A CONCEPTUAL MODEL OF SEDIMENT TRANSPORT AND DELIVERY FOR THE MONOCACY RIVER SUB-BASIN OF THE POTOMAC RIVER BASIN

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Report 87-11

September 1987

Interstate Commission on the Potomac River Basin
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Acknowledgments

Many individuals contributed both directly and indirectly to the completion of this work. Dr. Roland C. Steiner sustained the support and flexibility that allowed this work to be pursued. Dr. James A. Smith and Mr. Gerald Day provided many useful insights in adaptive estimation. Dr. Eliazbeth C. Casman, Dr. Fred Scatena, and Mr. Keith Brooks offerred thoughtful comments and careful review of an earlier version of this report. Dr. J. Hugh Ellis offerred an interesting discussion on the use of ARMAX models. The assistance which was received in completing this work is gratefully acknowledged. Responsibility for all assertions, errors and conclusions contained herein lies solely with the author.

This report resulted from the enthusiastic support and steadfast encouragement of original and creative work that was fostered at ICPRB by Paul W. Eastman. Those who had the privilege of knowing him are richer for the experience.

This report has been prepared by the Interstate Commission on the Potomac River Basin. Funds for this report were provided by the United States Government, the U.S. Environmental Protection Agency, and the signatory bodies of the ICPRB: Maryland, Pennsylvania, Virginia, West Virginia, and the district of Columbia.

Executive Summary

This report examines the role of storage and remobilization of sediment in determining the sediment load delivered to the Potomac river from the Monocacy River sub-basin. A conceptual model of sediment transport and delivery is developed that estimates the relative availability of sediment in the stream and channel system, and its contribution to the load delivered to the Potomac. If storage and remobilization of eroded material is not significant, widespread implementation of best management practices to control soil losses could be expected to have a rapid, measurable effect in reducing the load of sediment and sediment associated nutrients delivered to the Potomac Estuary. If a significant fraction of the observed sediment load is derived from the remobilization of sediment in storage, the downstream benefits from sediment control practices on upstream watersheds may be slow to appear, as eroded material slowly works its way through the stream channel system.

The observed suspended sediment load is the result of both erosive processes which generate sediment, and transport and storage processes which mobilize sediment, delivering this material to the mouth of the watershed. As drainage area increases, intermediate storage increasingly influences the net delivery of sediment from a watershed. The conceptual model emphasizes channel processes in which storage and remobilization of sediment by streamflow influences the delivery of suspended sediment to the mouth of the watershed. The success of a channel oriented model in accurately reproducing the delivery of suspended sediment at the mouth of a watershed, supports a storage dominated interpretation of sediment delivery.

- o Despite the high sediment yield observed in the Monocacy River Basin, the delivery of suspended sediment to the Potomac from the Monocacy River appears to be strongly influenced by the storage and remobilization of sediment, rather than the erosion and loss of material from upland areas.

- o Although the annual sediment yield to the Potomac from the Monocacy River Basin is over 300 tons/mi^2, the annual production of sediment in the corridor adjoining the main channel of the Monocacy, (accounting for only 15% of the area of the basin) has been estimated at over 1700 tons/mi^2.

- o On an annual basis most of the sediment produced in the Monocacy Basin is not delivered to the Potomac. Once mobilized, this material is retained in the basin in intermediate storage sites such as the deposits in channel margins, floodplains, and terraces.

- The Soil Conservation Service classification of soil groups in Frederick County identifies floodplains and terraces as topographic features with distinct soil groups. Soils of the floodplains and terraces are not derived from underlying bedrock. These soils are derived from alluvial material that was delivered to the fluvial system but has not yet been transported out of the basin.

- Storage of sediment appears to be a common feature of Appalachian watersheds in the Piedmont. Field estimates suggest that 90 percent of the sediment eroded in the southern Piedmont over the last 200 years is still above the Fall Line.

- Daily observations of suspended sediment delivery for the Monocacy River at Jug Bridge show hysteresis effects both within storm events and seasonally. Sediment hysteresis indicates the exhaustion of sediment sources as available sediment is mobilized faster than it can be replenished.

- Using the conceptual model, supply limited conditions of sediment transport can be identified. Supply limited conditions occur when the available sediment is depleted by both transport, removing material from the basin, and sedimentation, reducing the availability of sediment by returning mobilized material to storage.

- The conceptual model indicates sediment delivery during the summer is characterized by relative oversupply of sediment during capacity limited flow conditions. The seasonal shift to capacity limited conditions suggests that high spring flows not only deliver high sediment loads to the Potomac, but also replenish sediment stored in channel margins, stream banks and floodplains.

A Conceptual Model of Sediment Transport and Delivery for the Monocacy River Sub-Basin of the Potomac River Basin

1. Introduction

This report describes a conceptual model of the transport and delivery of suspended sediment developed for the Monocacy River sub-basin of the Potomac River Basin. On basins in which spatial scale precludes detailed process level description, measurement, and modeling, conceptual models can be useful tools for evaluating the availability and delivery of suspended sediment. The conceptual model emphasizes channel processes in which storage and remobilization of sediment by streamflow influences the delivery of suspended sediment to the mouth of the watershed.

The observed suspended sediment load is the result of both erosive processes which generate sediment, and transport and storage processes which mobilize sediment, delivering this material to the watershed outlet. As drainage area increases, intermediate storage increasingly influences the net delivery of sediment from a watershed. The relative effects of erosion versus storage and transport become increasingly difficult to separate for large watersheds. The success of a channel oriented model in accurately reproducing the delivery of suspended sediment at the mouth of a watershed, supports a storage dominated interpretation of sediment delivery.

The load contributed from channel processes can be limited by a combination of the availability of sediment, and the capacity of the fluvial system to transport available material. These limitations on sediment delivery can shift dynamically, both within a storm event, as well as seasonally. A conceptual model can be used to infer the relative importance of availability and transport capacity throughout the year.

Section 2 describes the physiographic setting of the Monocacy River Basin, noting topographic and geologic features influencing sediment production. The influence of sediment storage on sediment delivery is considered in section 3, noting the importance of storage on large basins in the Appalachian Piedmont. For the Potomac River, the role of sediment storage is briefly reviewed, along with similar evidence for the Monocacy River. Several models of the daily record of suspended sediment at the USGS gage at Jug Bridge, Maryland, are developed and compared in section 4. The sediment rating curve is estimated, and compared to a flow segmented model of delivery. The conceptual model is developed as an adaptive version of the flow segmented model. Using the conceptual model, the availability of sediment from conceptual channel storage is estimated dynamically.

The results suggest net sediment delivery from the Monocacy River is strongly influenced by the storage and remobilization of sediment. Storage on the basin scale could be manifested in aggradation of channels and flood plains and could be verified in the field. The existence of a detailed set of surveyed crossections for the main channel of the Monocacy River offers the opportunity to evaluate the significance of changes in channel storage through a directed field survey.

2. The Monocacy River: Geology, Geomorphology and Soils

The Monocacy River Basin, draining over 900 square miles of Maryland and Pennsylvania, is fairly typical of the eastern Appalachians of the Mid-Atlantic (Figure 1). Bounded to the west by Catoctin Mountain in the Blue Ridge, and to the east by the drainage divide of the Potomac River Basin, the geology of the Monocacy River reflects three distinct physiographic provinces, trending northeast to southwest, paralleling the Appalachians. The Piedmont, the Triassic Plain and Limestone Valley, and the easternmost Appalachians in the Blue Ridge are distinctly recognizable within the Monocacy River drainage.

The eastern and southeastern third of the basin drains the Piedmont Plateau. In the Piedmont, a mature, well developed drainage network has dissected the complex metamorphic bedrock. Steep slopes and incised valleys are common. Typical elevations range from 400 to 700 feet, with a local maximum over 1200 feet at Sugarloaf Mountain. The Limestone Valley that outcrops north of Frederick and extends below Buckeystown to the Potomac is bounded to the east by the Piedmont, and to the north and west by the extensive red shales and sandstones of the Triassic plains extending into southern Pennsylvania. To the west, the eastern slopes of Catoctin Mountain in the Blue Ridge form the divide between the Monocacy River and Catoctin Creek.

In the Soil Survey of Frederick County, the Soil Conservation Service (USDA 1960) has classified the soils found in the Monocacy River basin into distinct soil groups. Each soil group can be associated with general physiographic and topographic characteristics of the basin. Four general topographic groups have been identified in each of the major physiographic provinces of the basin. Within each topographic group, recognizable soil series reflect differences in parent material and drainage (Figure 2).

Topographically, the SCS distinguishes soils of the uplands, soils of the colluvial fans and slopes, soils of the old stream terraces, and the soils of the bottom lands and floodplains. Within these topographic groups, distinct soil groups develop under the geologic influence of the physiographic province in which they are found. The upland soils are generally derived from the deep weathering of underlying bedrock, and differences directly reflect differences in parent material. For example, the Edgemont and Chandler series are upland soils of the Blue Ridge, while the Braddock and Norton series are found on the adjoining colluvial foot slopes, formed from metabasaltic debris.

The severely eroded soils of the Penn series in the Triassics, and the Manor series in the Piedmont are among the most extensive soils in the basin, and may serve as the source for much of the sediment that eventually reaches the Potomac from the Monocacy River. The Penn-Readington series are upland soils developed in the residuum from Triassic sandstones and shales. The Penn loams are often found on steep slopes, and are often severely eroded. The Manor and Linganore series are generally associated with slopes of 15 to 25 percent, ranging up to 55 percent in some locations.

The identification of floodplain and terrace sites as significant topographic features for soil classification is an important feature of the SCS soil survey. Soils in the terraces and floodplains tend to be formed from one dominant parent material. This parent material is not, however, the underlying bedrock. These soils are derived from alluvial material that was delivered to the fluvial system, but has not yet been transported out of the basin. Sediment produced from the valleys and uplands is currently being stored in these deposits. The duration of this storage will be related to the topography of the stream channel, and the frequency with which flows can mobilize these deposits. The ability of infrequent high flows to mobilize this reservoir of stored sediment is an important feature of the Monocacy River that is emphasized in the conceptual model developed in this report.

The next section describes the influence of sediment storage on the delivery of sediment. The occurrence of significant storage on large watersheds in the mid-Atlantic drainage is briefly discussed. Indications of significant storage on the Potomac are also reviewed, along with similar evidence for the Monocacy River. Finally, features of the Monocacy River to be incorporated in the conceptual sediment model are summarized.

3. Sediment Production and Delivery: the effect of storage

Sediment, the result of erosion, is produced through the mobilization and transport of particulate material from the land surface. Sheet erosion refers to the detachment of particles by raindrops and the subsequent removal of these particles by prechannel or overland flow. At a given site, the mass of material mobilized is related to the kinetic energy of precipitation and the duration of the storm, as well as the physical characteristics of the source material. During a single storm over bare soil up to 100 tons of soil may be detached over a single acre (Gottschalk 1964). The rate at which detached particles are removed from small plots will depend on the velocity and discharge of overland flow.

The suspended sediment observed at the mouth of a watershed is the result of both the production of sediment through erosion, and the net export of eroded material from the basin to receiving waters donwstream. Soil losses through sheet and rill erosion can be estimated from empirically developed relationships, such as the Universal Soil Loss Equation (USLE). The USLE estimates the average annual loss (tons per acre) at a site using soil properties, relief, cropping practices, and meteorology. These estimates apply to field scale erosive processes and do not account for sedimentation or channel erosion above the watershed mouth. The sediment delivery ratio is the ratio of sediment delivered at a downstream point, to the gross erosion over the entire drainage above that point.

On an annual basis, a substantial fraction of the material produced through sheet and rill erosion that is removed from the land does not reach the watershed mouth. This material is deposited within the stream and channel network and remains in storage as bank and channel deposits, overbank deposits on floodplains, as well as the accumulating deposits in ponds, lakes and reservoirs. The sediment delivery ratio, (the fraction of eroded material which is transported out of a drainage) generally decreases as drainage area increases. For a watershed the size of the Monocacy River, Shoemaker and Miller (1986) indicate an appropriate sediment delivery ratio would be about 0.04. This suggests that most of the sediment produced annually on agricultural fields in the Monocacy River would not be delivered to the Potomac, but retained in storage within the basin.

Significant storage of suspended sediment within river basins, and the low delivery ratios this implies, appears to be a common feature of the Appalachian Piedmont. For 10 major drainage basins of the southern Piedmont, Trimble (1977) has estimated the areal weighted mean delivery ratio as 6 percent. In examining sediment delivery along the Atlantic coast, Meade (1982) cites additional evidence of headland storage on the order of centuries in the Piedmont. In the Maryland Piedmont, Costa (1975), in examining a 60 square mile basin, estimated that only 34 percent of the material eroded since 1700 had been transported out of this relatively small drainage despite its higher expected delivery ratio. Trimble (1975) estimates that 90 percent of sediment eroded in the southern Piedmont over the last 200 years is still above the Fall Line.

3.1 Storage in the Potomac

Significant storage appears to affect the delivery of sediment in the Potomac River basin as well. For the Potomac, Trimble (1977) notes the delivery ratio to Potomac tidewater has been estimated at 5 percent (USDA 1967). Smith and Shoemaker (1984) report evidence of significant storage in the region. Their sediment budget for a reach of the main channel above the Fall Line suggests that storage in large channels can significantly alter the delivery of sediment that is eventually transported out of headland drainages. Field evidence of storage in the channel of the Potomac River is provided by Shoemaker and Miller (1986).

3.2 Storage as Aggradation

The storage of large volumes of material could be expected to result in the aggradation of stream channels and floodplains. Linganore Creek, an 88 square mile watershed in the Monocacy River Basin, offers evidence of significant storage through aggradation. Linganore Creek has a history of severe erosion on the steep slopes of the Manor soil group. One third of the basin has slopes greater than 15% while two-thirds of the slopes are in excess of 8%. Tributaries tend to be entrenched in steep, narrow valleys. The Creek meanders over an aggrading floodplain, and out of bank flows are common. High rates of sediment production in the tributaries deliver large volumes of material to the main channel, which is building its bed, banks and floodplains in response to this apparent oversupply of sediment. In describing Linganore Creek in this setting, the USDA (1951) observes that the aggrading channel is periodically scoured by high flows with an approximate recurrence interval of 10 years. The sediment load delivered to the main stem of the Monocacy River during these infrequent events is estimated to exceed the cumulative load delivered by all of the intervening events. This suggests the delivery of sediment from Linganore Creek to the Monocacy River, is dominated by storage and channel processes, not sheet and rill erosion.

3.3 Storage in the Monocacy River Basin

Storage and channel processes play a dominant role in influencing the overall delivery of suspended sediment from the Monocacy River. In evaluating the Monocacy River, Kuss and Ballard (1982) have examined sediment production in the so-called Monocacy River corridor (Table 1). This corridor covers roughly 15% of the basin, and generally follows the main channel, including the flood plain. Using the USLE, the average annual sediment yield is estimated as 1,726 tons per square mile, for an average annual production of 265,142 tons. No sediment delivery ratio was estimated for this area. Nevertheless, sediment produced in this corridor adjoining the main channel is much more likely to be transported out of the basin than contributions from upland tributaries and more distant sources.

As a comparison, the average annual sediment load calculated from 20 years of daily suspended sediment observations at the USGS gage at Jug Bridge is 327 tons per square mile or 267,160 tons per year for the 817 square mile drainage. The Jug Bridge gage, located 17 miles upstream from the confluence with the Potomac does not represent the entire channel of the Monocacy River so the sediment loads are not strictly comparable. Nevertheless, the similarity is striking. If the average annual production for the Monocacy River corridor was applied to the entire drainage above Jug Bridge the average annual load would be about 1.4 million tons. The observed load is less than 20% of this number. Since sediment production in the 155 square miles of the Monocacy River corridor is equivalent to the annual load at Jug Bridge, the gross sediment production for the entire Monocacy River drainage above Jug Bridge must be substantially greater than the observed load.

3.4 Modeling Sediment Delivery to Jug Bridge.

In modeling the daily suspended sediment load at Jug Bridge, emphasis was placed on storage and channel processes rather than sheet and rill erosion. Emphasis on channel and storage processes suggests a model driven by streamflow rather than precipitation. Although the duration and intensity of precipitation is crucial to understanding sheet and rill erosion in plot scale studies, storage effects on the basin scale obscure storm related processes at the watershed mouth.

The size of the watershed makes detailed physical process models infeasible. For small watersheds sediment budgets can be constructed and verified in the field (Lehre 1982). Accounting for the many paths of production, transport and storage can be a formidable task (Dietrich et al. 1982) which grows in complexity with the size of the watershed. Shoemaker and Miller (1986) have succeeded in estimating the mass of sediment in storage for a 40 km section of the main channel of the Potomac River. On this scale, an entire field season was devoted to gathering what amounted to one snapshot of one component of storage. Field evaluation of the total storage in the Monocacy River would require a major effort.

As an alternative, the important components of the system can be represented conceptually (Bhowmik et al. 1984) in a lumped parameter model. Conceptual models represent the dominant, aggregate physical processes without detailed physical process parameterization. Moore (1984) has developed a model of sediment yield that emphasizes the variation in the supply of sediment on the basin scale. Using general functions of availability, removal, and translation, dynamic process interaction is incorporated in this hourly model. The model is calibrated to observed sediment yield data rather than field measurements of detachment and transport. Zingales et al. (1984) have developed a conceptual model for nonpoint source pollution in which the watershed response is modeled as a cascade of continuously stirred tank reactors. Although conceptual models can be related to general physical processes (Park and Mitchell 1982), conceptual storages or concentrations represent abstractions for the watershed and cannot be measured in the field. On basins in which spatial scale precludes detailed process level description, measurement, and modeling, conceptual models can be useful tools for evaluating the availability and transport of sediment.

4. The Conceptual model

The conceptual approach utilized was to represent the basin-scale changes in availability and storage of sediment. The distinction between channel, bank and overbank processes was integrated into the structure of the conceptual model. The magnitude as well as the frequency of these processes differ significantly. The recurrence interval of flow was used to suggest the threshold at which each process contributed to the total load observed at the watershed mouth.

The basin is treated as an idealized river cross-section. As the digitized crossections in Figure 3 suggest, this ideal channel has a main bed and channel, a flood plain and higher terraces, each corresponding to greater flow recurrence intervals. Each feature corresponds to a reservoir of stored sediment. The volume, and storage constants for each reservoir may differ by several orders of magnitude. Their contribution to the total load is linked to streamflow.

4.1 Supply Limited vs Capacity Limited Sediment Transport

As streamflow increases, the capacity for mobilization (critical shear stress) and transport of sediment within the channel increases. The observed load is a combination of this capacity and the availability of sediment. Exhaustion of contributing sources has been called on to explain the well known hysteresis loops observed in storm event sediment delivery. The memory of the system is inferred to be greater than individual events, and contributing area effects have been suggested by Walling and Webb (1982) as explaining seasonal patterns of sediment delivery as well.

Hysteresis in suspended sediment records (different loads at the same flow) indicates the observed sediment load is a function of both sediment availability and the capacity for transport of the fluvial system. This distinction (availability versus capacity) is suggested by the relationship between the maximum daily suspended sediment concentration, and the maximum daily sediment load (Table 2) for each of the 21 years of observation at the USGS gage at Jug Bridge.

The maximum daily load for the period of record (water years 1961 - 1981) is 134,000 tons/day. This extreme of the record was associated with a daily average discharge of 74,000 cfs, caused by Tropical Storm Agnes in June of 1972. The observed sediment concentration of 942 mg/l was the greatest concentration observed in water year 1972. For the entire period of record, the sediment concentration of 942 mg/l was the seventh **lowest** daily maximum concentration observed in any year. Thirteen of the twenty-one years of record had higher daily maximum suspended sediment concentrations than that observed during Tropical Storm Agnes.

The highest daily average suspended sediment concentration was observed in 1969. The concentration of 1510 mg/l observed on July 21 of that year was associated with a daily average flow of only 1530 cfs (only 2% of the flow during Tropical Storm Agnes). The observed sediment load was only 6700 tons/day (only 5% of the load observed during Tropical Storm Agnes). One week later on July 28, 1969 the lowest annual maximum daily load for the period of record, 7210 tons/day, was observed. The observed sediment concentration of 1220 mg/l was the second highest concentration observed in 1969, and was over 30% higher than the maximum concentration of sediment observed during Tropical Storm Agnes.

The association of the highest daily average suspended sediment concentration with the lowest annual daily maximum sediment load, indicates an oversupply of sediment compared to the capacity of the river for mobilization and transport. The relatively low concentration of sediment associated with the maximum daily load generated by Tropical Storm Agnes, indicates that, despite the extreme mass of sediment delivered during this event, the observed load was limited by the supply of sediment, not by the transport capacity of the river. If sediment had been available at the higher concentrations observed during the lower flow events of July 1969, the observed sediment load would have been 50-60% larger.

These two extremes from the historical record clearly demonstrate that sediment loads can be limited by the supply of sediment, (June 1972) or by the capacity of the stream to mobilize and transport the available sediment (July 1969). The availability and limitation of sediment will reflect the recent history of the stream system, and will change throughout the year. For small agricultural watersheds in Baltimore County, Md., Scatena (1987) reports small rills and swales become clogged with leaves in the fall, trapping eroded soil, making fall sediment loads delivery limited. The extreme concentrations observed during two relatively small flow rises in 1969 indicates that a significant amount of sediment was readily available for transport when the high concentration of July 21 was observed. Furthermore, this source of sediment had not been exhausted by the moderate rise on July 21, producing another extremely high observation of suspended sediment concentration one week later.

Increasing streamflow transports suspended sediment which continues to be entrained by turbulent eddies. Channel storage is mobilized as critical shear causes incipient motion on the stream bed. As these sources are exhausted, sediment concentrations, and loads may decrease, even as streamflow and transport capacity remains unchanged. The peak of sediment delivery may lag or lead the peak flow within storm events.

As flow continues to increase to bankfull stages, bank storage becomes available, contributing to total load. The availability of bank material can change rapidly in response to moisture content, and the dissipation of energy on the roughness elements of the bank. The bankfull, or "channel forming" discharge has an inherently different time-scale, and magnitude, compared to the stream bed processes which dominate flows with short recurrence intervals. The recurrence interval for the bankfull discharge has been estimated between 1 and 2 years (Leopold et al. 1964). This suggests the majority of daily flows and loads are not reflecting processes involving stream bank storage. The fundamentally different processes and time scales associated with bank storage, suggested a distinct conceptual reservoir for the stream bank "component" of load, linked to bankfull discharge.

As flow rises above bankfull, the vast storage of adjoining floodplains becomes available for both deposition, and remobilization. The low depositional velocities associated with flood plains generally produce fine grained deposits which tend to be transported for considerable distances, once entrained. (See Scatena 1986 for an alternate view associated with an extreme event). Flood plain storage is inaccessible for long periods of time, with the potential to contribute substantial quantities of readily transported material if mobilized. The processes and response of flood plain sediment storage, would therefore be expected to display threshold effects at appropriate flows, particularly on the basin scale where flood conditions could be anticipated simultaneously over much of the watershed.

At higher flow, the out of bank deposits associated with terraces would become active. This storage site might be accessed by flows with 10 to 100 year recurrence intervals (see Figure 3). The erosive processes mobilizing this source material would include lateral scouring and under cutting, and failure of steep slopes. This material would likely reflect the source material, as well as more deeply weathered alluvial material.

To represent this channel oriented description of sediment availability, the conceptual model is developed as a series of linked, interacting storage sites or reservoirs. The contribution of each reservoir is related to the recurrence interval of the average daily flow. Since the streamflow and suspended sediment record for the Monocacy River at Jug Bridge is used to calibrate and verify the model, the flow duration relationship at the USGS gage at Jug Bridge was used to conceptually segment channel processes according to flow (Figure 4). The aggregation of storage, erosion and transport from all of the stream and tributary channels throughout the Monocacy River, into one set of "channel reservoirs" is the conceptual simplification used in this model.

Although the recurrence interval of flows at Jug bridge was used to determine when each conceptual reservoir contributed to observed suspended sediment load, seasonal effects were not explicitly included in this model. Smith and Shoemaker (1983) have noted the difference in observed sediment loads at equivalent flows in winter and spring. They suggest this may be a reflection of the seasonal availability of sediment in the basin related to freeze thaw processes. This seasonal hysteresis effect, is a commonly observed feature of suspended sediment records. Walling and Webb (1982) observe similar seasonal effects. They suggest however, this observation may be explained by dilution effects, and differences in sediment contributions from baseflow and storm flow . For small watersheds, increases in sediment loads during storms represent material derived from sheet and rill erosion. This material contributes to the load through overland flow. In observing sediment hysteresis in both storm event, and seasonal records, Walling and Webb suggest the apparent exhaustion effects are related to varying availability in different contributing areas.

This interpretation suggests the utility of a model with separable dynamically changing, conceptual sources of sediment. Linking each of these conceptual reservoirs allows the relative importance, as well as the separability of these effects to be judged, without incorporating seasonality in the model structure explicitly. The fidelity with which such a model predicts load in different seasons may provide insight into the nature of aggregate processes controlling watershed sediment delivery throughout the year.

The conceptual elements included in this representation were:

Conceptual Reservoir	**Flow**	**Annual Probability of Exceedance**
stream bed and channel	base flow	> 99%
stream bank	500 cfs	50%
flood plain and overbank	3000 cfs	5%
terrace and upland	> 3000 cfs	< 5%

These idealized elements of a conceptual channel emphasize riverine channel processes, deemphasizing the importance of sheet and rill erosion as well as interill erosion and splash effects.

4.2 Structure of the Model

The performance of any conceptual model of aggregate, basin scale sediment processes should, at a minimum, be no worse than the standard sediment rating curve (Leopold et al. 1964, Ferguson 1986). An inability to at least reproduce the performance of this simple log-log regression would cast doubt on the value of the additional computational burden and detail required in more complex models. As a first step, the log-log rating curve was constructed, and used as a standard for comparison.

The log-log regression of load and flow was used as a simple predictor of daily suspended sediment load for water year 1973.

The best fit (least squares) linear model was estimated as :

$$\ln(\text{load}(t)) = -5.43 + 1.5 \ln(\text{flow}(t))$$

This model is a reasonably good predictor of sediment load from flow, explaining 75 percent of the variance of the suspended sediment record. The additive error in the log-log model becomes multiplicative in the untransformed sediment load prediction. Predictions from the rating curve tend to be biased (Ferguson 1986), underestimating the daily load at high flows (Figure 5.)

Extensive exploratory data analysis suggested a simple structural model better captured the explanatory power of segmented flow. The log of daily load in tons L(t) is modeled as:

$$L(t) = c_0 + c_1F(1) + c_2F(2) + c_3F(3)$$

The F(i)'s represent segments in the piecewise linearization of flow.

$$F(i) = Q(i) \qquad \text{for } Q(i) < T(i)$$

$$= T(i) \qquad \text{for } Q(i) \geq T(i)$$

where:

$$Q(i) = Q(t) - \sum F(j)$$

Q(t) is the average discharge on day t

and F(0)=0

T(i) represents the i-th threshold value, separating components of flow in each of the conceptual channel segments. As indicated in Figure 4, the flow thresholds used in the model were chosen to correspond to segmenting of flow with T(1) = 500, and T(2) = 3000 cfs.

The c_k's can be thought of as the availability of sediment from the k-th conceptual reservoir. The flow in this conceptual channel segment, F(i), indicates the capacity for transport. The load is constructed from the availability of sediment and the flow based capacity for transport in each conceptual reservoir. For comparative purposes, the flow stratified model, referred to as the "non-adaptive" model, was evaluated. The model is considered non-adaptive because constant or mean values are used for the sediment availability of each reservoir.

The conceptual model is developed as an adaptive model in which the availability of sediment from each conceptual reservoir is estimated dynamically. At each time step, the availability of sediment from each of the conceptual reservoirs is inferred from observed rates of sediment production and streamflow. The current realization of the vector of c_k's, $\mathbf{C}(t)$, is calculated as the conditional expectation of $\mathbf{C}(t)$, accounting for both the previous estimate of $\mathbf{C}(t)$, $\mathbf{C}(t-1)$, and the error of the forecast from the previous period (Table 3). The derivation of the estimation equations is presented in detail in appendix A. In this way the availability of sediment over the basin is inferred from the observed sediment load.

4.3 Infrequent high flows

Flow intervals were chosen to correspond to identified recurrence intervals from the streamflow record at Jug Bridge. The relative frequency of occurrence of these flows reflects the frequency with which each conceptual sediment reservoir contributes to the observed load. As a consequence, the highest flow segment, corresponding to that segment of flow in excess of 3000 cfs, was zero most of the year. The infrequent character of flows in this range, raises questions about the variability of the reservoir behavior on the time scale of interest. Walling and Webb (1982) suggest that sediment concentrations are constant for the highest components of flow. In contrast to alluvial deposits which are reworked at varying recurrence intervals, upland deposits may form largely in place, over bedrock and much older alluvial material, and do not rely on infrequent riverine deposition to replenish this storage site. With respect to the infrequent stream flows that access this storage site, the total volume of sediment may not be limiting. The apparent lack of exhaustion at high flows observed by Walling and Webb (1982) is a feature incorporated in the current model as a constant value for c(3).

4.4 Calibration

Calibration of the conceptual model, the non-adaptive model, and the log-log rating curve was performed using one year of flow and suspended sediment data. Water year 1973 was judged to be a representative year for calibration purposes. The total load for water year 1973 was approximately 240,000 tons, comparing favorably to both the long term average annual load at this site, as well as the USLE derived estimate of annual sediment production in the river corridor. Flows in water year 1973 were above average, and the maximum discharge of 10100 cfs has a probability of exceedance of less than 0.5 percent. The presence of such an extreme event was judged useful for a calibration data set. In addition, graphs of load versus flow for twelve storm events of varying magnitude were inspected (Figure 6). In every case a hysteresis loop showing sediment exhaustion through the storm event was observed. This suggested the rises in load were not atypical, and gave no reason to reject this record for calibration purposes.

The coefficients c_k were estimated using linear regression. The parameters estimated with standard linear regression were used in the non-adaptive model. Model predictions were significantly better than those of the standard log-log rating curve for the same calibration data set, explaining 78% of the variance in the sediment record. The non-adaptive model also underpredicts the peaks of daily load (Figure 7). Nevertheless, it is better at predicting the peak loads and, consequently, better fits the cumulative load for the calibration year (Figure 8).

Unlike the rating curve, and the non-adaptive model, in which unique coefficients are estimated, the adaptive conceptual model continually updates the coefficient estimates based on the observed sediment load. While the initial coefficient estimates from the non-adaptive model are supplied to the conceptual model, these coefficients change continuously. In any time period the current coefficients of the conceptual model reflect the history of the system, representing the best estimate of the relative availability of sediment in each conceptual channel segment.

Comparisons for the 1973 calibration period are presented for the non-adaptive model, the rating curve, and the conceptual model in Figures 7 and 8. The rating curve underestimates the peaks in sediment load and under-predicts the total annual load by 120,000 tons. Although the non-adaptive model also tends to under-predict sediment load, its predictions are significantly better for high load events. The total annual load for the calibration year is underestimated by only 30,000 tons or roughly 13%. A striking feature of both the rating curve and the non-adaptive model is the overestimation of load during recession flows. The load during these flows makes an insignificant contribution to the total annual load, and poor predictive power during these conditions will not be reflected in the cumulative loads. Nevertheless the consistent error of the predicted load during recession flows suggests a physical process is not being captured by either model.

In contrast, the conceptual model gives a good fit of both the peak loads and the lowest loads on the falling limb of hydrographs (Figure 9). A tendency to over-predict some peak loads is evident in the adaptive model, and the annual load estimated with the conceptual model is about 20,000 tons (8%) too high (Figure 10). The ability of the conceptual model to accurately predict the load during declining flows indicates a fundamental improvement over the non-adaptive model, which is not necessarily evident from the prediction of annual load.

4.5 Verification

The calibrated models were further evaluated using flow and sediment data from Water Year 1974 to verify performance under different conditions. 1974 was a year of above average streamflow with below average sediment production. Despite the average daily flow of 941 cfs half of the average daily flows in Water Year 1974 are less than 550 cfs. The annual sediment load of only 160,675 tons is less than 65% of the long term average. Nevertheless several extreme events occurred in this otherwise below average year, which accounted for most of the annual load. The maximum daily average flow of 26,800 cfs has a recurrence interval in excess of 50 years. This combination of extreme high flow events in a year of below average sediment delivery and discharge represented unusual conditions compared to the calibration year of 1973. The use of Water Year 1974 for verification was therefore judged to provide a useful test of the general predictive power of the three models.

In the verification year, as in the calibration period, the rating curve is biased and underpredicts the total load by nearly 50%. The peak sediment delivery is underestimated, while baseflow sediment production is overestimated (Figures 11b,12b). The tendency to overestimate baseflow load is reversed after day 240 in both calibration and verification years. This suggests a seasonal change in the availability of sediment in the late summer and early fall. The non-adaptive model, performs slightly better than the rating curve in following base flow recession sediment loads, and continues to over predict suspended sediment delivery for the largest events in the verification data set. While overestimating the load during peak flows, the non-adaptive model underestimates the availability of sediment during the late summer early fall period (days 240 to 365) as in the calibration year. This again suggests a seasonal change in availability, which neither the rating curve nor the non-adaptive model can explain.

In the low load verification year, the non-adaptive model overestimates two extreme events resulting in an error of 300,000 tons (130% of the average annual load) in the estimate of total load. The unusually low sediment production of the verification year appears to be related to low availability of sediment during extreme events. The non-adaptive model treats availability as constant, and consequently vastly overestimates the annual load (Figure 12a)

The adaptive, conceptual model appears superior to both the rating curve and the non-adaptive model. Though overestimating peak loads, as in the calibration year, these errors are modest for the conceptual model. The annual load from the conceptual model is within 5% of the observed load, compared to errors of

50% and 270% for the rating curve and non-adaptive model respectively. As in the calibration year, the conceptual model accurately reproduces the observed loads at low flows. These low loads account for only a small fraction of the total cumulative load and are not critical to estimating the annual sediment load. Nevertheless, the accuracy with which the conceptual model reproduces the observed load at the lowest flows suggests a superior representation of compound delivery processes for the drainage basin. The strength of the conceptual model in capturing both baseflow sediment delivery as well as total load is indicated by the verification of Water Year 1974.

4.6 Discussion

Using the conceptual model, the relative load contribution from each conceptual reservoir can be examined throughout the year. The contribution represents the combination of sediment availability and transport capacity. The apparent seasonal change in availability in late summer and fall can be examined in greater detail by examining the load contributions from the baseflow and bankfull conceptual channel segments in Figure 13. After day 240 the baseflow contribution to load remains uniformly high through the end of the year. The bankfull contribution is declining through day 270 and remains relatively low until the sequence of high flows beginning on day 340.

The higher suspended sediment production observed in the late summer and early fall suggests a relative increase in the availability of sediment. The conceptual contributions indicate a shift to channel contributions from bank storage. This could occur as lower flows reworked bank material that had been mobilized by preceding high flows. Such a cascading pathway for the transfer of sediment between storage sites has been suggested by Zingales et al. (1982) for sediment associated nutrients, by Verhoff et al. (1979) for total phosphorous, and by Troutman (1980) and Turgeon (1973) for individual particles. For this low-flow period, channel sediment does not appear to be limiting in the conceptual model. The sediment loads observed at this time may be limited by the capacity of flow to carry the available material. Capacity limited transport during this period is consistent with the underestimation of both the rating curve and the non-adaptive model.

Capacity limited transport during this period is also suggested by the conceptual sediment availability. Figure 14 compares the relative availability of sediment from channel and bank reservoirs. In contrast to channel contributions which increase relative to bank contributions, both channel and bank availability remain high during this period. The relatively high availability in the bank reservoir with a relatively low overall contribution from the bank suggests available bank material is not being mobilized by streamflow. Figures 13 and 14 indicate the bank contribution after day 270 is a combination of low flow and high availability, suggesting capacity limited sediment delivery.

The distinction between availability and transport capacity can be inferred from the differences in annual extremes of load and concentration. Figure 15 is a smoothed plot of the relative availability of sediment for flows up to 500 cfs. The increased availability of sediment is apparent after day 240. For the low flow period at the end of Water Year 1974, the conceptual model suggests sediment delivery is capacity limited. If this is the case, we would expect to see higher concentrations of suspended sediment, despite the low overall load. The concentration of suspended sediment is indeed above the annual average for this period, consistent with a capacity-limited interpretation of sediment delivery. Table 4 compares the daily concentration of suspended sediment from January through April to concentrations from May through August. In both the calibration year, 1973, and the verification year 1974, the concentration of suspended sediment was significantly higher later in the year. Comparing the first and third quartiles of the distribution of suspended sediment, (Q1 and Q3 in Table 4) the distributions are clearly different. In both years the third quartile for the January through April concentrations is **less** than the first quartile for May through August concentrations.

This seasonal pattern in concentrations is consistent with the seasonal change in the availability of sediment indicated by the conceptual model. Although maximum loads are generally observed during spring runoff events, the seasonal pattern of sediment availability seen in the conceptual model suggests a significant amount of mobilized sediment does not leave the basin, but remains readily available.

The performance of the model is consistent with a storage dominated mechanism for producing suspended sediment at the mouth of the Monocacy River. The significant bank and channel storage this suggests would be manifested in aggradation of banks and floodplains. This could be tested through field surveying crossections of the main channel to the level of the terraces. A series of crossections surveyed at a site through time would allow changes in storage to be estimated. For the Monocacy River a detailed set of crossections was taken by the U.S. Army COE in June 1975. Nearly 100 such crossections were surveyed. A resurvey of these same sites would give a relatively comprehensive evaluation of changes in storage in the river corridor. Such a resurvey could be easily accomplished in a field season. The high resolution that could be expected from 100 crossections taken over a distance of only 47 river miles provides a unique opportunity to characterize storage changes for a large watershed.

5. Conclusion

The observed suspended sediment load for a large watershed is the result of the aggregation of complex processes of varying spatial and temporal scales. The episodic transport of eroded material obscures storm related sediment production at the mouth of a watershed. The sediment delivery ratio decreases with drainage area. For drainage areas greater than 100 square miles, storage of up to 90% of the material eroded annually is not unreasonable. The importance of storage on the watershed scale has been demonstrated in the southern Piedmont as well as the Piedmont of Maryland. Evidence suggests storage of sediment significantly influences the net export of sediment from both the Monocacy River and the Potomac.

A conceptual model of suspended sediment delivery for the Monocacy River basin has been constructed and verified. On basins for which spatial scale precludes detailed field and process measurements, conceptual models may provide the only means to estimate the availability and delivery of sediment. The conceptual model treats suspended sediment as though it is derived exclusively from fluvial storage. Conceptual reservoirs representing channel, bank, overbank, and terrace storage are used to model the effective availability of sediment over the entire 817 square mile drainage above Jug Bridge. Streamflow alone, rather than precipitation, is used to drive the production of sediment for the basin reflecting the dominance of channel processes over sheet and rill erosion incorporated in this model.

Despite the unusually high sediment yield observed at the mouth of the Monocacy River Basin, most of the sediment produced annually on the Monocacy, does not leave the Basin. The delivery of suspended sediment at the watershed mouth is a combination of the availability of sediment, and the capacity of the river system to mobilize and transport the available sediment. Using the conceptual model, transport-limited and availability limited sediment delivery can be distinguished. The ability to recognize transport-limited conditions allows the anticipation of large loads under suitable flow conditions.

The success of the channel oriented conceptual model in reproducing historical sediment loads reinforces the view that sediment delivery from the Monocacy River Basin is dominated by storage and channel processes, not sediment production on upland agricultural land. The conceptual model indicates a seasonal pattern of capacity limited transport in the summer and early fall. Despite the large sediment loads observed during spring runoff events, a significant mass of sediment remains available for transport under suitable flow conditions.

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Appendix A: Adaptive Estimation

This section describes the adaptive estimation procedure used to update sediment availabilities at each time step. The presentation and estimation procedures presented here are taken from Meinhold and Singpurwalla (unpublished manuscript). Sediment availability is estimated recursively with each new observation of suspended sediment load. The new estimate can be derived using Bayes Theorem. The likelihood of the previous day's prediction error could be applied to the posterior distribution of the availability vector, C(t), to derive the posterior distribution of C(t) for the next prediction step. In practice this calculation is often impractical. Alternatively the posterior distribution can be analytically calculated as the conditional distribution of C(t) given the prediction error, e(t), using a general result from multivariate statistics. A linear transformation of C(t) can be used to make it independent of e(t). The distribution of the transform of C(t) readily yields the posterior distribution of C(t) we desire.

1. Bayes Theorem

Consider the model:

$$L(t) = C(t)*F(t) + v(t)$$

where:

L(t): is the log of the daily load
C(t): is the current vector of sediment availability
F(t): is the piecewise linearized vector of daily average flow
v(t): is a random error distributed as ~N(0,V)

The one period prediction error, e(t), is the difference between the observed load at time t, OL(t), and the predicted load, L(t):

$$e(t) = OL(t) - L(t)$$

We wish to use the prediction error to update our estimate of C(t). From Bayes Theorem:

$$f(C(t)|e(t)) = \frac{f(e(t),C(t))}{\int_{\text{all } C(t)} f(C(t),e(t))\, dC(t)} = \frac{e(t)|C(t))*f(C(t))}{\int_{\text{all } C(t)} f(e(t),C(t))\, dC(t)} \quad (1)$$

This would hold true if we knew the joint distribution of e(t) and C(t) at each time step.

In practice we have an estimate of sediment availability prior to time t, C(t-1), which we use to predict the sediment load in time period t:

$$L(t) = F(t) * C(t-1).$$

This gives a prediction error e(t):

$$e(t) = OL(t) - L(t)$$

$$= OL(t) - F(t) * C(t-1).$$

Using e(t) and Bayes Theorem we update the estimate of the coefficient vector C using the additional information contained in the observed load. This updated estimate is then used to predict tomorrow's load. The adaptive model is actually an iterative prediction procedure, alternating between backward inference and forward prediction.

In terms of Bayes Theorem, f(C(t-1)) is the PRIOR distribution of the estimate of C. We want to form:

f(C(t)|e(t))- the POSTERIOR distribution of the estimate of C, having observed e(t).

$$f(C(t)|e(t)) \,.\, f(e(t)|C(t-1)) * f(C(t-1) \qquad (2)$$

where :

f(e(t)|C(t-1)) is the LIKELIHOOD of observing e(t) given the PRIOR distribution of C(t-1).

From equation (1) the constant of proportionality in (2) would be:

$$\left[\int_{\text{all } C(t-1)} f(e(t),C(t))\, dC(t-1)\right]^{-1} \qquad (3)$$

2. Estimating the Posterior

An alternative to evaluating the integral (3) for all possible states of C(t-1) at each time step, is to attempt to calculate the posterior distribution of (C(t)|e(t)) directly. A general result from Anderson(1984) makes this calculation possible. Consider a random vector X that is normally distributed with mean vector m and covariance matrix S. This vector can be arbitrarily partitioned as X(1) and X(2), normally distributed as:

$$\begin{bmatrix} X(1) \\ X(2) \end{bmatrix} \sim N \begin{bmatrix} m(1) \\ m(2) \end{bmatrix} \begin{bmatrix} S(1,1) & S(1,2) \\ S(2,1) & S(2,2) \end{bmatrix} \qquad (4)$$

For this arbitrary partition, a linear transformation can be found so that :

$$Y(1) = X(1) + B*X(2)$$

and

$$Y(2) = X(2)$$

are independent. For such a transformation:

$$f(Y(1)|Y(2)) = f(Y(1)|X(2))$$

is just the marginal distribution of Y(1) by independence. A transformation allows the conditional distribution f(X(1)|X(2)) to be recovered directly without evaluating the integral (3). This result holds if we can find the matrix B such that Y(1) and Y(2) are independent. This requires the covariance of Y(1) and Y(2) to equal zero. Letting E{...} denote expectation,

$$0 = E\{(Y(1)-E\{Y(1)\}) * (Y(2)-E\{Y(2)\})'\}$$

Setting Y(1)=X(1)+B*X(2) and Y(2)=X(2) and evaluating E{Y(1)} and E{Y(2)} this reduces to :

$$0 = E\{[(X(1)-m(1))+B(X(2)-m(2))]*[X(2)-m(2)]'\}$$

Taking expectations of the product yields:

$$0 = S(1,2) + B * S(2,2)$$

or

$$B = -S(1,2) * S(2,2)^{-1}$$

so

$$E\{Y(1)\} = m(1) - S(1,2) * S(2,2)^{-1}$$
$$E\{Y(2)\} = m(2)$$
$$Cov\{Y(1),Y(2)\} = 0$$

We only need to evaluate the mean and variance of Y to completely specify the conditional distribution of Y(1) and Y(2), which is the marginal distribution of Y(1).

The variance if Y(1) =

$$V(Y(1)) = E\{(Y(1)-E\{Y(1)\}) * (Y(1)-E\{Y(1)\})'\}$$

$$= E\{[(x(1)-m(1))+B*(x(2)-m(2))] * [(x(1)-m(1))+B*(x(2)-m(2))]'\}$$

$$= S(1,2)*B' + S(1,1) + B*S(2,1) + B*S(2,2)*B'$$

$$V(Y(1) = S(1,1) - S(1,2)*S(2,2)^{-1}*S(2,1) \qquad (5)$$

Thus:

$$\begin{bmatrix} Y(1) \\ Y(2) \end{bmatrix} \sim \left[\begin{matrix} m(1)+B*m(2) \\ m(2) \end{matrix} \right., \left. \begin{matrix} S(1,1)-S(1,2)*S(2,2)^{-1}*S(2,1) & 0 \\ 0 & S(2,2) \end{matrix} \right]$$

For the conditional distribution

f(Y(1)|Y(2)) = f(Y(2)|X(2))=F(Y(1)):

$$f(Y(1)|X(2)) \sim \qquad (6)$$

$$N[m(1)-S(1,2)*S(2,2)*m(2),\ S(1,1)-S(1,2)*S(2,2)*S(2,1)]$$

To recover the conditional distribution of X(1), f(X(1)|X(2)), we recall that X(1)=Y(1)+B*X(2). For the conditional distribution, the condition X(2)=x(2) refers to a particular realization of the random variable X(2) and is therefore a constant. Evaluating the conditional distribution if X(1) is therefore equivalent to evaluating the conditional distribution of Y(1) plus the constant B*X(2), where B*X(2) here refers to conditioning on a realization of X(2). Adding a constant to a random variable will not change its variance, and will increase its expectation by the value of the constant. Therefore the distribution f(X(1)|X(2)) (which is equivalent to F(Y(1)+B*X(2)|X(2)) will have the same variance as in (6) above, with a mean increased by B*X(2);

$$f(X(1)|X(2)) \sim$$

$$N[\ m(1)+B*(X(2)-m(2)),\ S(1,1)-B*S(2,1)\ ] \qquad (7)$$

So, knowing the covariance $\begin{matrix} S(1,1) & S(1,2) \\ S(1,2) & S(2,2) \end{matrix}$ and the mean $\begin{matrix} m(1) \\ m(2) \end{matrix}$

allows the conditional distribution f(X(1)|X(2)) to be estimated for an arbitrary partition of random vector X. Using this result we return to the conceptual sediment model, and attempt to form the posterior distribution of C(t) as a conditional distribution f(C(t)|e(t)) as in (7).

3.Posterior distribution of C(t)

The adaptive model is represented as:

$$L(t)=C(t) * F(t) +v(t) \qquad v(t) \sim N(0,V) \qquad (8a)$$

Considering C(t-1) as the state of the system at time t-1, we allow for the incorporation of state dynamics in the form:

$$C(t) = G(t) * C(t-1) + w(t) \qquad w(t) \sim N(0,W) \qquad (8b)$$

Moore (1984) has postulated differentiable state transition functions in a conceptual model of hourly sediment load. Where such functions can be determined apriori, they can be incorporated in the adaptive conceptual model in the G(t) matrix. In the model developed in this paper, G(t) is taken as the identity matrix, reflecting our apriori ignorance of systematic temporal changes in aggregate sediment availability over the basin. From this structural model we attempt to estimate the POSTERIOR distribution $f(C(t)|e(t))$ using the results of the previous section.

Following the approach outlined above, we consider C(t) and e(t) as jointly normal random variables:

$$\begin{bmatrix} e(t) \\ C(t) \end{bmatrix} \sim N\left[\begin{bmatrix} m(1) \\ m(2) \end{bmatrix}, \begin{bmatrix} S(11) & S(12) \\ S(21) & S(22) \end{bmatrix}\right] \qquad (9)$$

We need to evaluate each of the parameters in equation (9). From equation (8b):

$$m(2)=E\{C(t)\}=G(t)*C(t-1) \qquad (10)$$

$$S(2,2)=Var\{C(t)\}=G(t)*S(C)*G(t)'+W=R(t) \qquad (11)$$

where S(C) is the estimate of the covariance of C prior to the transformation (8b).

from equation (7) we expect

$$E\{e(t)|C(t)) = m(1) + S(1,2)*S(2,2)^{-1} (C(t)-m(2))$$

but from (8)

$$(e(t)|C(t)) \sim N(OL(t)-L(t), V)$$

$$\sim N(F(t)(C(t)-G(t)*C(t-1)), V) \qquad (12)$$

thus

$$m(1) + S(1,2)*R(t)^{-1}*(C(t)-m(2)) = F(t)*(C(t)-G(t)*C(t-1)$$

or

$$m(1) + S(1,2)*R(t)^{-1} = F(t) \qquad \text{using (10)}$$

which is satisfied if

$$m(1) = 0 \qquad (13)$$
$$S(1,2) = F(t)*R(t) \qquad (14)$$

Finally,

$$Var(e(t)|C(t)) = V \qquad \text{from (12)}$$

but

$$Var(e(t)|C(t)) = S(1,1) - B*S(2,1) \qquad \text{from (7)}$$

so

$$V = S(1,1) - B*S(2,1)$$

or

$$S(1,1)=V + B*S(2,1)$$
$$=V + S(1,2)*S(2,2)^{-1}*S(2,1) \qquad (15)$$

Substituting (10),(11),(13)-(15) into (9) yields

$$\begin{pmatrix} C(t) \\ e(t) \end{pmatrix} \sim N \left(\begin{pmatrix} G(t)*C(t-1)' \\ 0 \end{pmatrix}, \begin{pmatrix} R(t) & F(t)*R(t)' \\ R(t)*F(t)' & V+S(1,2)*S(2,2)^{-1}*S(2,1) \end{pmatrix} \right)$$

Using (7),

$$E\{C(t)|e(t)\}=G(t)*C(t-1)'+ S(1,2)*S(2,2)^{-1}*e(t) \qquad (16)$$

or

$$E\{C(t)|e(t)\}=G(t)*C(t-1)'+$$
$$F(t)*R(t)'*[V+F(t)*R(t)*F(t)']^{-1}*e(t)$$

In this way the estimate of C(t) is updated at each time step using the prediction error e(t), flow F(t), and the estimated variances from the flow segmented model.

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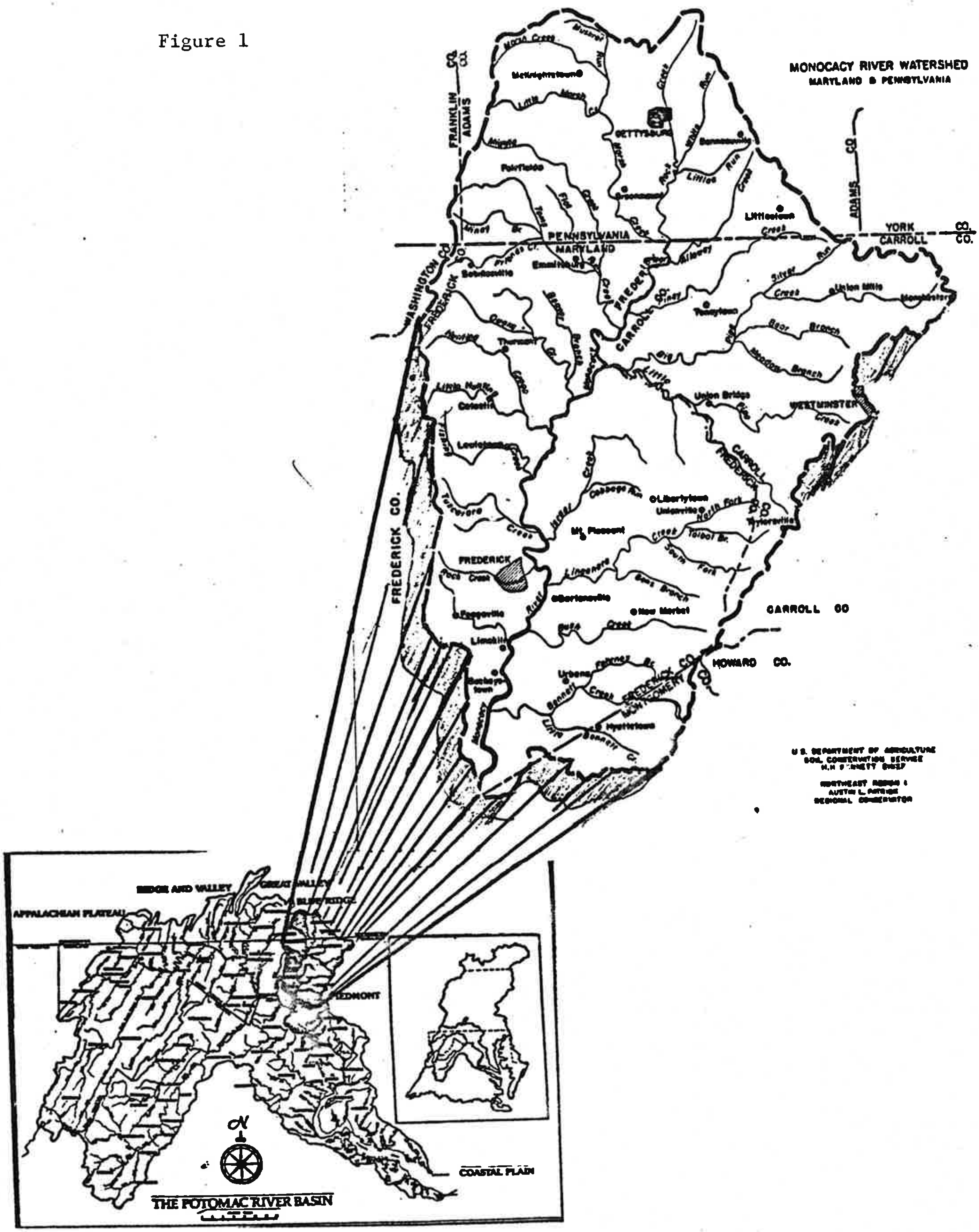
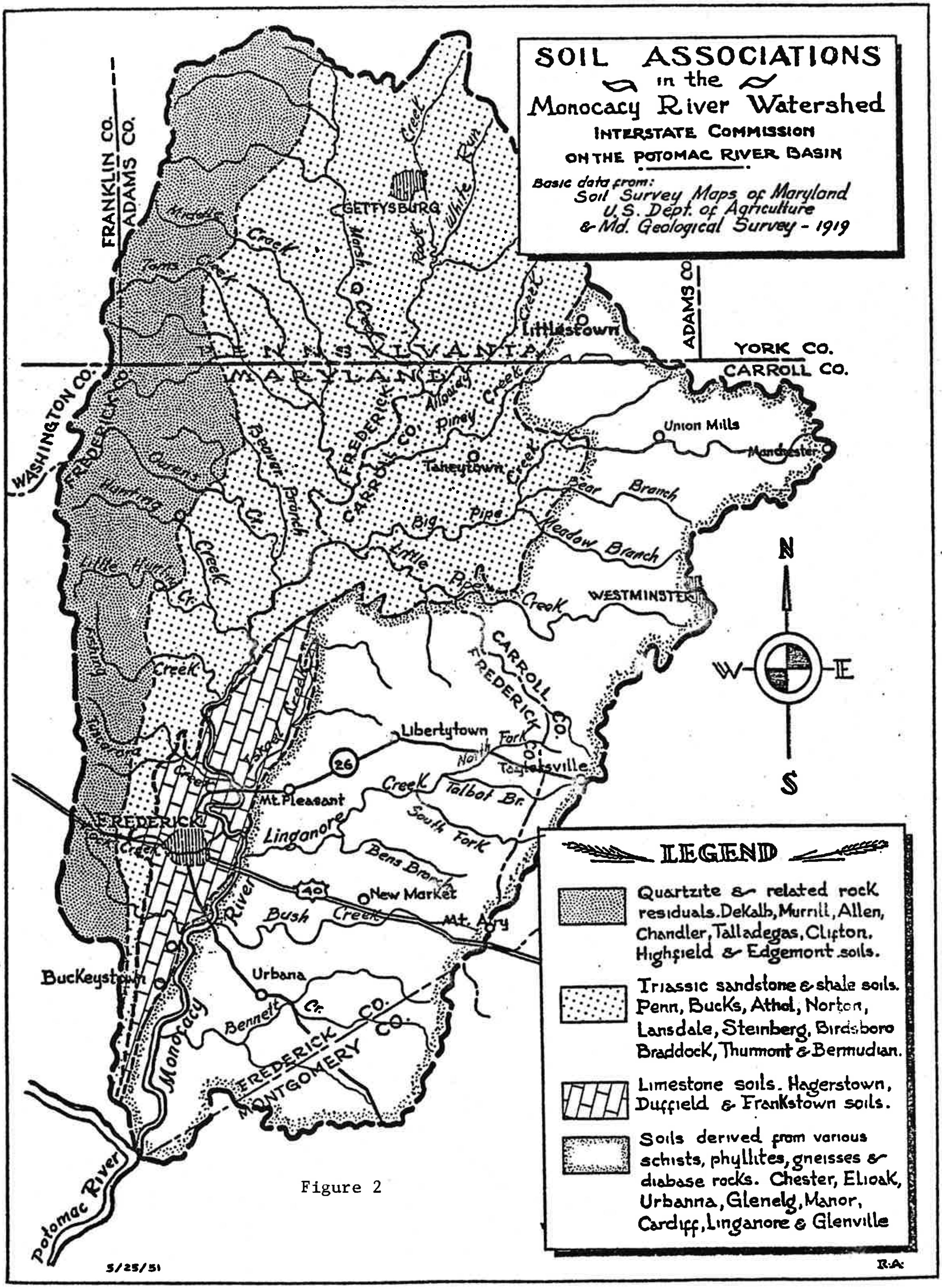
Meinhold, R.J. and N.D. Singpurwalla. Understanding the Kalman Filter; Unpublished Manuscript, The George Washington University, School of Engineering and Applied Science, Institue for Reliability and Risk Analysis

Figure 1


MONOCACY RIVER WATERSHED
MARYLAND & PENNSYLVANIA
FRANKLIN CO.
ADAMS CO.
GETTYSBURG
Littlestown
PENNSYLVANIA
MARYLAND
YORK CO.
CARROLL CO.
ADAMS CO.
WASHINGTON CO.
FREDERICK CO.
Emmitsburg
Taneytown
Union Mills
Manchester
Union Bridge
WESTMINSTER
CARROLL
FREDERICK
FREDERICK CO.
Libertytown
Unionville
Mt. Pleasant
FREDERICK
Rock Creek
New Market
CARROLL CO
HOWARD CO.
Urbana
Hyattstown
FREDERICK CO.
MONTGOMERY CO.
U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
RIDGE AND VALLEY
GREAT VALLEY
BLUE RIDGE
APPALACHIAN PLATEAU
PIEDMONT
COASTAL PLAIN
THE POTOMAC RIVER BASIN


SOIL ASSOCIATIONS
in the
Monocacy River Watershed
INTERSTATE COMMISSION
ON THE POTOMAC RIVER BASIN
Basic data from:
Soil Survey Maps of Maryland
U.S. Dept. of Agriculture
& Md. Geological Survey - 1919
LEGEND
Quartzite & related rock residuals. DeKalb, Murrill, Allen, Chandler, Talladegas, Clifton, Highfield & Edgemont soils.
Triassic sandstone & shale soils. Penn, Bucks, Athol, Norton, Lansdale, Steinberg, Birdsboro Braddock, Thurmont & Bermudian.
Limestone soils. Hagerstown, Duffield & Frankstown soils.
Soils derived from various schists, phyllites, gneisses & diabase rocks. Chester, Elioak, Urbanna, Glenelg, Manor, Cardiff, Linganore & Glenville
FRANKLIN CO.
ADAMS CO.
GETTYSBURG
Littlestown
ADAMS CO.
YORK CO.
CARROLL CO.
PENNSYLVANIA
MARYLAND
WASHINGTON CO.
FREDERICK CO.
CARROLL CO.
Union Mills
Manchester
Taneytown
WESTMINSTER
Libertytown
Mt. Pleasant
FREDERICK
New Market
Mt. Airy
Urbana
Buckeystown
Monocacy River
Potomac River
MONTGOMERY CO.
N
W
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S
26
40
5/25/51
R.A.

Figure 2

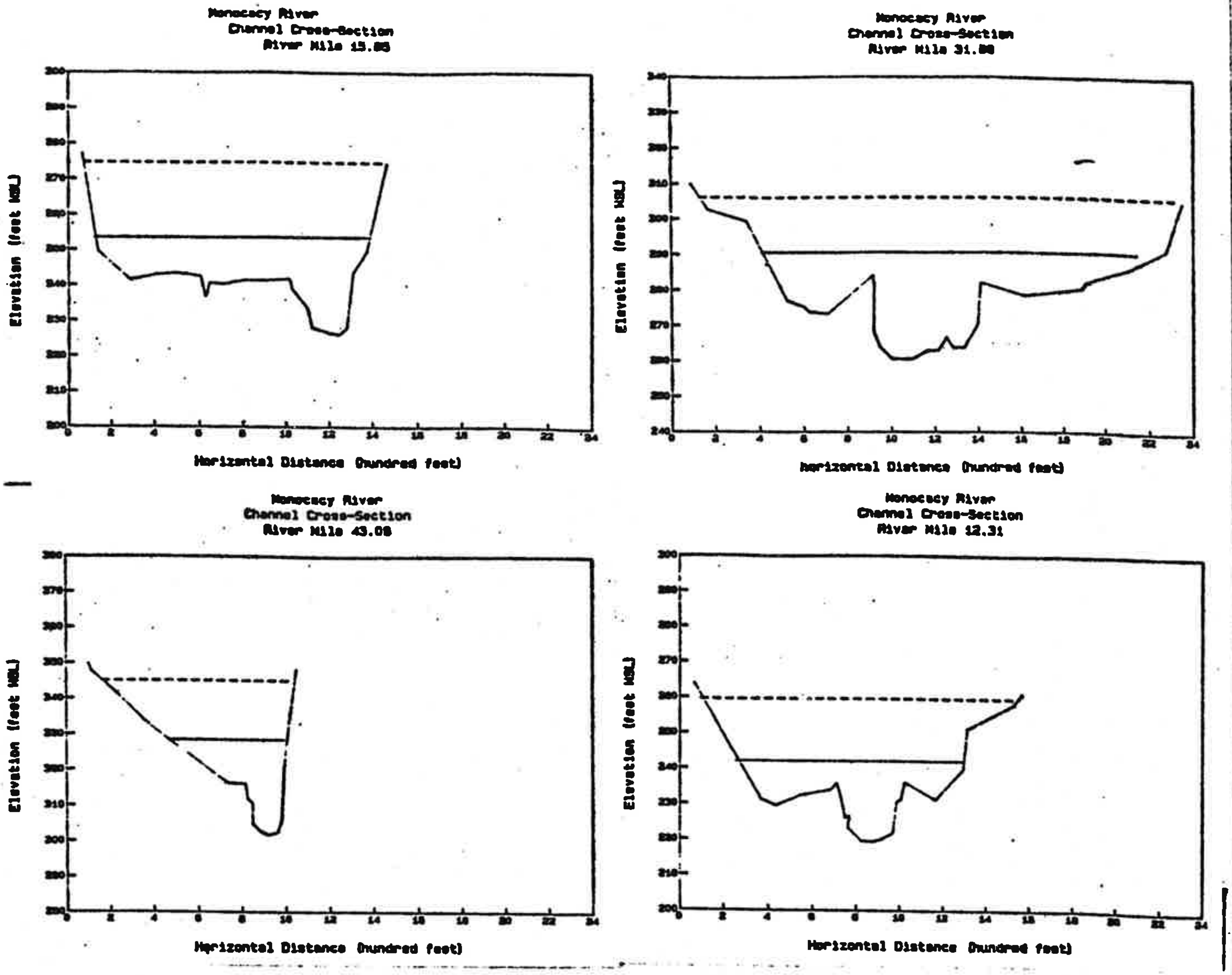
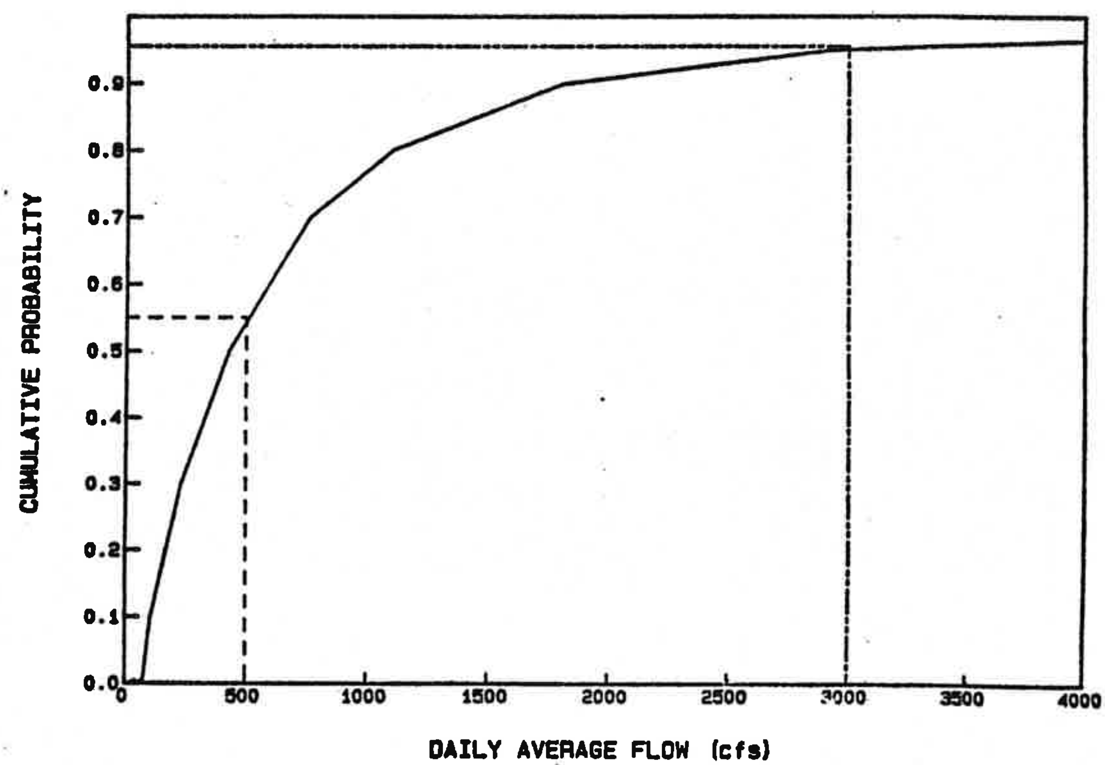


Figure 3 Digitized Cross sections of the main channel, Monocacy River

100 year flood ——————

500 year flood — — — — —

Figure 4



CUMULATIVE PROBABILITY
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
500
1000
1500
2000
2500
3000
3500
4000
DAILY AVERAGE FLOW (cfs)

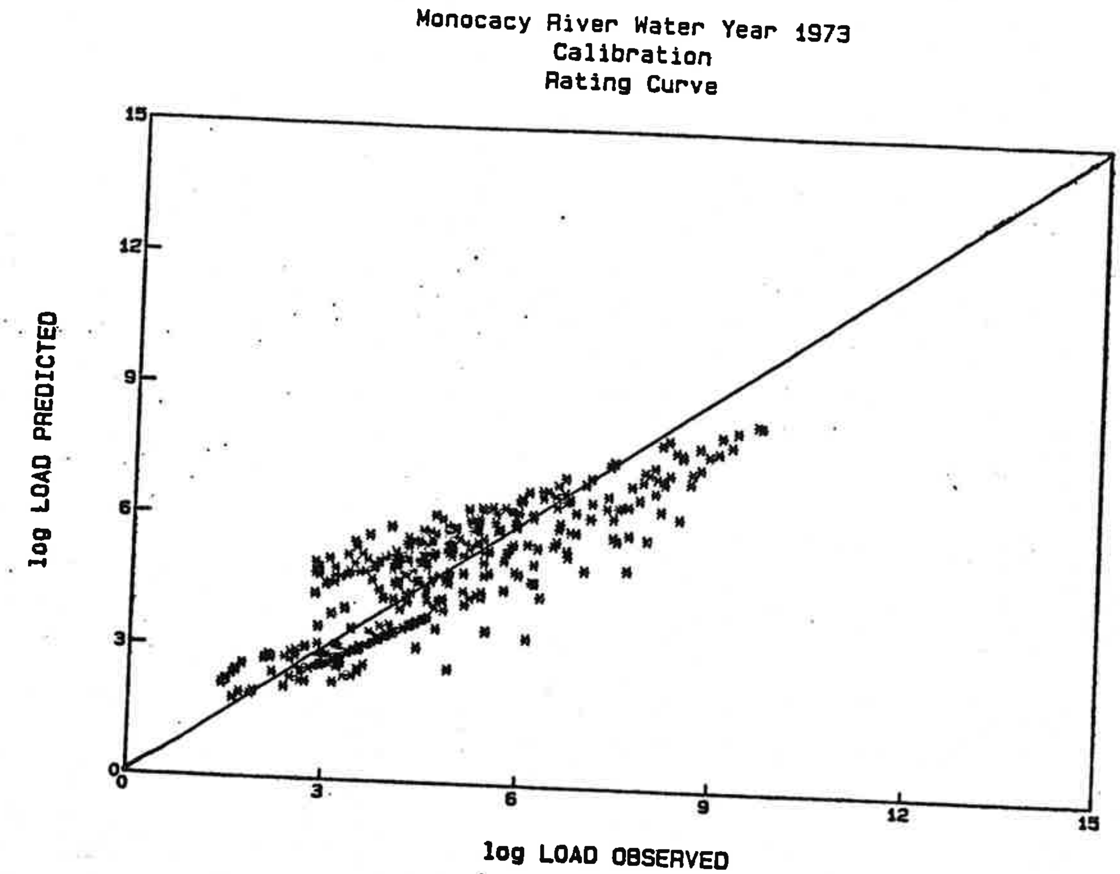

Monocacy River Water Year 1973
Calibration
Rating Curve
log LOAD PREDICTED
log LOAD OBSERVED
0
3
6
9
12
15
0
3
6
9
12
15

Figure 5

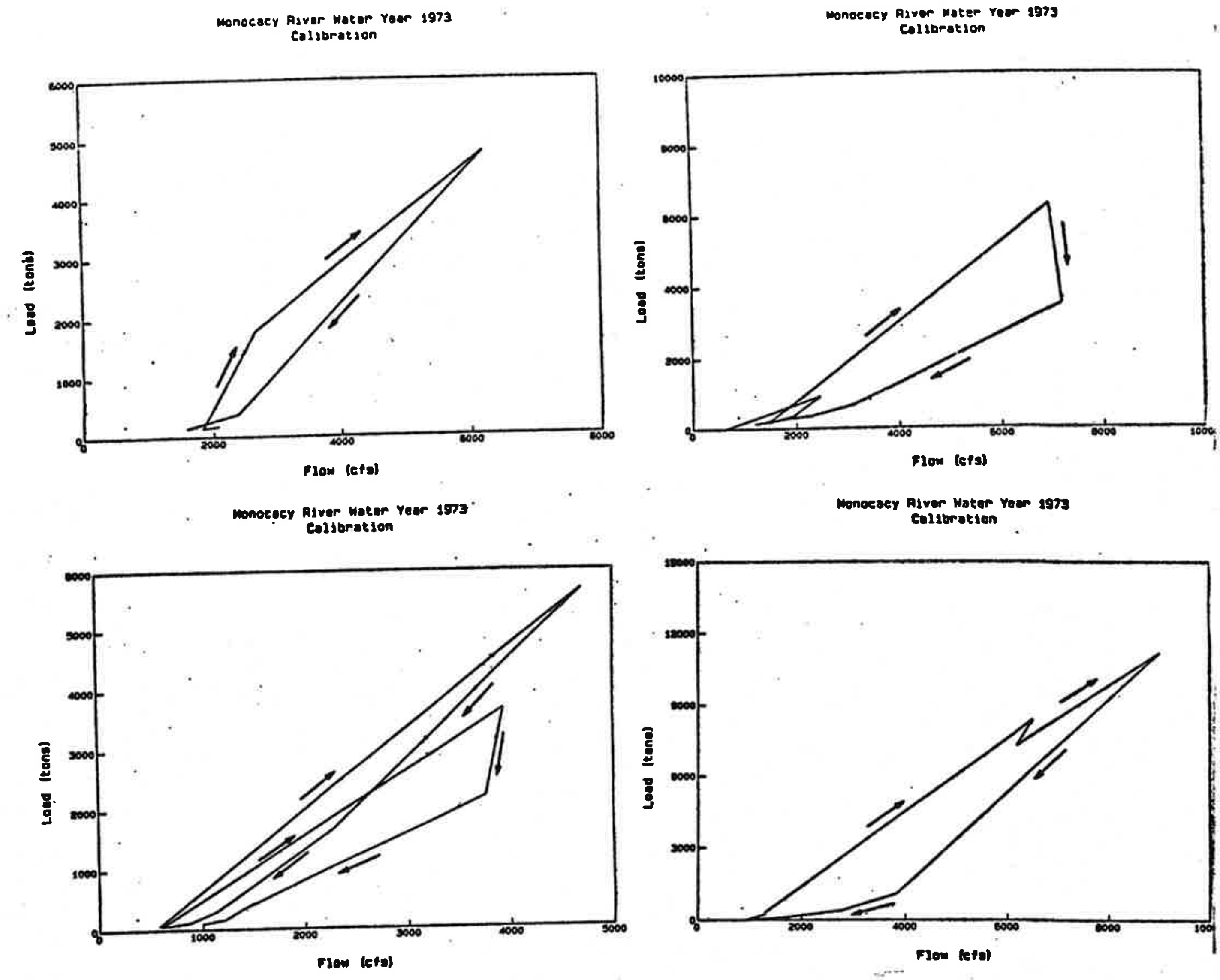
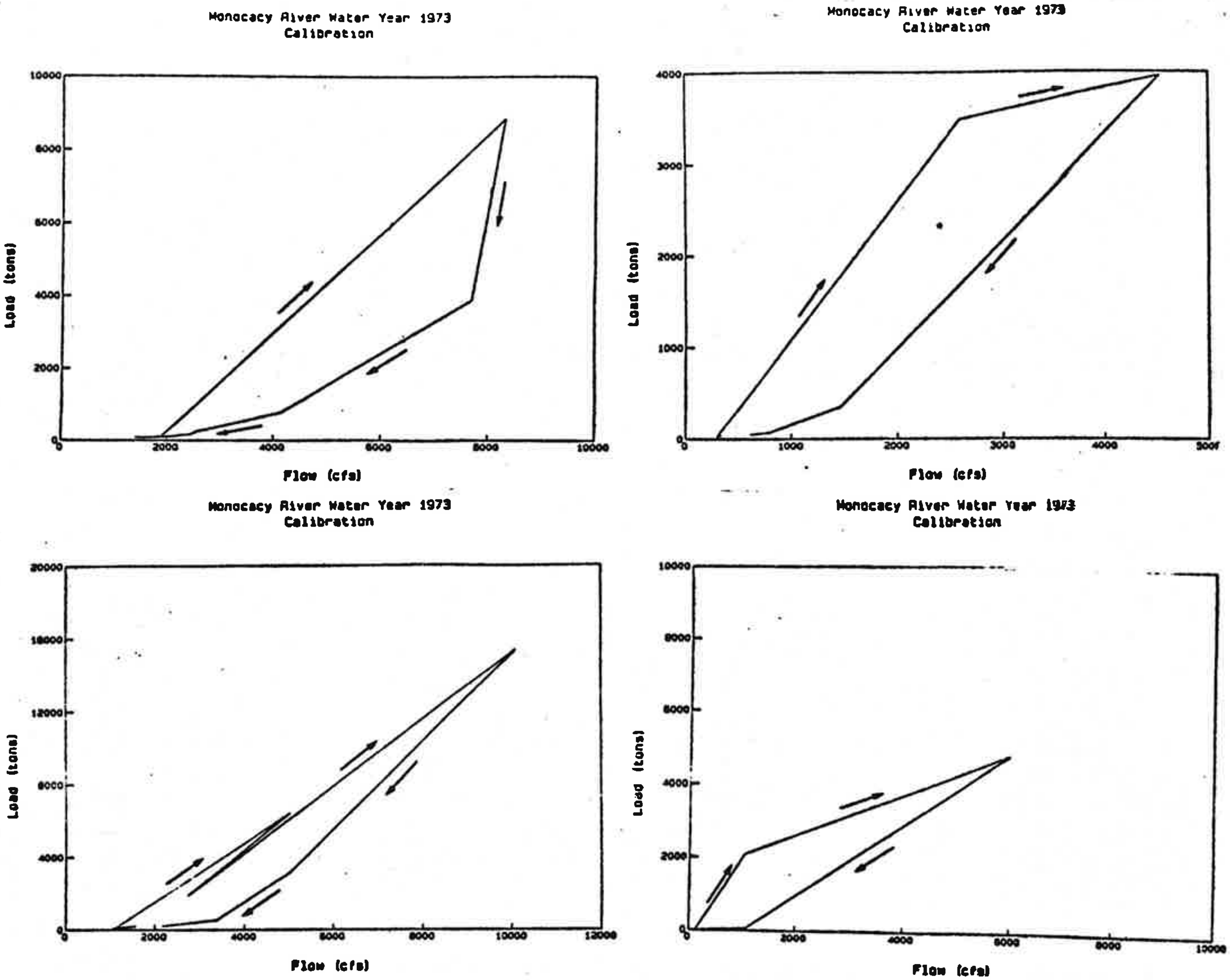

Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)
Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)
Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)
Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)

Figure 6 a-d



Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)
Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)
Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)
Monocacy River Water Year 1973
Calibration
Load (tons)
Flow (cfs)

Figure 6 e-h

Figure 7

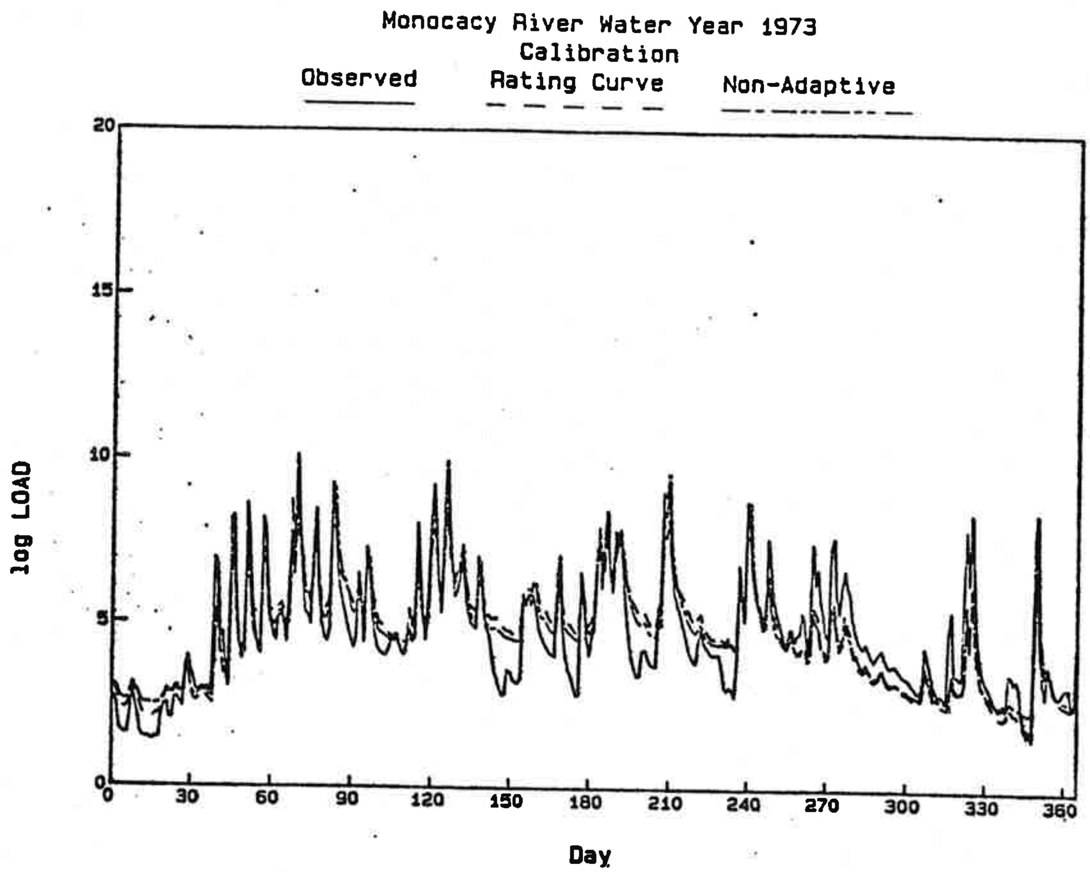


Figure 8

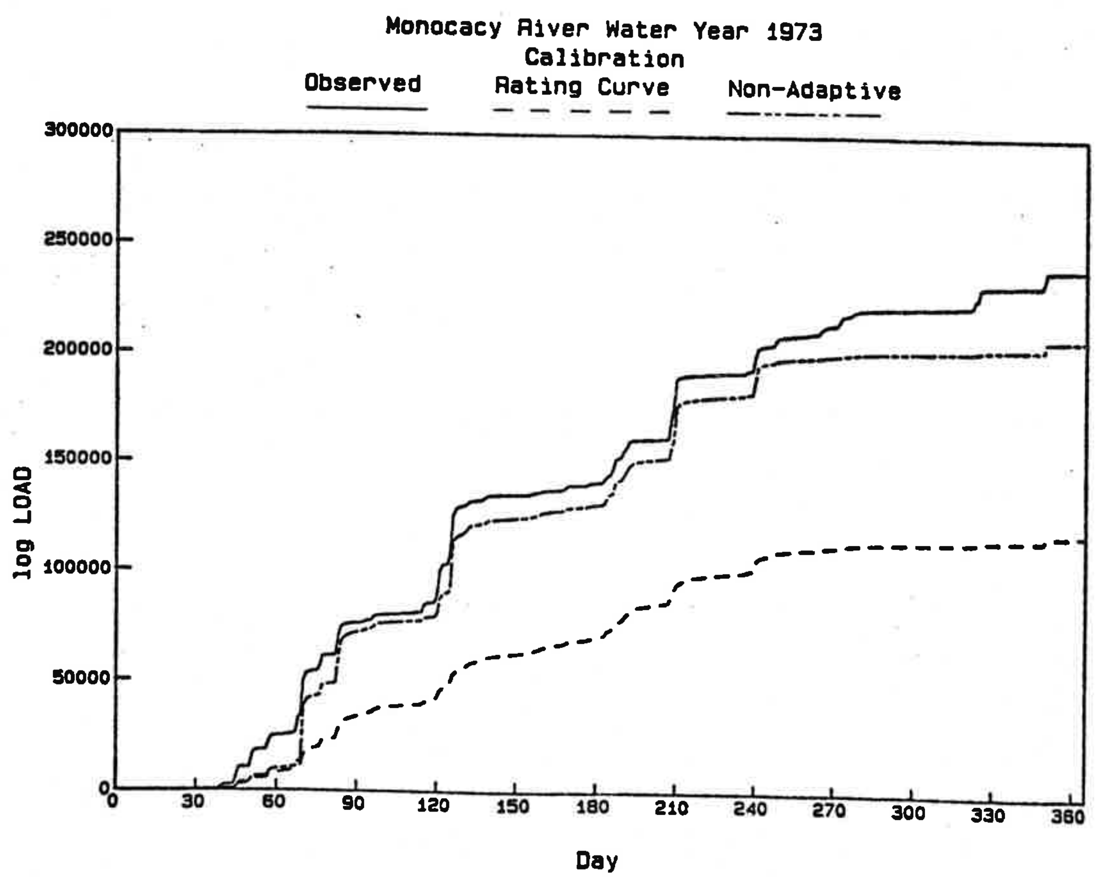


Figure 9a

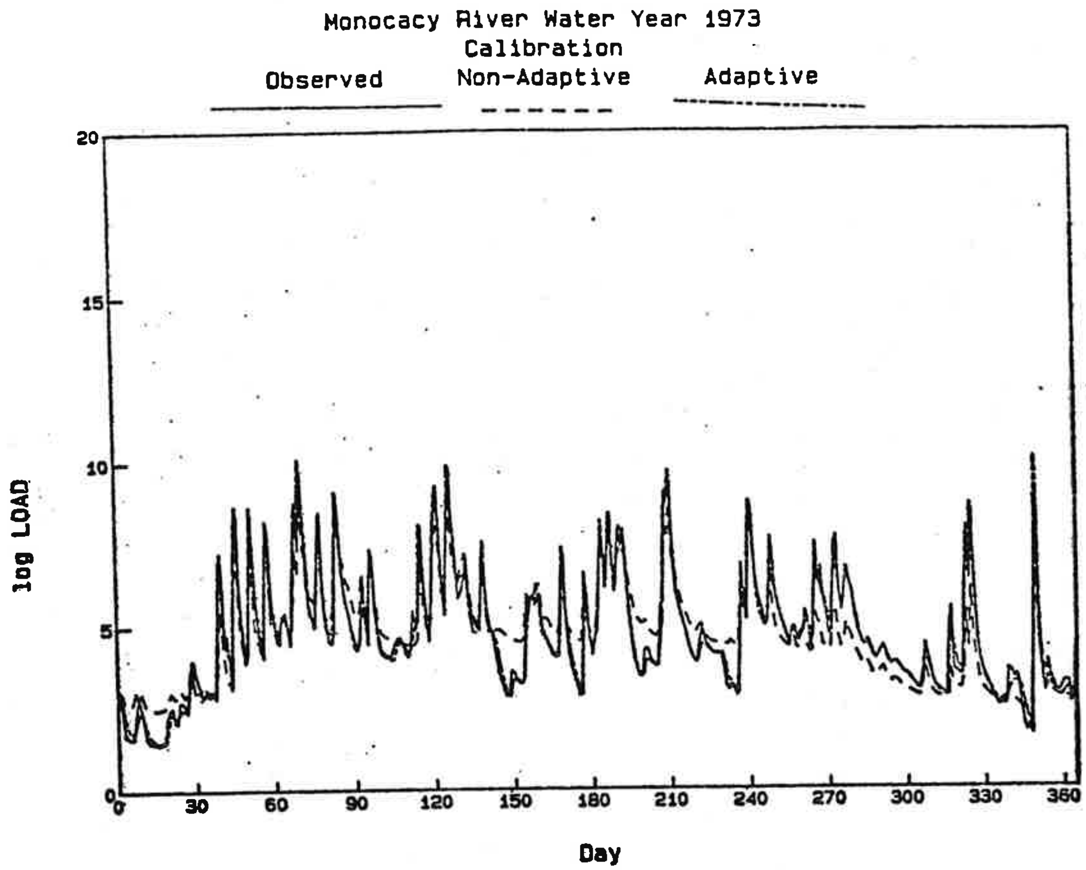


Figure 9b

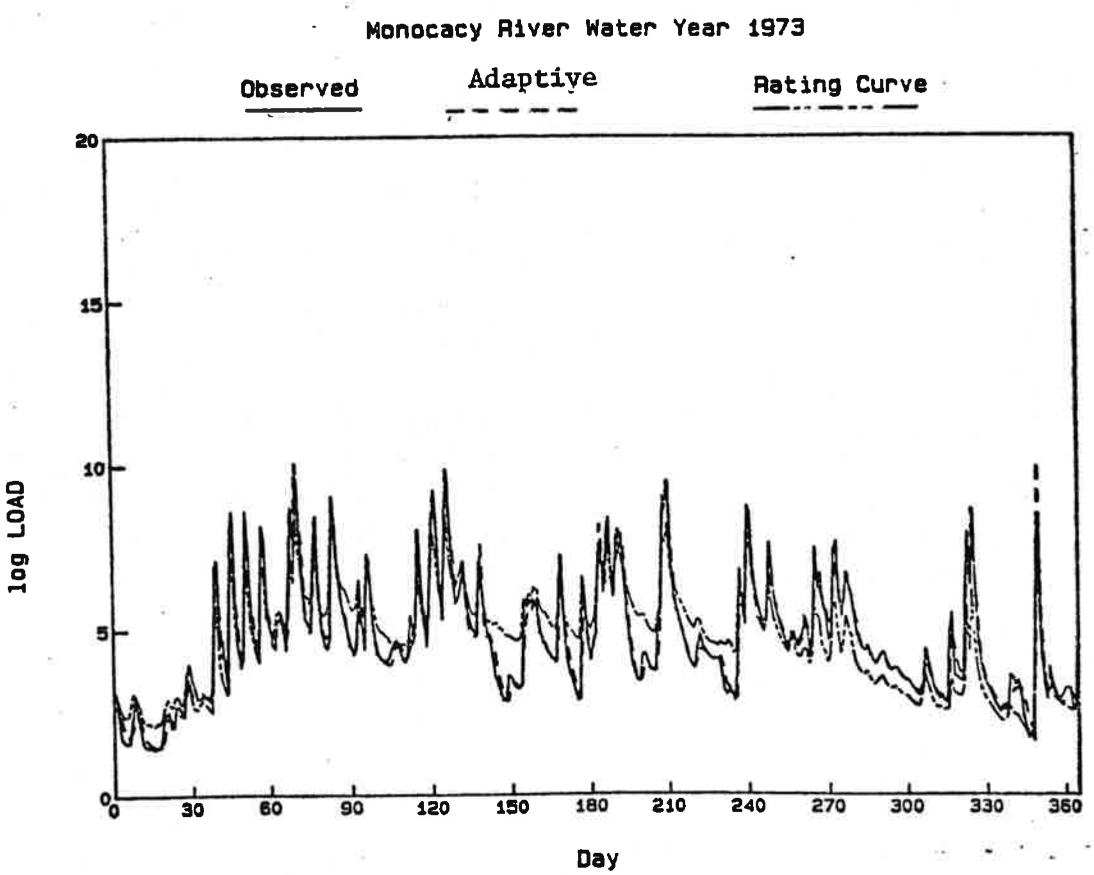


Figure 10a

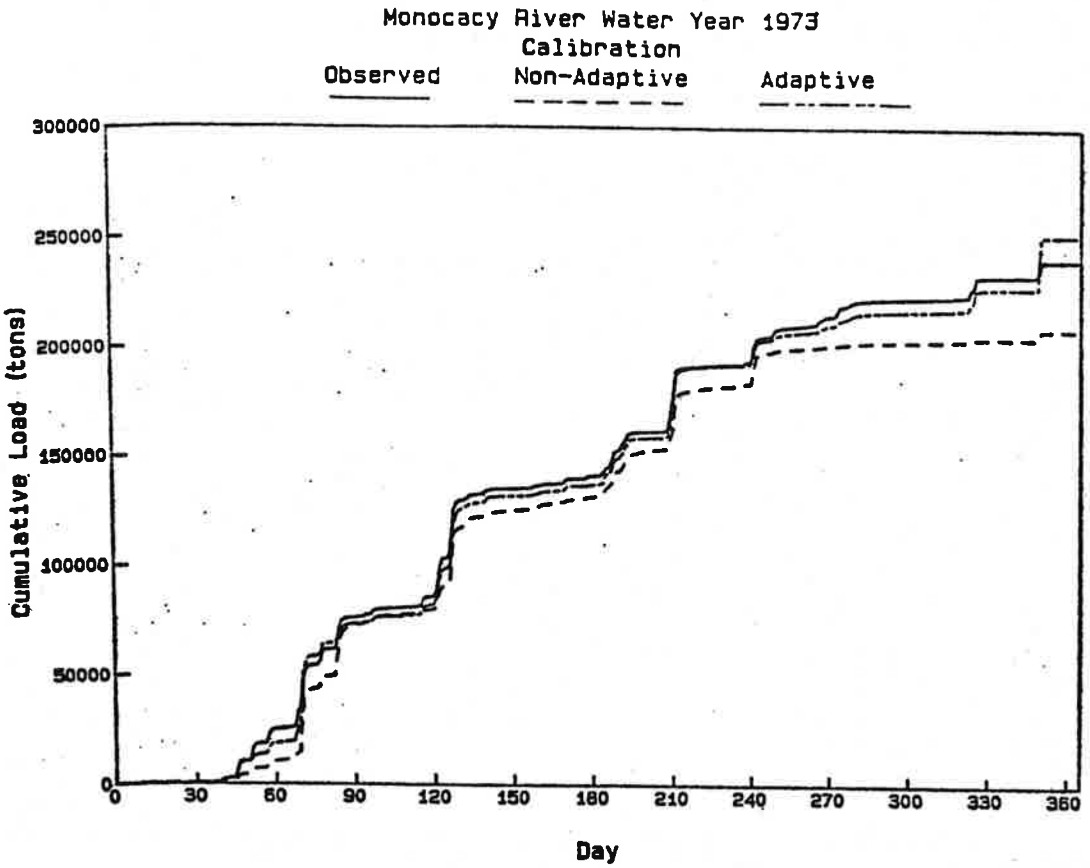


Figure 10b

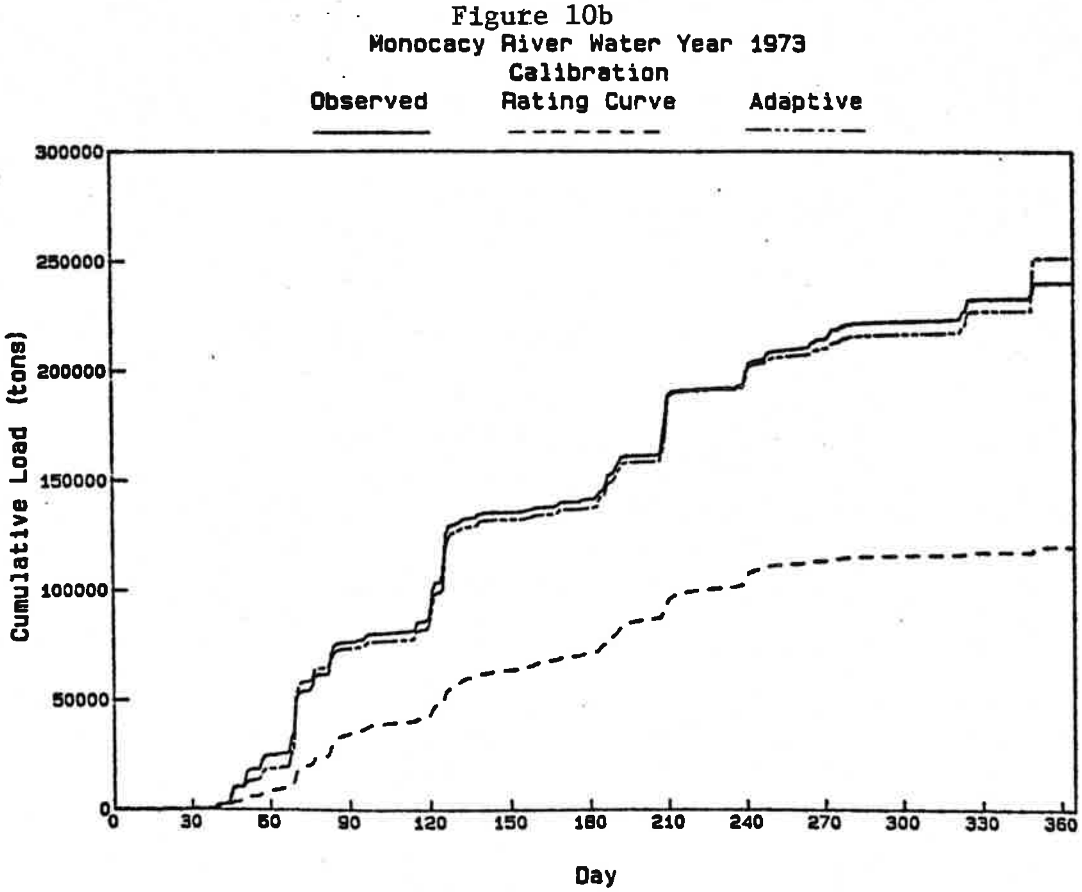


Figure 11a

Monocacy River Water Year 1974
Validation

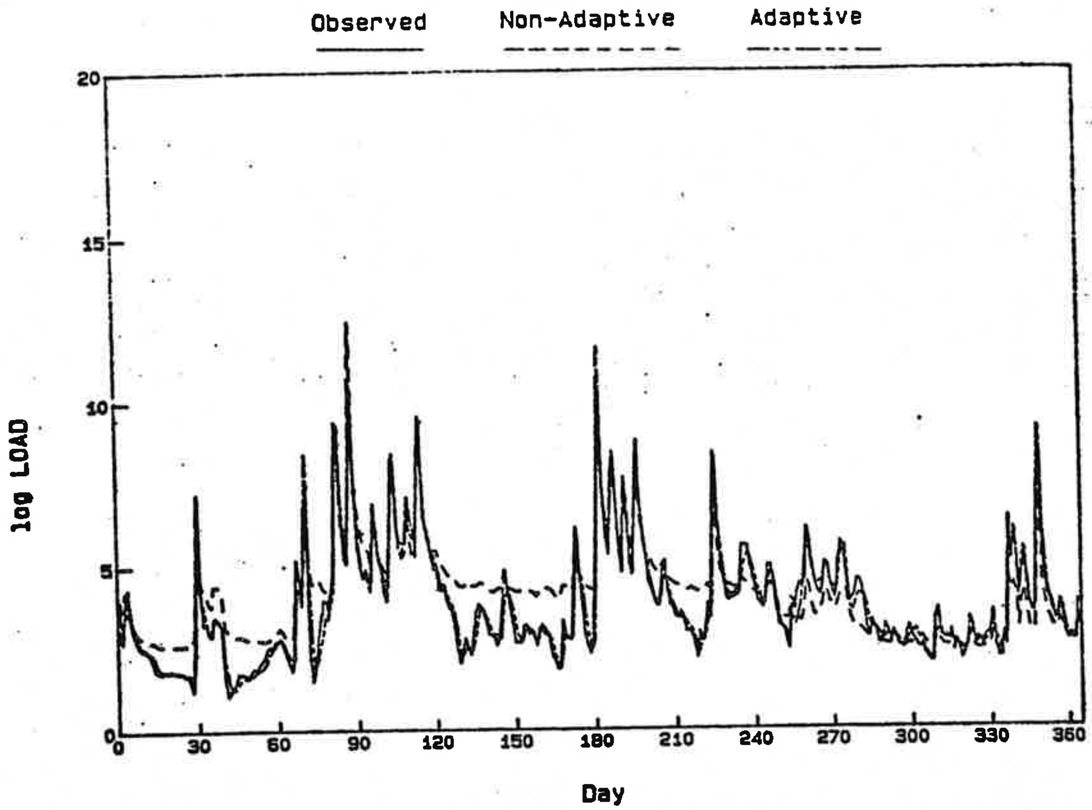


Figure 11b

Monocacy River Water Year 1974
Validation

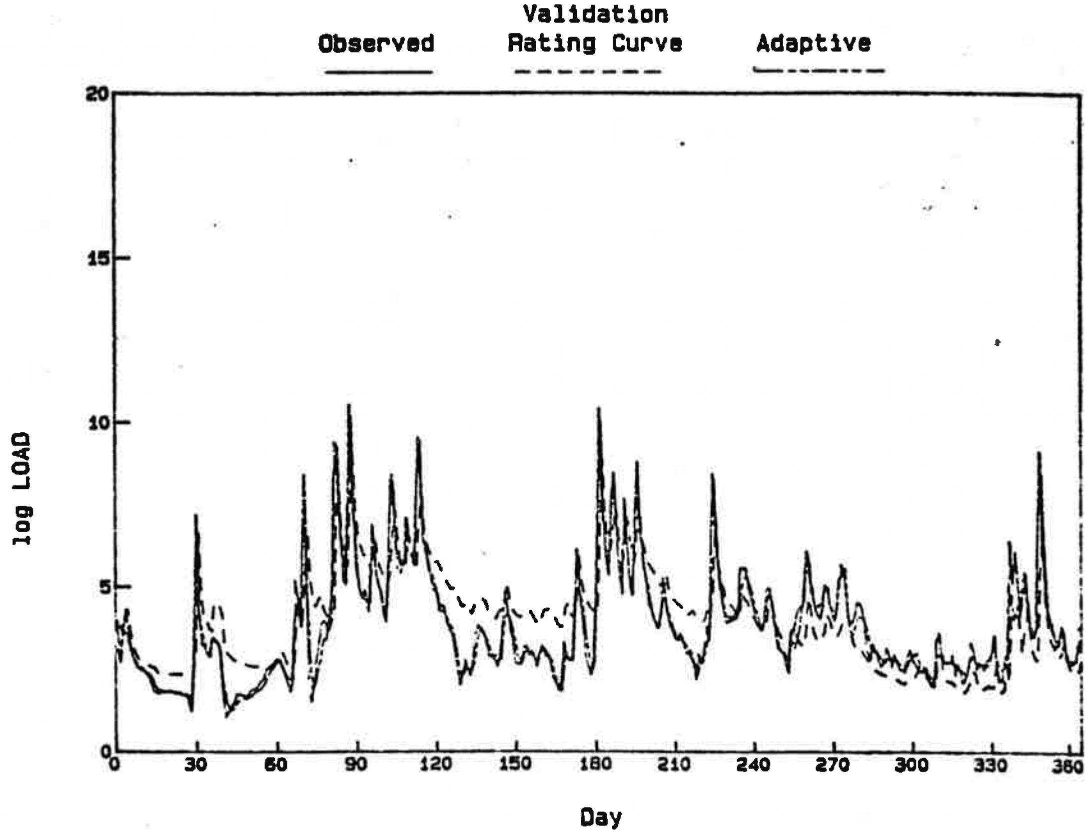


Figure 12a

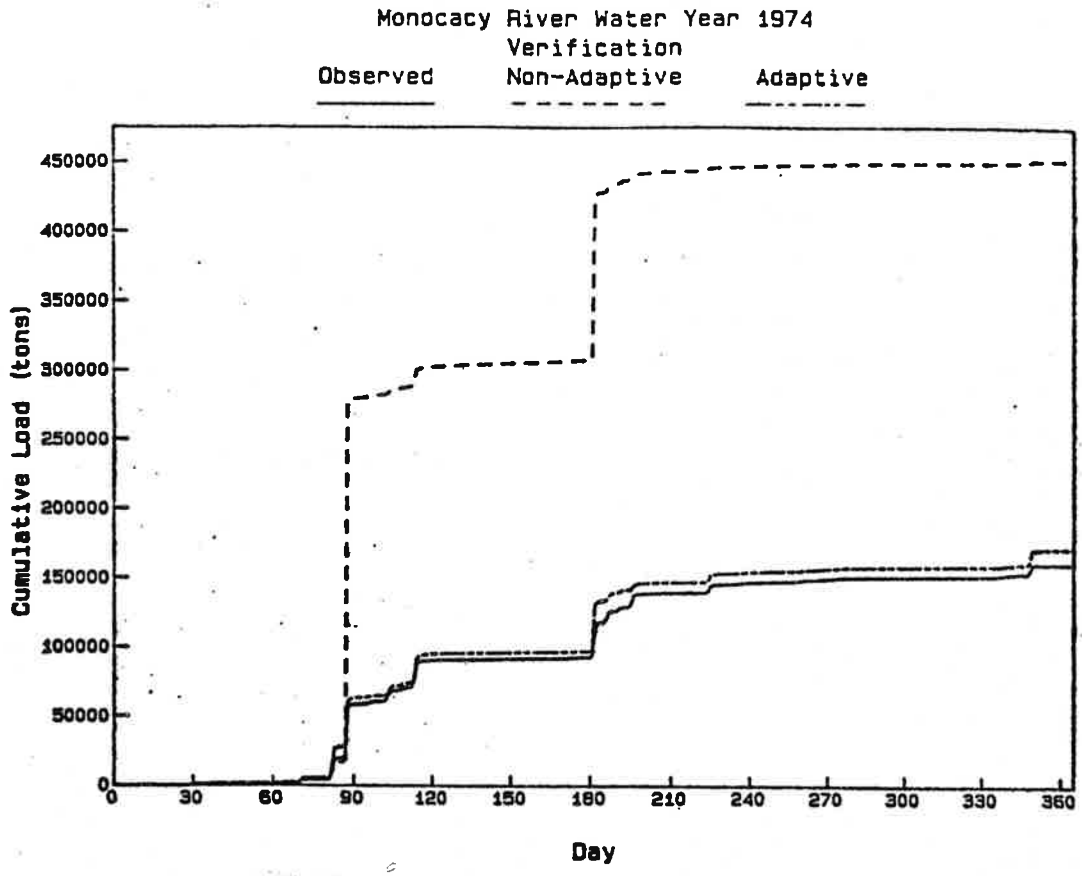


Figure 12b

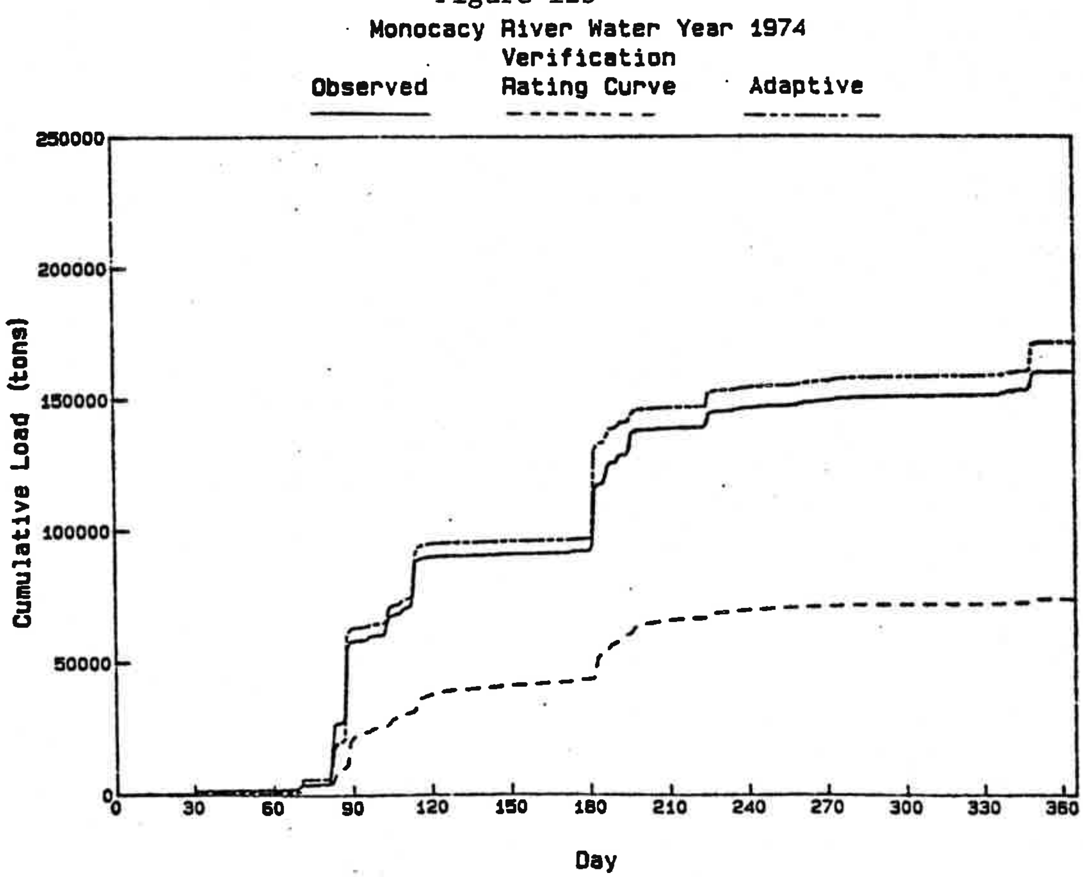


Figure 13a
Conceptual Storage Contributions
Stream Bed
Water Year 1974

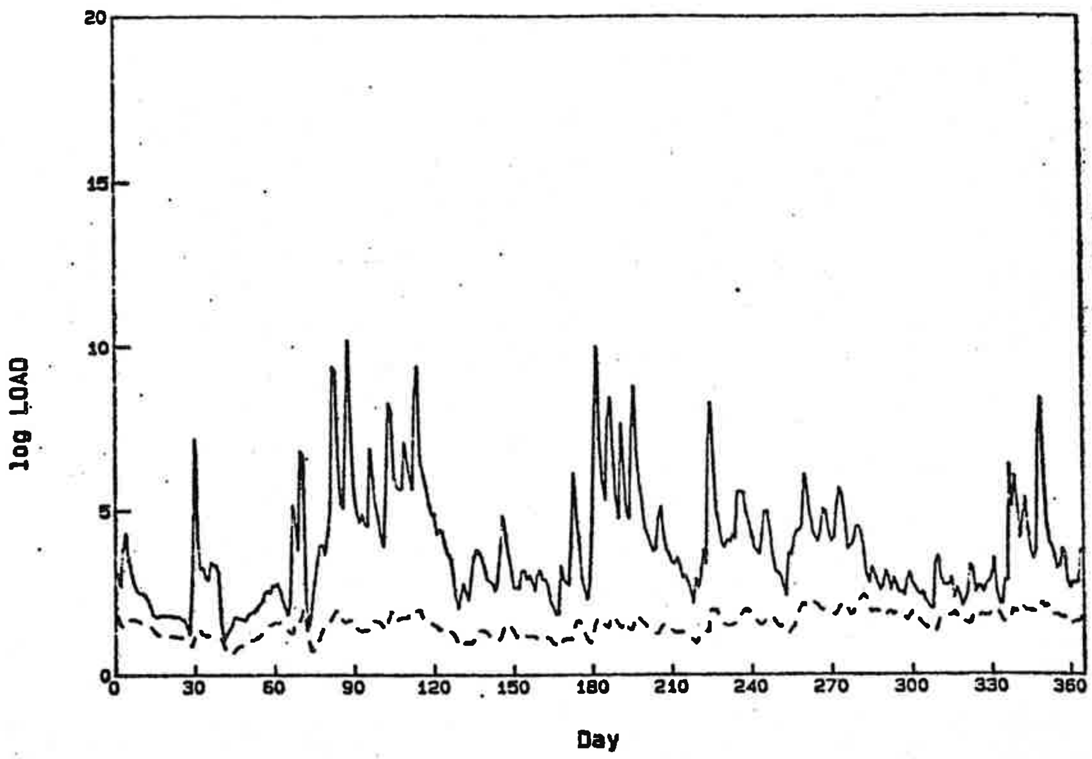


Figure 13b
Conceptual Storage Contributions
Bankfull Channel
Water Year 1974

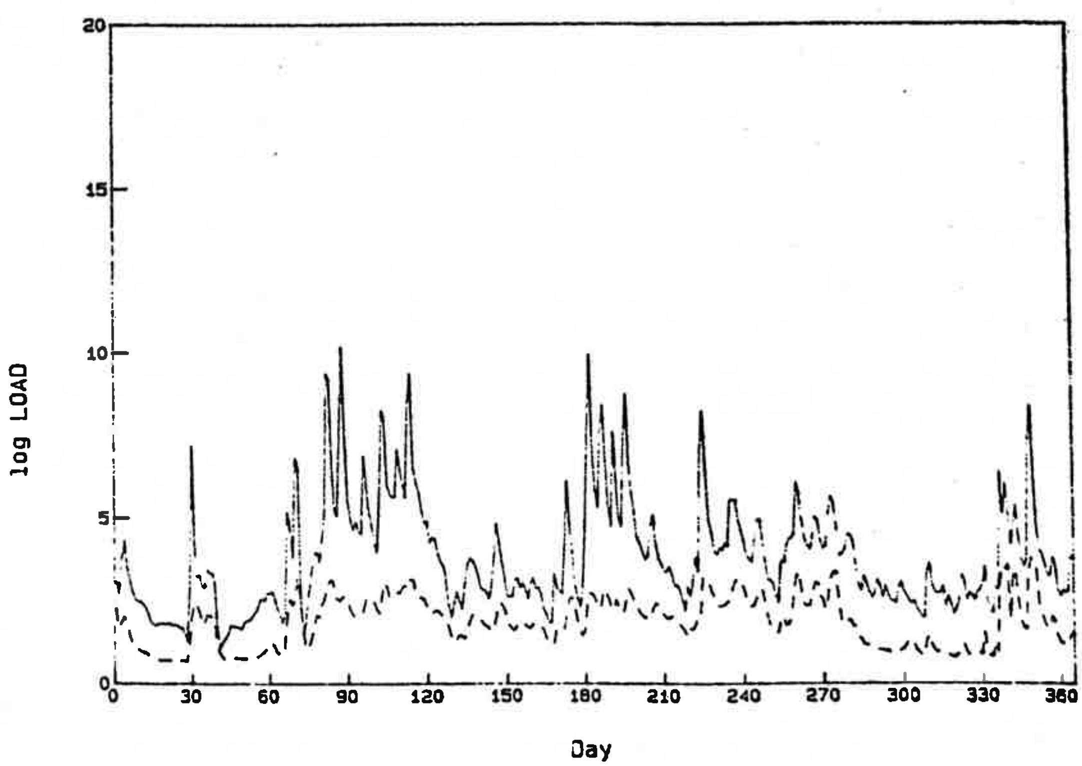
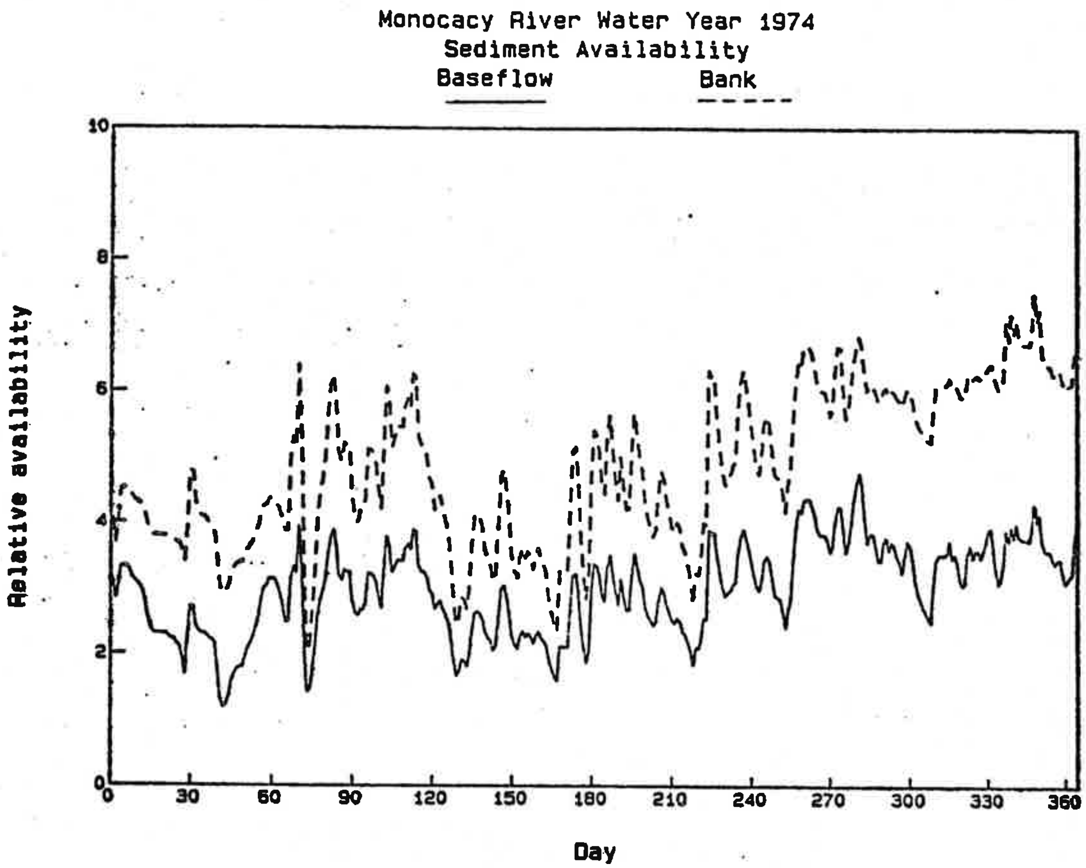


Figure 14



Monocacy River Water Year 1974
Sediment Availability
Baseflow
Bank
Relative availability
0
2
4
6
8
10
0
30
60
90
120
150
180
210
240
270
300
330
360
Day

Figure 15

Monocacy River
500 cfs Availability
Water Year 1974

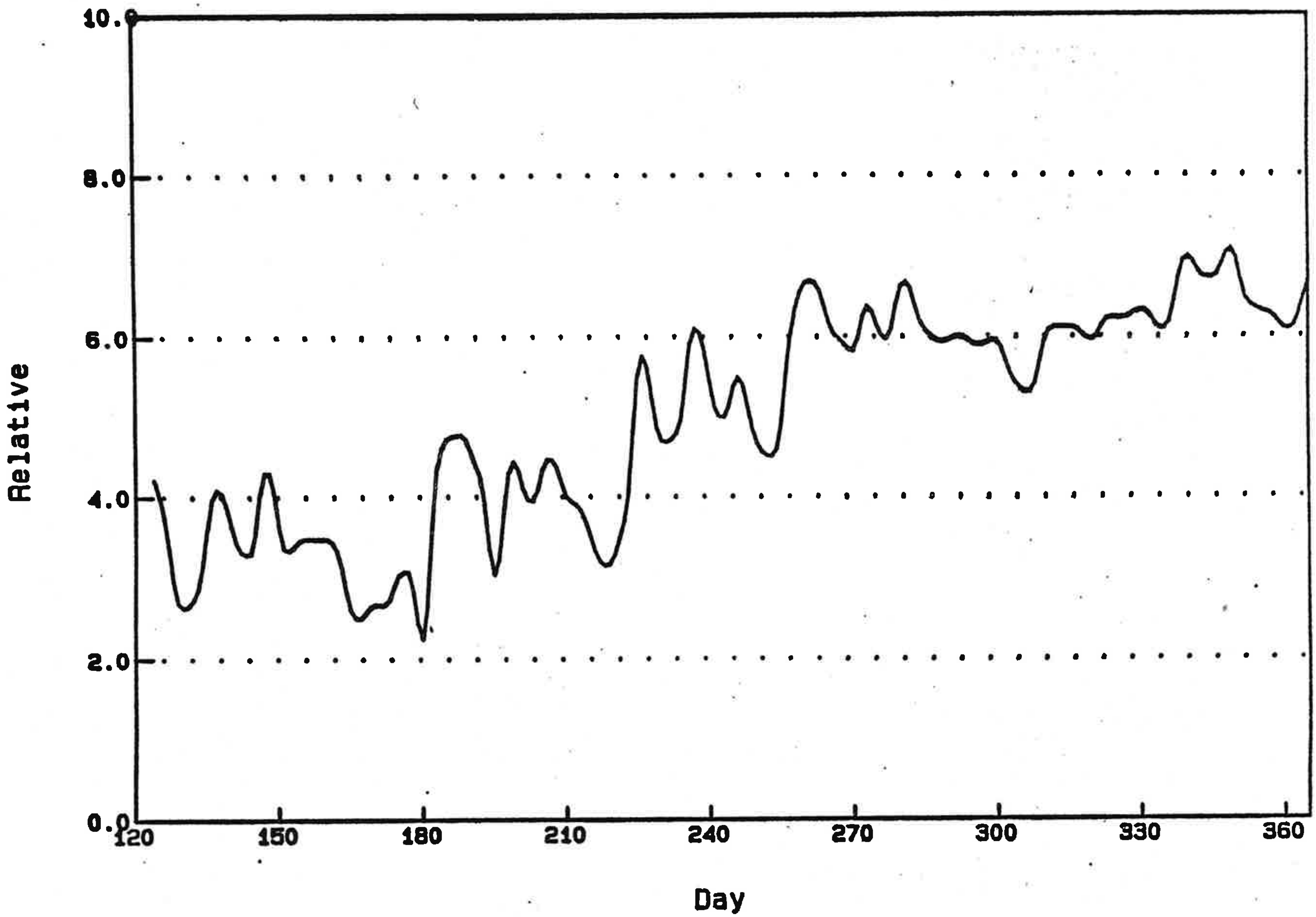


Table 1

	Monocacy Corridor (Kuss & Ballard 1982)	Monocacy Basin
Area	155 mi^2	817 mi^2
Yield	1,725 tons/mi^2	327 tons/mi^2
Annual	265,142 tons (production)	267,160 tons (load)

Table 2

	22 June 1972	21 July 1969
Flow	74,000 cfs	1,530 cfs
Load	134,000 tons(1)*	6,700 tons
Concentration	942 mg/l (14)*	1,510 mg/l (1)*

*( ) - rank of annual maximum

Table 3

RATING CURVE

$$\ln(\mathrm{LOAD}) = a_0 + a_1 \ln(Q)$$

FLOW SEGMENTED

$$\ln(\mathrm{LOAD}) = b_0 + b_1 q_1 + b_2 q_2 + b_3 q$$

ADAPTIVE

$$\ln(\mathrm{LOAD}) = c_0 + c_1(t) q_1 + c_2(t) q_2 + c_3(t) q_3$$

$$\sum_{i=1}^{3} q_i = Q$$

Table 4

SUSPENDED SEDIMENT CONCENRATION *

	mean	Q1	Q3
1973			
Jan.–April	20.5	12	35
May–August	61.5	43	70
1974			
Jan.–April	18.7	10	24
May–August	56	31	70

* mg/l