well as the possibility of overtopping the dam, leading to structural failure.

The choice between conflicting risks in making flood control release decisions is formulated as a multiobjective optimization problem. The operating choices at each decision point can be represented as a tradeoff of maximum reduction in flood discharge and maximum use of flood storage. Instead of identifying a single flood control release, the operator can choose from a set of non-inferior releases representing the range of risky alternatives that are available.

The stochastic sequential nature of real-time operation is represented as a two-stage recourse problem. Real-time forecast information is used to describe the current risks from hydrologic uncertainty. The two-stage problem seeks the release decision for the current period that maximizes the total expected benefits over the operating horizon. The two-stage recourse algorithm uses current forecast information to approximate the hydrologic state of the system in each time period. Recursive approximation of system hydrology using forecast information offers an alternative to the exponential growth in problem size associated with full state-space representations of complex hydrologic systems.

The combination of stochastic optimization and multiobjective programming is a unique feature of this work. At each decision point the reservoir operator is presented with a set of probabilistic operating alternatives that are non-inferior with respect to their first moments. Stochastic real-time operation is viewed as a controllable mapping of stochastic hydrologic inputs into a joint probability density function of system outputs. Each first moment non-inferior solution prescribes a current period release and a conditional estimate of the resulting joint density function of benefits.

The reservoir operator chooses a distribution of benefits, by choosing the current period's release decision. The stochastic multiple objective two-stage algorithm developed here provides the operator with both the first moment non-inferior set of feasible releases as well as the estimated joint density function of system benefits resulting from each release. The use of quantile locating criteria such as chance constraints or expected value optimization is shown to provide an insufficient description of the risks associated with alternate operating decisions.

An operating simulation is used to demonstrate the operation of the multiple objective two-stage recourse feedback algorithm. The combined information in the first moment non-inferior set and the estimated joint density of system outputs provides superior criteria for choosing between the risky operating decisions available through a flood event.

FIGURE 7.1

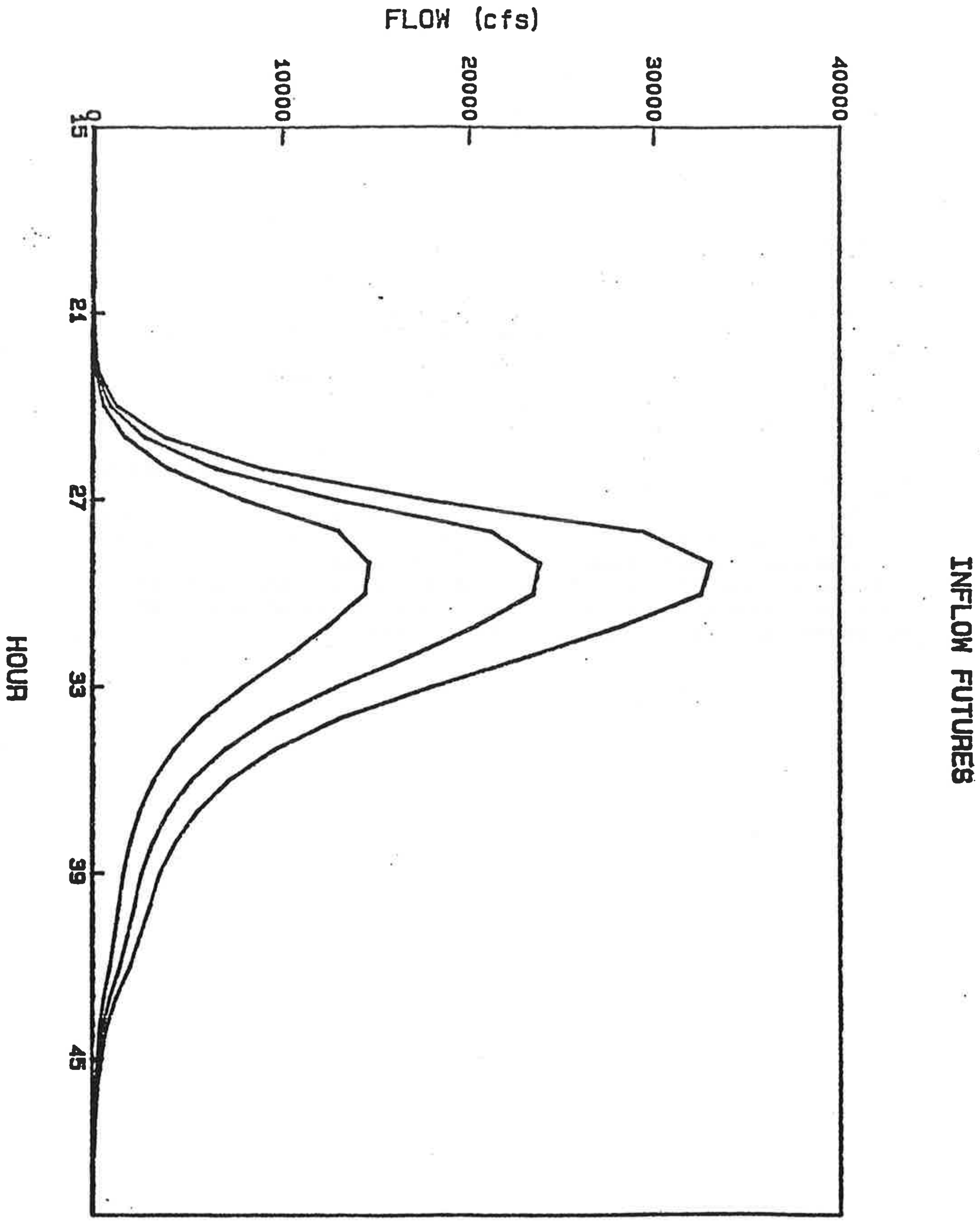

INFLOW FUTURES
FLOW (cfs)
0
10000
20000
30000
40000
15
21
27
33
39
45
HOUR

FIGURE 7.2

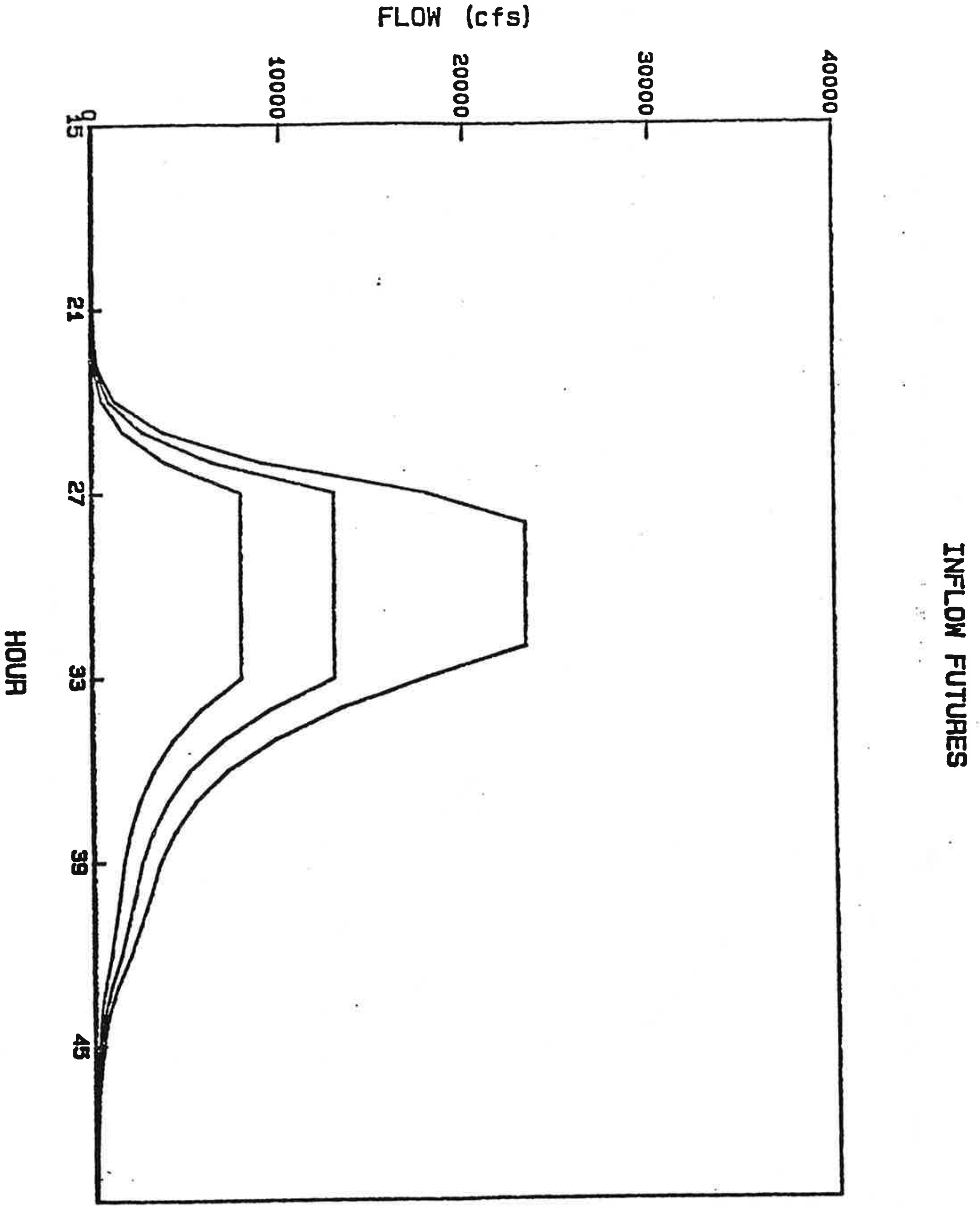

INFLOW FUTURES
FLOW (cfs)
0
10000
20000
30000
40000
HOUR
15
21
27
33
39
45

FIGURE 7.3

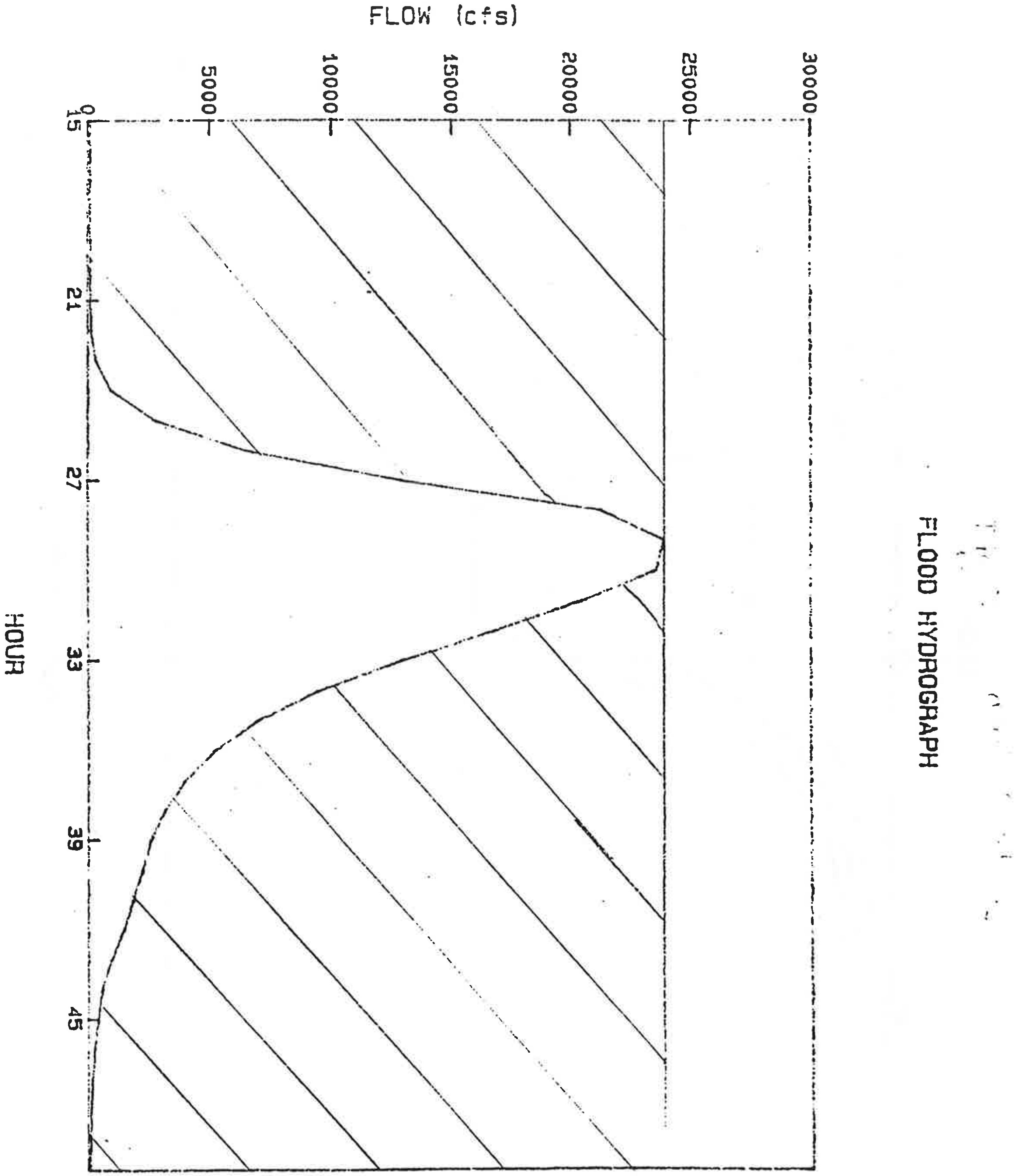

FLOOD HYDROGRAPH
FLOW (cfs)
0
5000
10000
15000
20000
25000
30000
HOUR
15
21
27
33
39
45

FIGURE 7.4

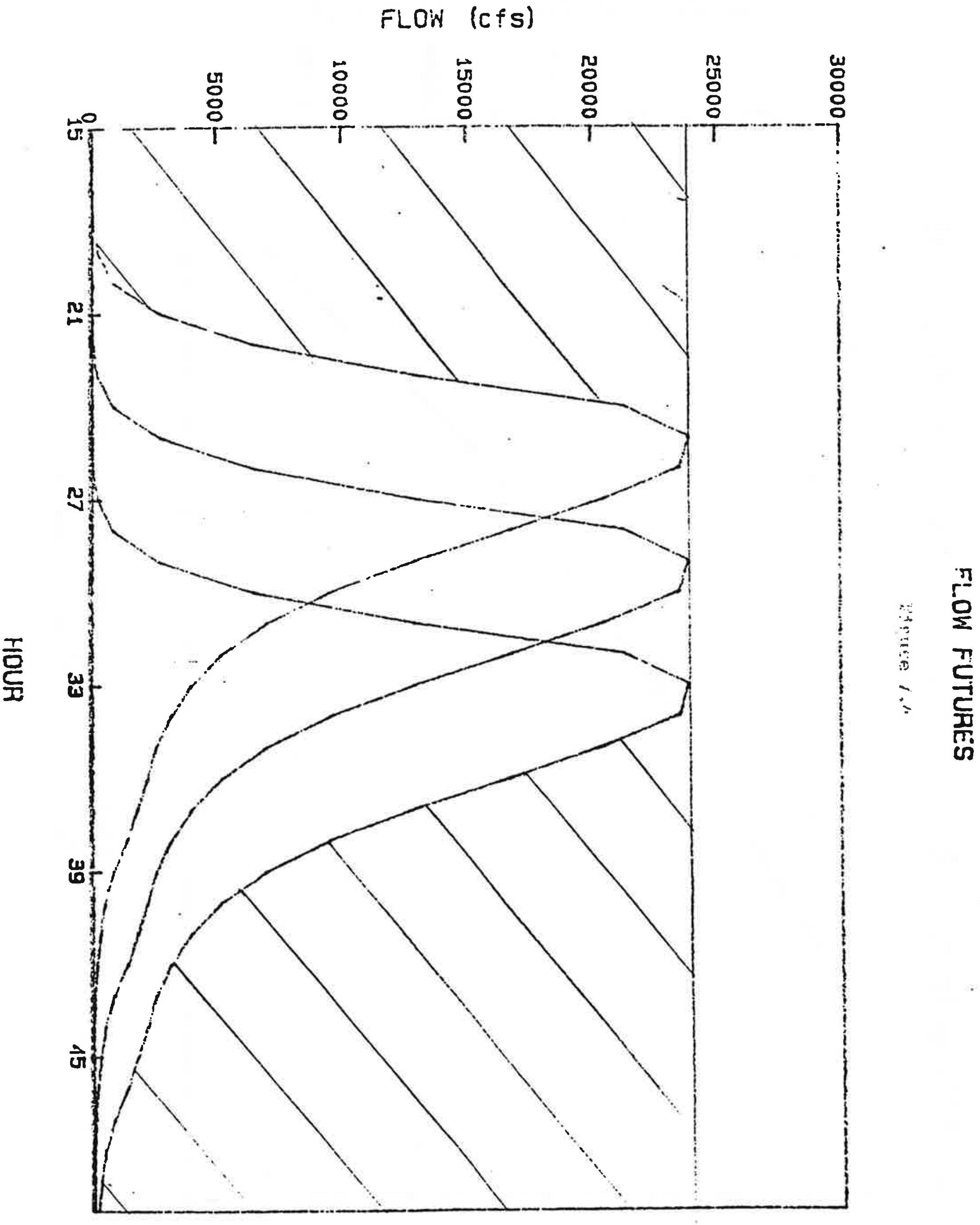

FLOW FUTURES
FLOW (cfs)
30000
25000
20000
15000
10000
5000
0
15
21
27
33
39
45
HOUR

FIGURE 7.5

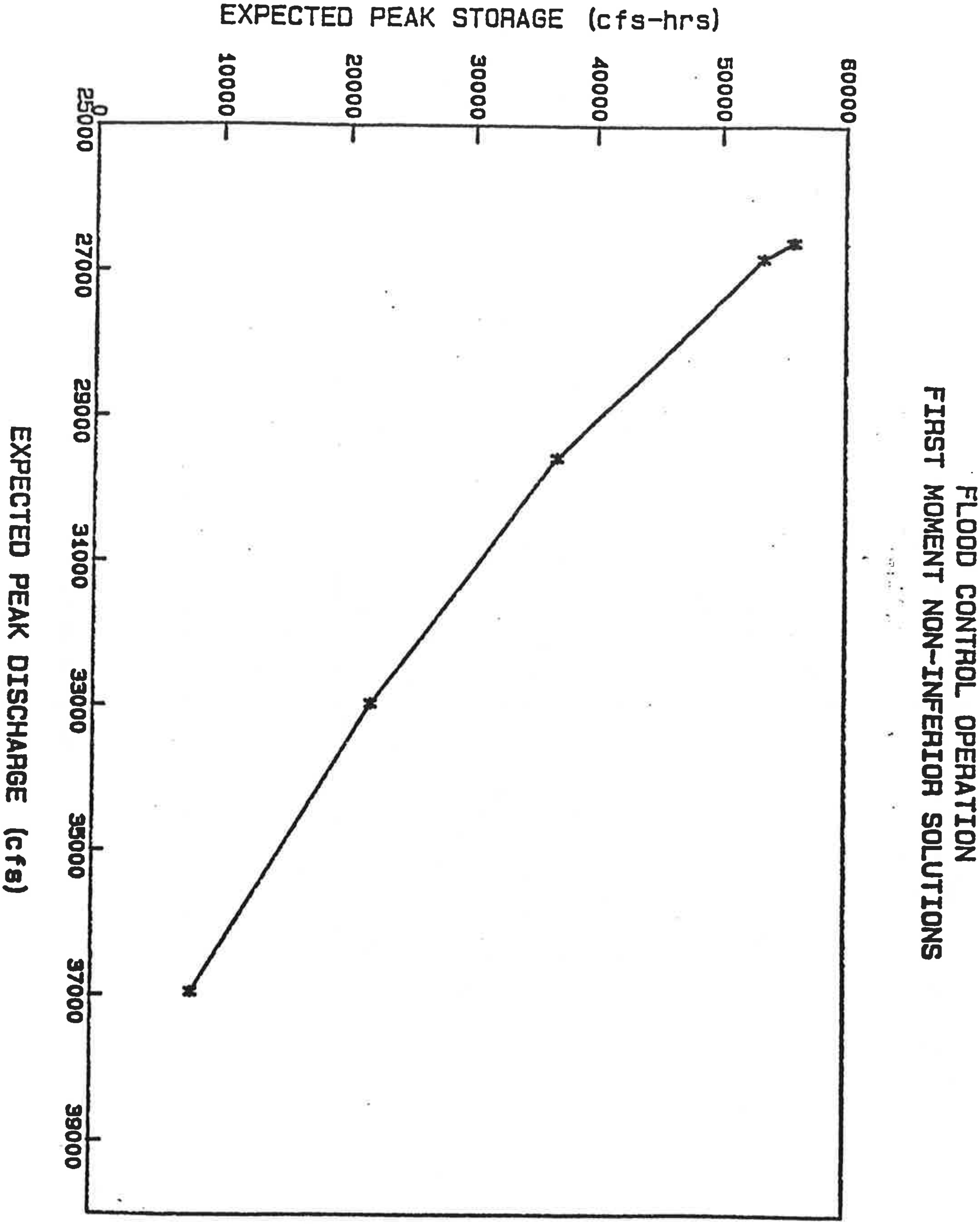

FLOOD CONTROL OPERATION
FIRST MOMENT NON-INFERIOR SOLUTIONS
EXPECTED PEAK STORAGE (cfs-hrs)
0
10000
20000
30000
40000
50000
60000
25000
27000
29000
31000
33000
35000
37000
39000
EXPECTED PEAK DISCHARGE (cfs)

Figure 7.6

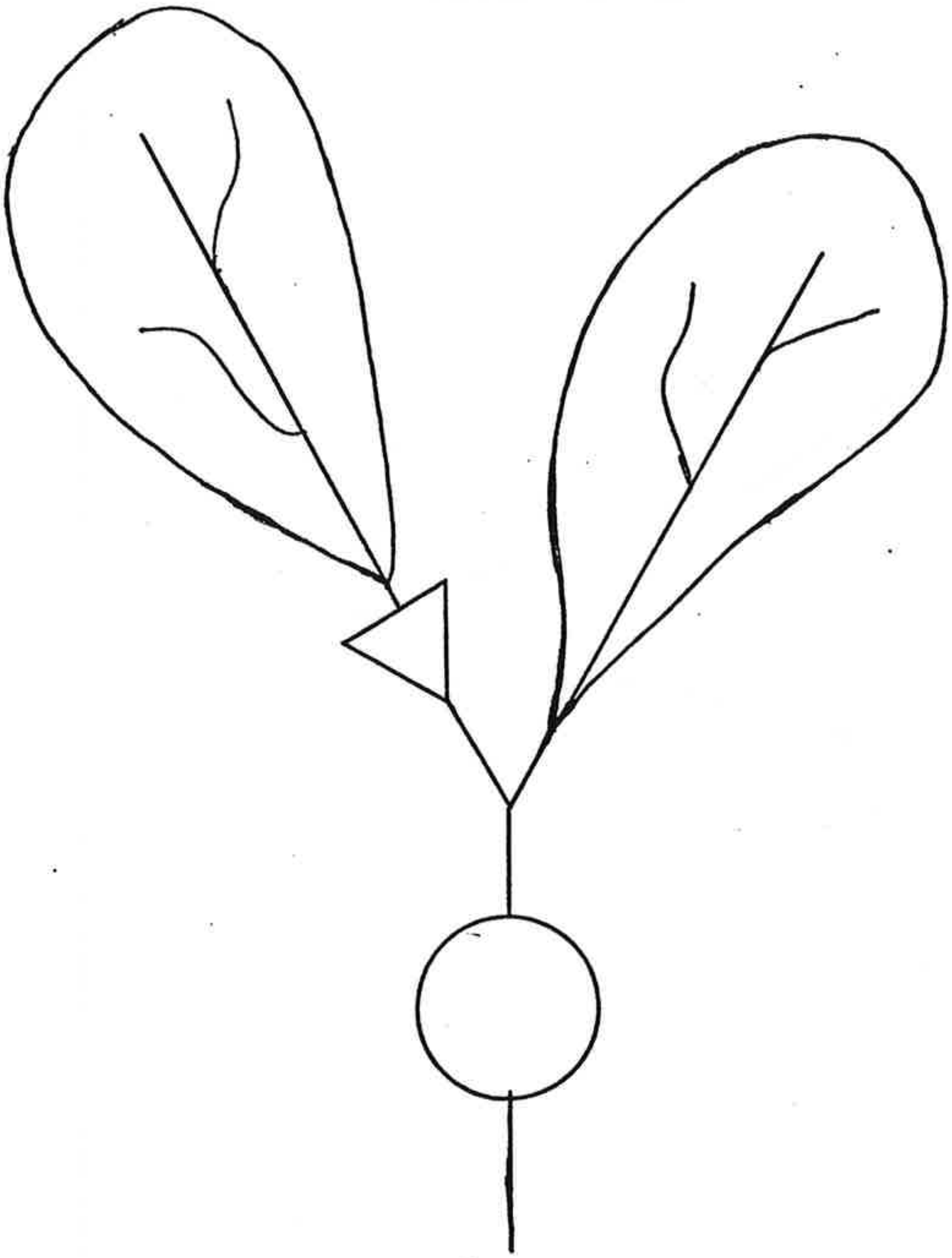


FIGURE 7.7

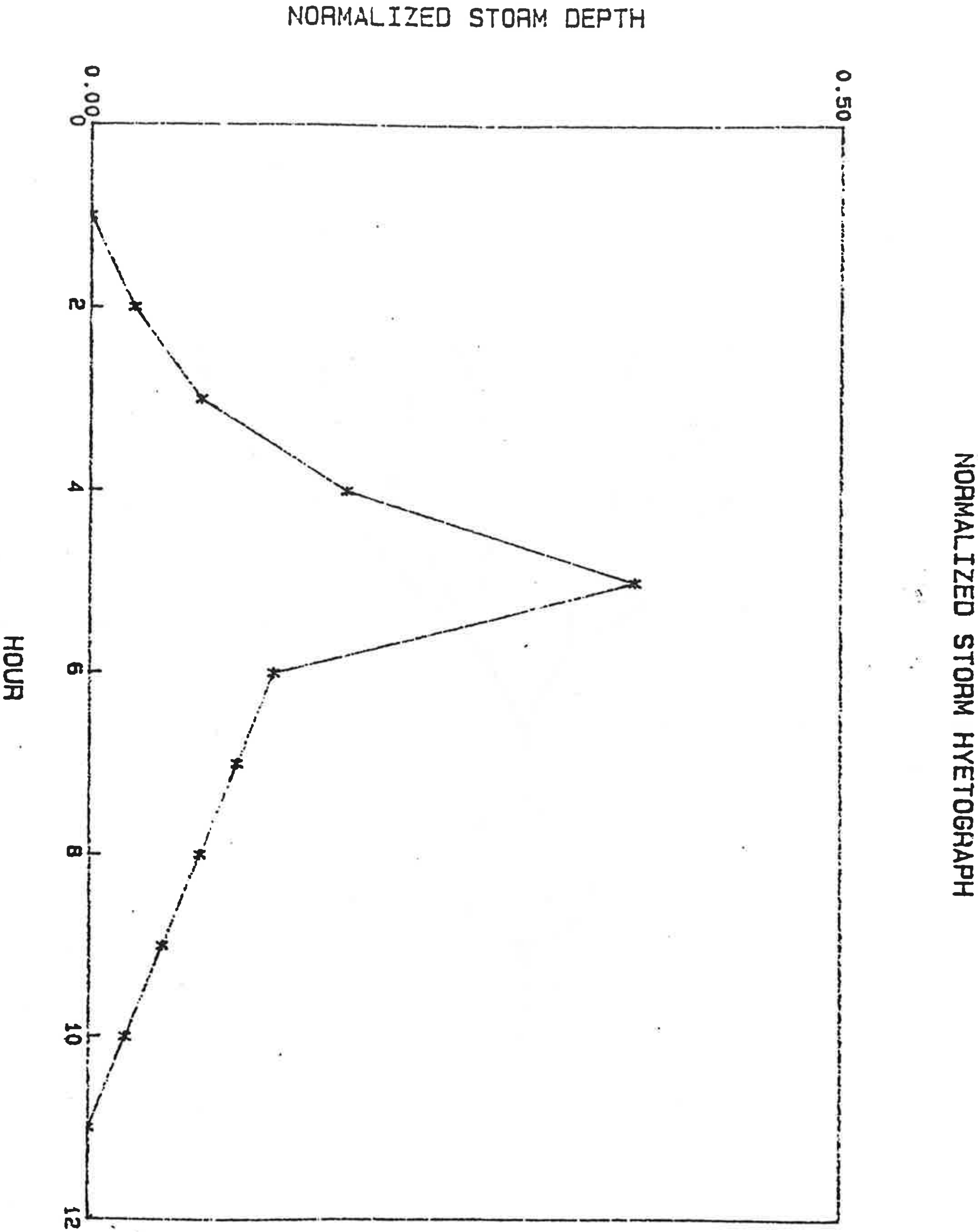

NORMALIZED STORM DEPTH
0.00
0.50
NORMALIZED STORM HYETOGRAPH
0
2
4
6
8
10
12
HOUR

FIGURE 7.8

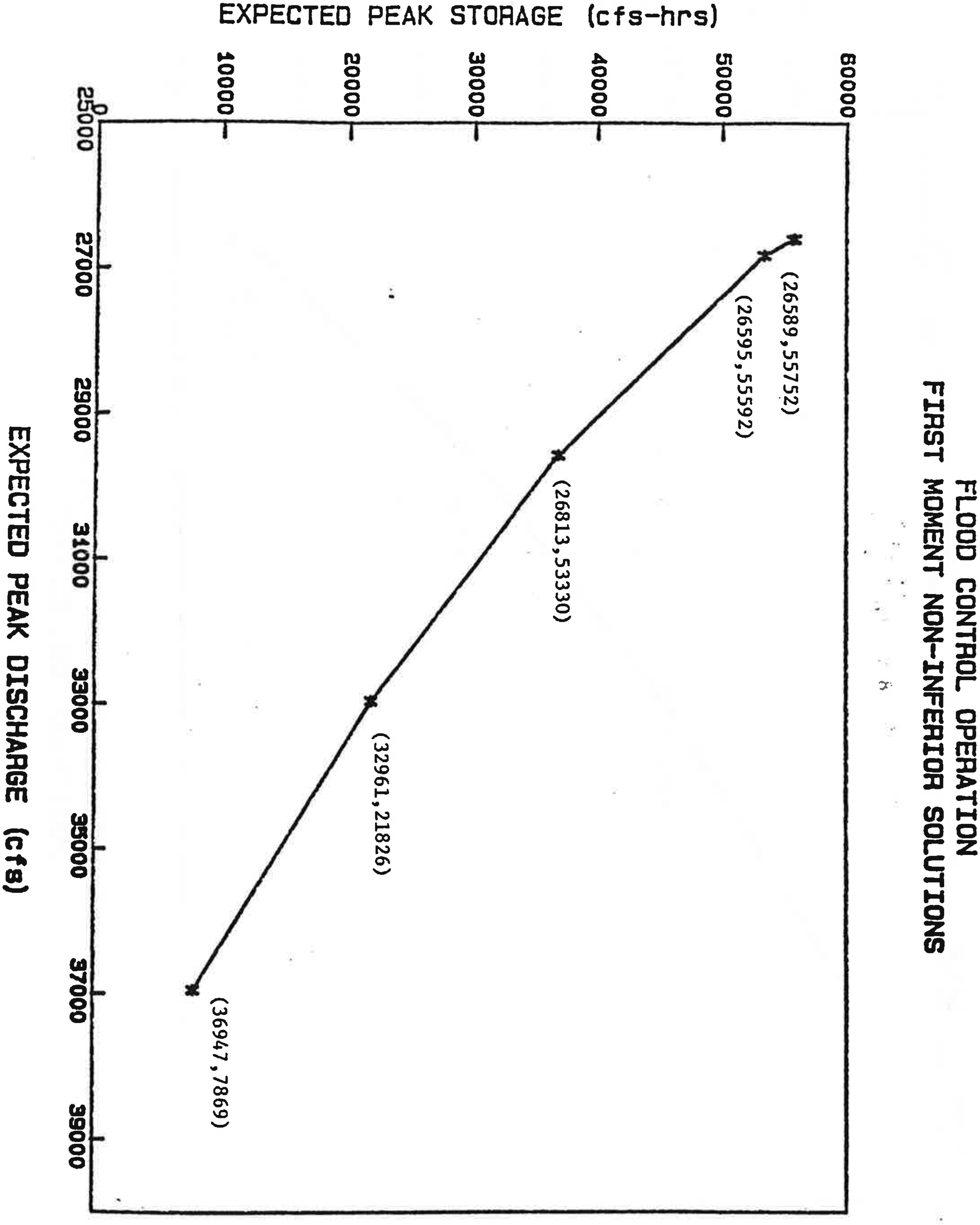
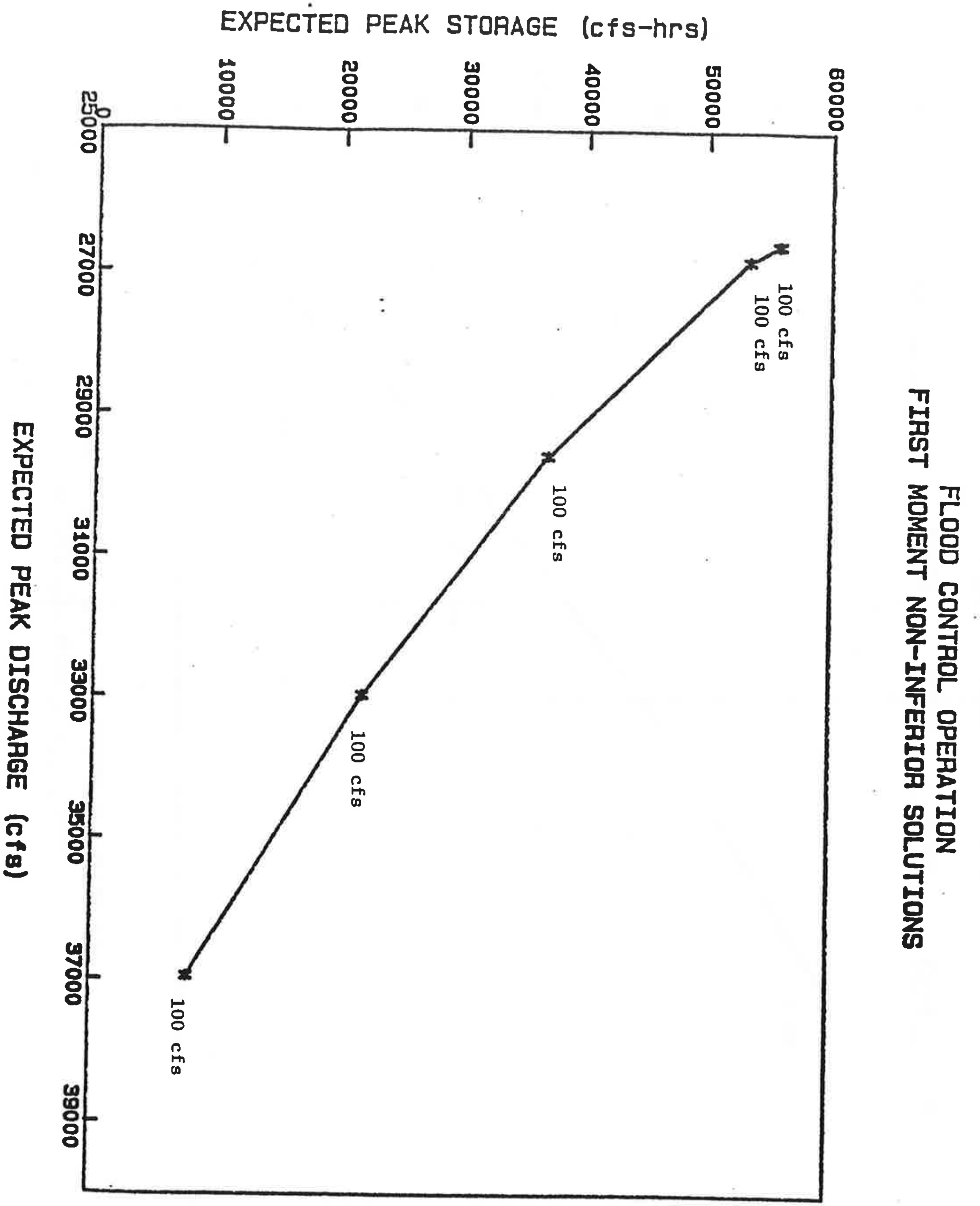

FLOOD CONTROL OPERATION
FIRST MOMENT NON-INFERIOR SOLUTIONS
EXPECTED PEAK STORAGE (cfs-hrs)
EXPECTED PEAK DISCHARGE (cfs)
0
10000
20000
30000
40000
50000
60000
25000
27000
29000
31000
33000
35000
37000
39000
(26589,55752)
(26595,55592)
(26813,53330)
(32961,21826)
(36947,7869)

FIGURE 7.9



FLOOD CONTROL OPERATION
FIRST MOMENT NON-INFERIOR SOLUTIONS
EXPECTED PEAK STORAGE (cfs-hrs)
0
10000
20000
30000
40000
50000
60000
25000
27000
29000
31000
33000
35000
37000
39000
EXPECTED PEAK DISCHARGE (cfs)
100 cfs
100 cfs
100 cfs
100 cfs
100 cfs

FIGURE 7.10

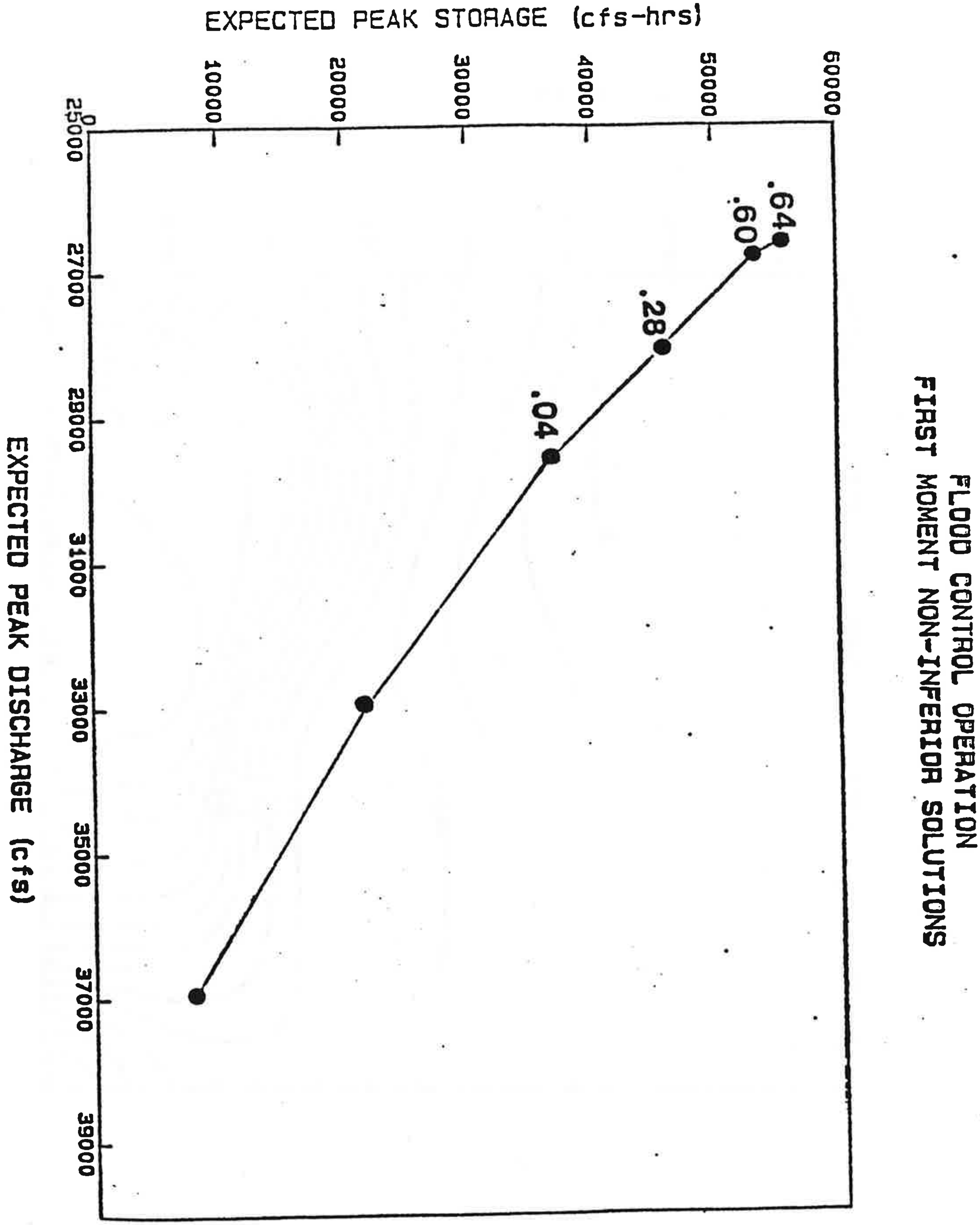

FLOOD CONTROL OPERATION
FIRST MOMENT NON-INFERIOR SOLUTIONS
EXPECTED PEAK STORAGE (cfs-hrs)
0
10000
20000
30000
40000
50000
60000
25000
27000
29000
31000
33000
35000
37000
39000
EXPECTED PEAK DISCHARGE (cfs)
.64
.60
.28
.04

FIGURE 7.11

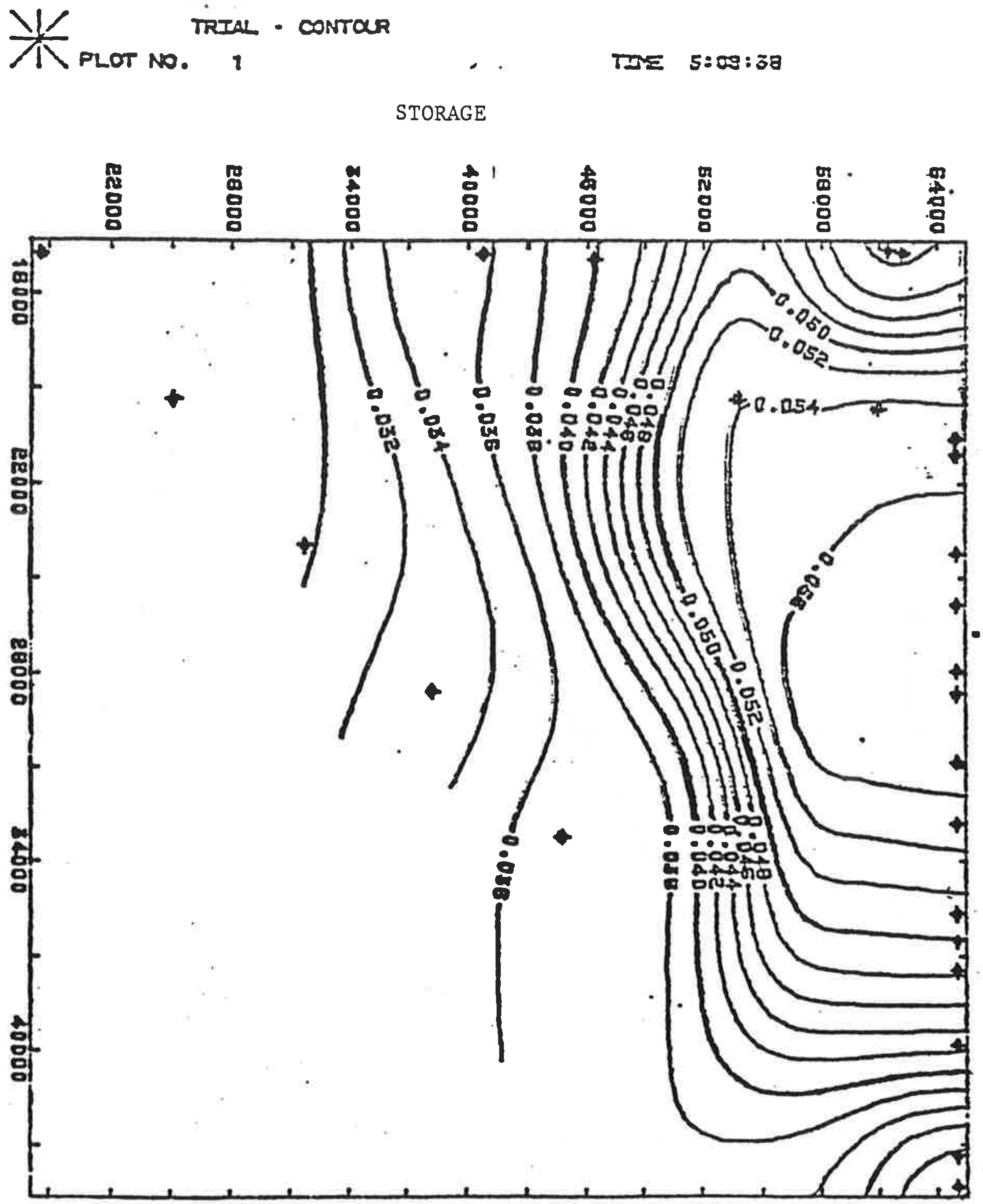

TRIAL - CONTOUR
PLOT NO. 1
TIME 5:03:38
STORAGE
0.032
0.034
0.036
0.038
0.040
0.042
0.044
0.046
0.048
0.050
0.052
0.054
0.058

FIGURE 7.12

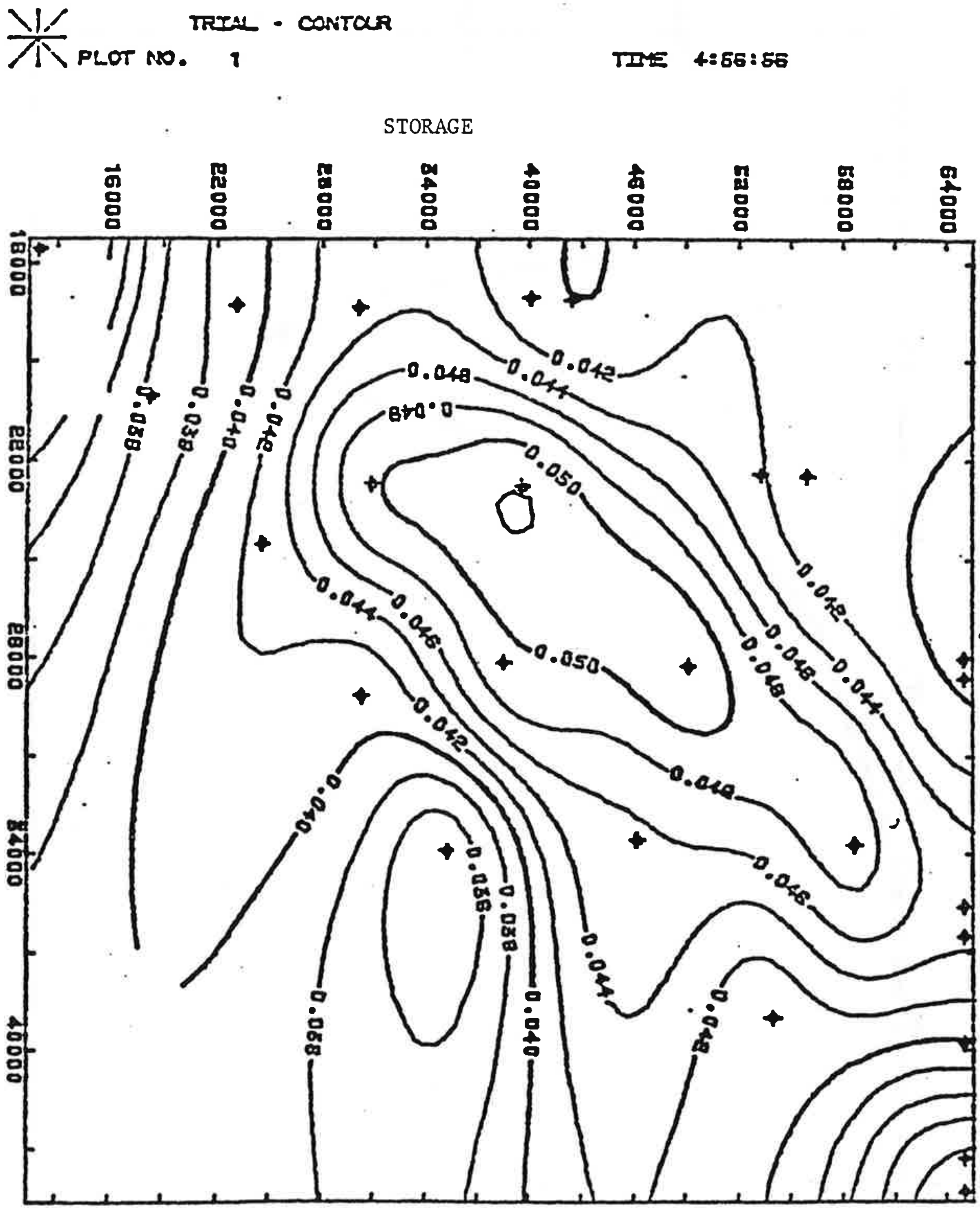

TRIAL - CONTOUR
PLOT NO. 1
TIME 4:56:56
STORAGE
16000
22000
28000
34000
40000
46000
52000
58000
64000
0.038
0.040
0.042
0.044
0.046
0.048
0.050

FIGURE 7.13

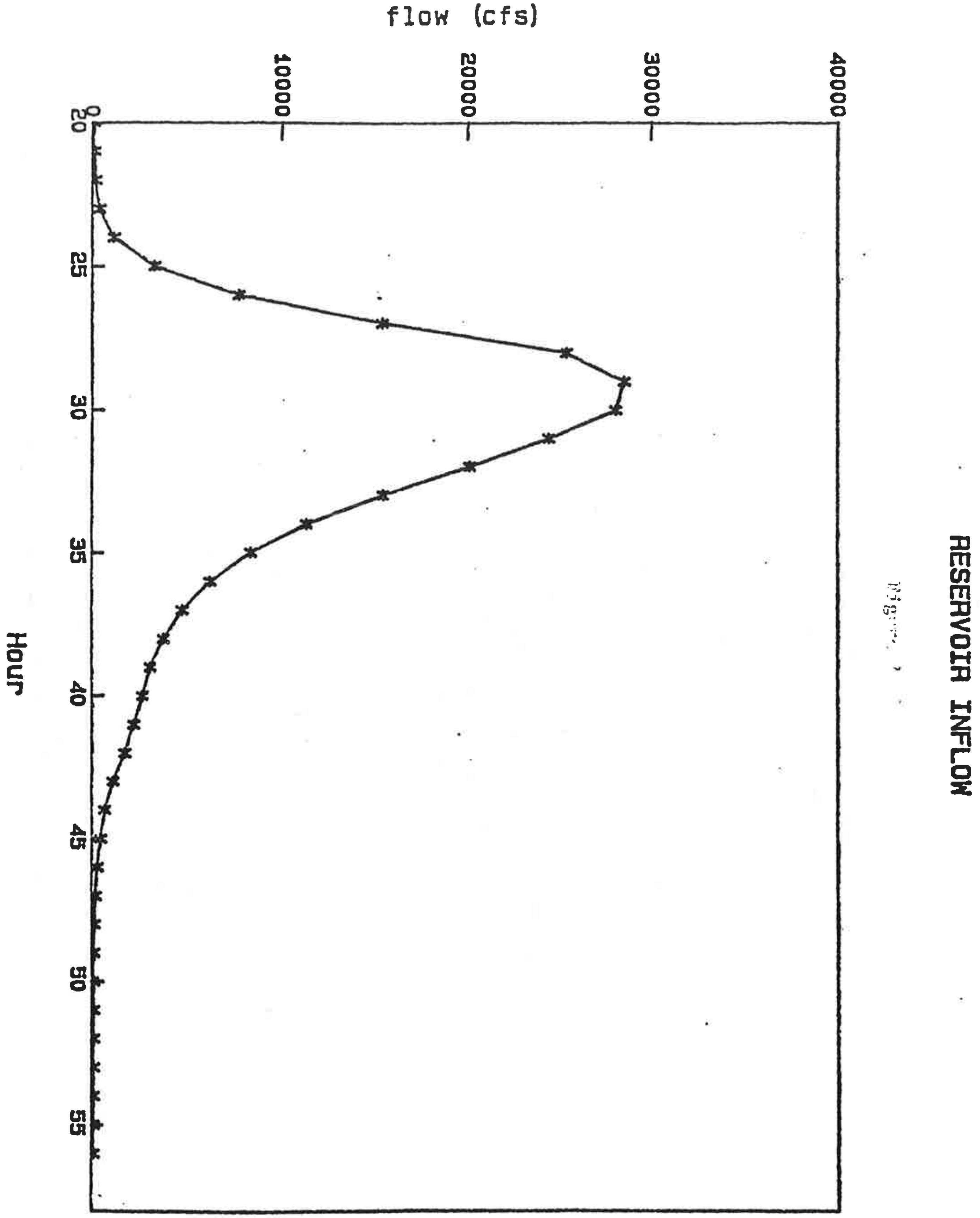

RESERVOIR INFLOW
flow (cfs)
Hour
0
10000
20000
30000
40000
20
25
30
35
40
45
50
55

FIGURE 7.14

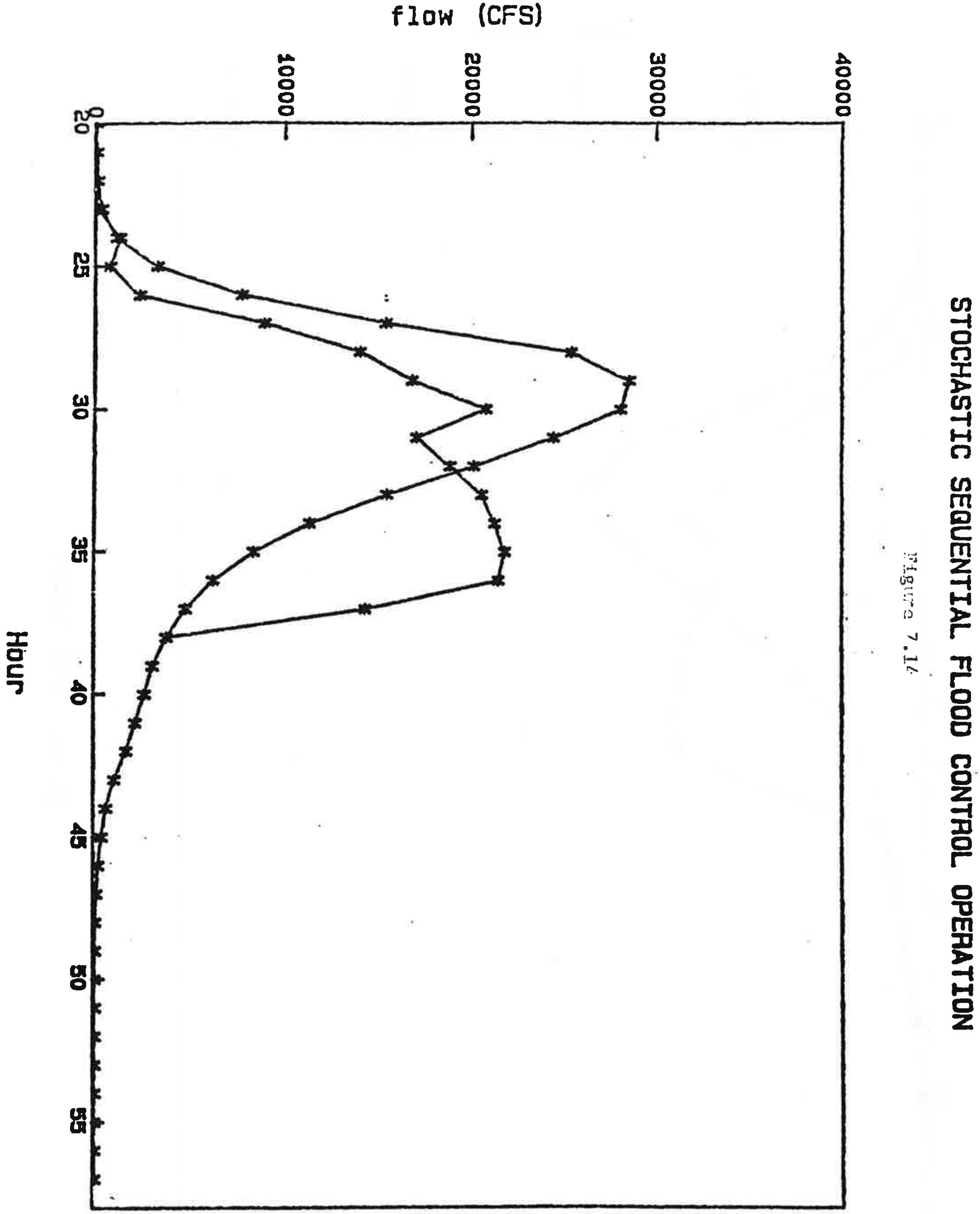
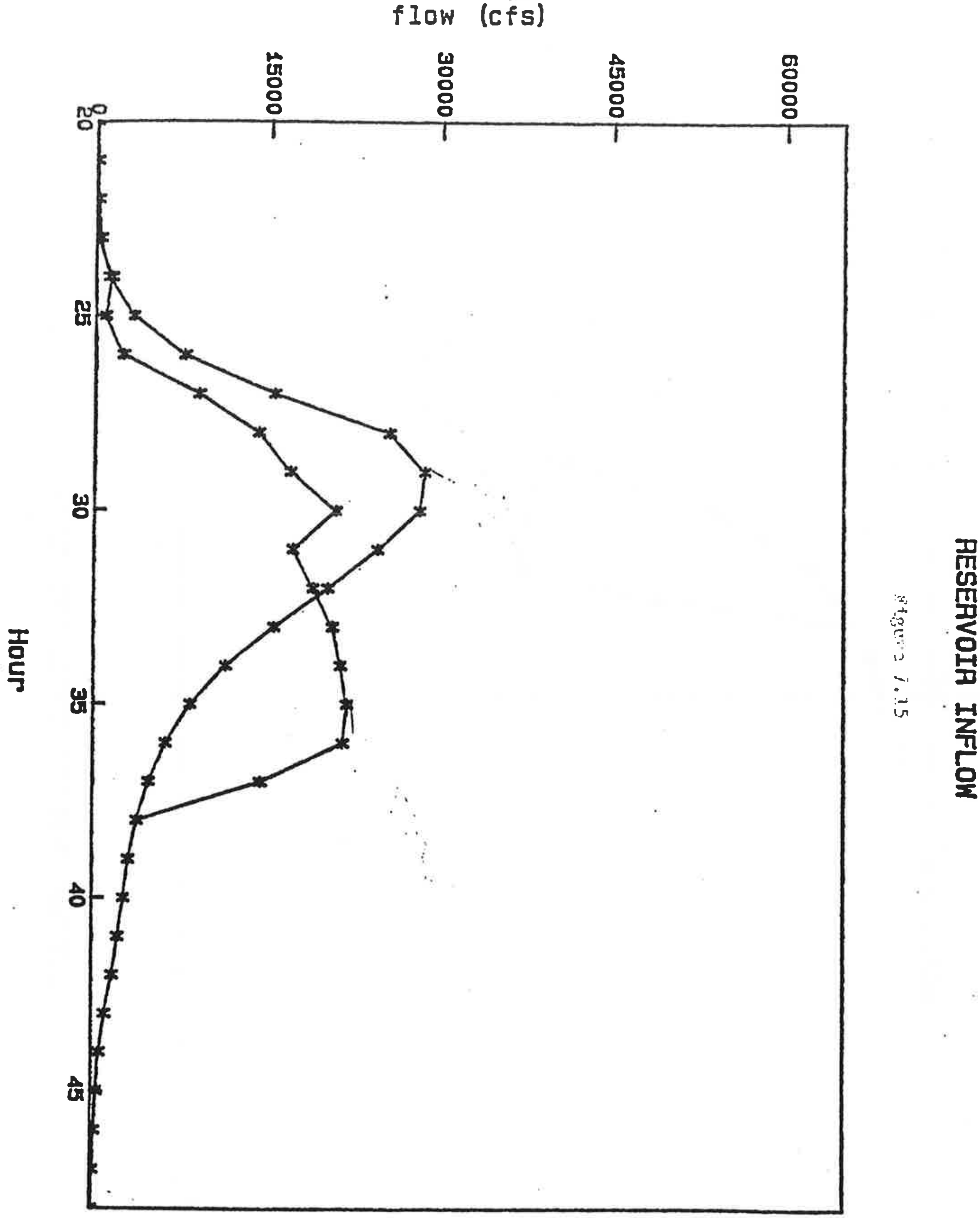

STOCHASTIC SEQUENTIAL FLOOD CONTROL OPERATION
Figure 7.14
flow (CFS)
0
10000
20000
30000
40000
20
25
30
35
40
45
50
55
Hour

FIGURE 7.15


flow (cfs)
60000
45000
30000
15000
0
20
25
30
35
40
45
Hour
RESERVOIR INFLOW
Figure 7.15

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